Mathematical Markup Language (MathML) Version 2.0

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Abstract

This specification defines the Mathematical Markup Language, or MathML. MathML is an XML application for describing mathematical notation and capturing both its structure and content. The goal of MathML is to enable mathematics to be served, received, and processed on the World Wide Web, just as HTML has enabled this functionality for text.

This specification of the markup language MathML is intended primarily for a readership consisting of those who will be developing or implementing renderers or editors using it, or software that will communicate using MathML as a protocol for input or output. It is not a User's Guide but rather a reference document.

This document begins with background information on mathematical notation, the problems it poses, and the philosophy underlying the solutions MathML proposes. MathML can be used to encode both mathematical notation and mathematical content. About thirty of the MathML tags describe abstract notational structures, while another one hundred provide a way of unambiguously specifying the intended meaning of an expression. Additional chapters discuss how the MathML content and presentation elements interact, and how MathML renderers might be implemented and should interact with browsers. Finally, this document addresses the issue of MathML entities (extended characters) and their relation to fonts.

While MathML is human-readable it is anticipated that, in all but the simplest cases, authors will use equation editors, conversion programs, and other specialized software tools to generate MathML. Several early versions of such MathML tools already exist, and a number of others, both freely available software and commercial products, are under development.

Status of this document

This is a W3C working draft for review by W3C members and other interested parties. It is a draft document and may be updated, replaced or obsoleted by other documents at any

time. It is inappropriate to use W3C working drafts as reference material or to cite them as other than ‘work in progress’. This is work in progress and does not imply endorsement by, or the consensus of, either W3C or members of the Math working group.

This document has been produced by the W3C Math Working Group.

A list of current W3C Technical Reports can be found at http://www.w3.org/TR.

It is expected that there will be at least two more Working Drafts, appearing at roughly one month intervals, before finalization of the Working Group’s proposed specification MathML 2.0.

The present draft is a revision of the earlier corrected W3C Recommendation MathML 1.01. It differs from it in that several chapters have been modified and one added. The introductory chapters 1 and Chapter 2 are almost unchanged in this draft. They remain essentially correct, but will later be revised to reflect the changes in the rest of the document when these have settled down.

Chapters 3 and 4 have been extended to describe new functionalities added, as well as smaller improvements of material already proposed. Chapter 5 has been newly written to reflect changes in the technology available. The major tables in chapter 6 are being regenerated to reflect an improved list of characters useful for mathematics. However, since the outcomes of several initiatives with respect to mathematics in Unicode are not yet clear, the main text of this chapter has not yet been revised, and the character tables are omitted. Chapter 7 has been completely revised. A new chapter 8 on the DOM for MathML has been added; the latter points to a new appendix E for a detailed listing.

The appendices have been reorganized into normative and non-normative groups. The former have draft updates. Appendices E and H are completely new.

Comments on this document should be sent to the public mailing list of the Math Working Group.

Contents

Chapter 1

Introduction

1.1 Mathematics and its Notation

A distinguishing feature of mathematics is the use of a complex and highly evolved system of two-dimensional symbolic notations. As J.R. Pierce has written in his book on communication theory, mathematics and its notations should not be viewed as one and the same thing [Pierce1961]. Mathematical ideas exist independently of the notations that represent them. However, the relation between meaning and notation is subtle, and part of the power of mathematics to describe and analyze derives from its ability to represent and manipulate ideas in symbolic form. The challenge in putting mathematics on the World Wide Web is to capture both notation and content (that is: meaning) in such a way that documents can utilize the highly-evolved notational forms of written and printed mathematics, and the potential for interconnectivity in electronic media.

Mathematical notations are constantly evolving as people continue to discover innovative ways of approaching and expressing ideas. Even the commonplace notations of arithmetic have gone through an amazing variety of styles, including many defunct ones advocated by leading mathematical figures of their day [Cajori1928]. Modern mathematical notation is the product of centuries of refinement, and the notational conventions for high-quality typesetting are quite complicated. For example, variables, or letters which stand for numbers, are usually typeset today in a special italic font subtly distinct from the usual text italic. Spacing around symbols for operations such as +, -, $\times$ and / is slightly different from that of text, to reflect conventions about operator precedence. Entire books have been devoted to the conventions of mathematical typesetting, from the alignment of superscripts and subscripts, to rules for choosing parenthesis sizes, to specialized notational practices for subfields of mathematics (for instance, [Chaundy1954], [Swanson1979], [Higham1993], or in the TEX literature [Knuth1986] and [Spivak1986]).

Notational conventions in mathematics, and printed text in general, guide the eye and make printed expressions much easier to read and understand. Though we usually take them for granted, we rely on hundreds of conventions such as paragraphs, capital letters, font families and cases, and even the device of decimal-like numbering of sections such as we are using in this document (an invention due to G. Peano, who is probably better known for his axioms for the natural numbers). Such notational conventions are even more important for electronic media, where one must contend with the difficulties of on-screen reading.

However, there is more to putting mathematics on the Web than merely finding ways of displaying traditional mathematical notation in a Web browser. The Web represents a fundamental change in the underlying metaphor for knowledge storage, a change in which

interconnectivity plays a central role. It is becoming increasingly important to find ways of communicating mathematics which facilitate automatic processing, searching and indexing, and reuse in other mathematical applications and contexts. With this advance in communication technology, there is an opportunity to expand our ability to represent, encode, and ultimately to communicate our mathematical insights and understanding with each other. We believe that MathML is an important step in developing mathematics on the Web.

1.2 Origins and Goals

1.2.1 The History of MathML

The problem of encoding mathematics for computer processing or electronic communication is much older than the Web. The common practice among scientists before the Web was to write papers in some encoded form based on the ASCII character set, and e-mail them to each other. Several markup methods for mathematics, in particular TEX [Knuth1986], were already in wide use in 1992, just before the Web rose to prominence, [Poppelier1992].

Since its inception, the Web has demonstrated itself to be a very effective method of making information available to widely separated groups of individuals. However, even though the World Wide Web was initially conceived and implemented by scientists for scientists, the capability to include mathematical expressions in HTML is very limited. At present, most mathematics on the Web consists of text with images (in GIF or JPEG format) of scientific notation, which are difficult to read and to author.

The World Wide Web Consortium (W3C) recognized that lack of support for scientific communication was a serious problem. Dave Raggett included a proposal for HTML Math in the HTML 3.0 working draft in 1994. A panel discussion on mathematical markup was held at the WWW Conference in Darmstadt in April 1995. In November 1995, representatives from Wolfram Research presented a proposal for doing math in HTML to the W3C team. In May 1996, the Digital Library Initiative meeting in Champaign-Urbana played an important role in bringing together many interested parties. Following the meeting, an HTML Math Editorial Review Board was formed. In the intervening years, this group has grown, and was formally reconstituted as the W3C Math working group in March 1997.

The MathML proposal reflects the interests and expertise of a very diverse group. Many contributions to the development of MathML deserve special mention, some of which we touch on here. One such contribution concerns the question of accessibility, especially for the visually handicapped. T.V. Raman is particularly notable in this regard. Neil Soiffer and Bruce Smith from Wolfram Research shared their experience with the problems of representing mathematics in connection with the design of Mathematica 3.0, which was an important influence in the design of the presentation elements. Paul Topping from Design Science also contributed his expertise in mathematical formatting and editing. MathML has benefited from the participation of a number of working group members involved in other mathematical encoding efforts in the SGML and computer-algebra communities, including Stephen Buswell from Stilo Technologies, Nico Poppelier from Elsevier Science, Stéphane Dalmas from INRIA, Sophia Antipolis, Stan Devitt from Waterloo Maple, Angel Diaz and Robert S. Sutor from IBM, and Stephen M. Watt from the University of Western Ontario. In particular, MathML has been influenced by the OpenMath project, the work of the ISO 12083 working group, and Stilo Technologies' work on a 'semantic' mathematics DTD fragment. The American Mathematical Society has played a key role in the development of

MathML. Among other things, it has provided two working group chairs: Ron Whitney led the group from May 1996 to March 1997, and Patrick Ion, who has co-chaired the group with Robert Miner from The Geometry Center, from March 1997 to the present.

Issue (): Add a sentence mentioning Ken Whistler.

The working group has benefited from the help of many people. We would like to particularly name Barbara Beeton, Chris Hamlin, John Jenkins, Ira Polans, Arthur Smith, Robby Villegas and Joe Yurvati for help and information in assembling the character tables in chapter 6, as well as Peter Flynn, Russel S.S. O'Connor, Andreas Strotmann, and other contributors to the www-math mailing list for their careful proofreading and constructive criticisms.

1.2.2 Limitations of HTML

The demand for effective means of electronic scientific communication is high. Increasingly, researchers, scientists, engineers, educators, students and technicians find themselves working at dispersed locations and relying on electronic communication. At the same time, the image-based methods that are currently the predominant means of transmitting scientific notation over the Web are primitive and inadequate. Document quality is poor, authoring is difficult, and mathematical information contained in images is not available for searching, indexing, or reuse in other applications.

The most obvious problems with HTML for mathematical communication are of two types.

Display Problems. Consider the equation $2^{2^x} = 10$. This equation is sized to match the surrounding line in 14pt type on the system where it was authored. Of course, on other systems, or for other font sizes, the equation is too small or too large. A second point to observe is that the equation image was generated against a white background. Thus, if a reader or browser resets the page background to another color, the anti-aliasing in the image results in white 'halos'. Next, consider the equation $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ shown with the equation's horizontal alignment axis above the tops of the lower-case letters in surrounding text.

This equation has a descender which places the baseline for the equation at a point about a third of the way from the bottom of the image. One can pad the image like this: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, so that the centerline of the image and the baseline of the equation coincide, but this causes problems with the inter-line spacing, which also makes the equation difficult to read. Moreover, center alignment of images is handled in slightly different ways by different browsers, making it impossible to guarantee proper alignment for different clients.

Image-based equations are generally harder to see, read and comprehend than the surrounding text in the browser window. Moreover, these problems become worse when the document is printed. The resolution of the equations will be around 70 dots per inch, while the surrounding text will typically be 300 or more dots per inch. The disparity in quality is judged to be unacceptable by most people.

Encoding Problems. Consider trying to search this page for part of an equation, for example, the '=10' from the first equation above. In a similar vein, consider trying to cut and paste an equation into another application; even more demanding is to cut and paste a sub-expression. Using image-based methods, neither of these common needs can be adequately addressed. Although the use of the `alt` in the document source can help, it is clear that highly interactive Web documents must provide a more sophisticated interface

between browsers and mathematical notation. Another problem with encoding mathematics as images is that it requires more bandwidth. By using markup-based encoding, more of the rendering process is moved to the client machine. Markup describing an equation is typically smaller and more compressible than an image of the equation.

1.2.3 Requirements for Mathematics Markup

Some display problems associated with including mathematical notation in HTML documents as images could be addressed by improving browser image handling. However, even if image handling were improved, the problem of making the information contained in mathematical expressions available to other applications would remain. Therefore, in planning for the future, it is not sufficient to merely upgrade image-based methods. To fully integrate mathematical material into Web documents, a markup-based encoding of mathematical notation and content is required.

In designing any markup language, it is essential to carefully consider the needs of its potential users. In the case of MathML, the needs of potential users cover a broad spectrum, from education to research, and on to commerce:

The education community is a large and important group that must be able to put scientific curriculum materials on the Web. At the same time, educators often have limited resources of time and equipment, and are severely hampered by the difficulty of authoring technical Web documents. Students and teachers need to be able to create mathematical content quickly and easily, using intuitive, easy-to-learn, low-cost tools.

Electronic textbooks are another way of using the Web which will potentially be very important in education. Management consultant Peter Drucker has recently been prophesying the end of big-campus residential higher education and its distribution over the Web [Drucker1997]. Electronic textbooks will need to be active, allowing intercommunication between the text and scientific software and graphics.

The academic and commercial research communities generates large volumes of dense scientific material. Increasingly, research publications are being stored in databases, such as the highly successful physics preprint server at Los Alamos National Laboratory. This is especially true in some areas of physics and mathematics where academic journal prices have been increasing at an unsustainable rate. In mathematics there are large collections at Duke, MSRI and SISSA, and on the AMS e-MATH server. In addition, databases of information on mathematical research, such as Mathematical Reviews and Zentralblatt für Mathematik, offer on the Web millions of records containing mathematics.

To accommodate the research community, a design for mathematical markup must facilitate the maintenance and operation of large document collections, where automatic searching and indexing are important. Because of the large collection of legacy data, especially TEX documents, the ability to convert between existing formats and new formats is also very important to the research community. Finally, the ability to maintain information for archival purposes is vital to academic research.

Corporate and academic scientists and engineers also use technical documents in their work to collaborate, to record results of experiments and computer simulations, and to verify calculations. For such uses, mathematics on the Web must provide a standard way of sharing information that can be easily read, processed and generated using commonly available, easy to use tools.

Another design requirement is the ability to render mathematical material in other media such as speech or braille, which is extremely important for the visually impaired.

Commercial publishers are also involved with mathematics on the Web at all levels from electronic versions of print books to interactive textbooks to academic journals. Publishers require a method of putting mathematics on the Web that is capable of high-quality output, robust enough for large-scale commercial use, and preferably compatible with their current, usually SGML-based, production systems.

1.2.4 Design Goals of MathML

In order to meet the diverse needs of the scientific community, MathML has been designed with the following ultimate goals in mind.

MathML should:

- Encode mathematical material suitable for teaching and scientific communication at all levels.
- Encode both mathematical notation and mathematical meaning.
- Facilitate conversion to and from other mathematical formats, both presentational and semantic. Output formats should include:
  - graphical displays
  - speech synthesizers
  - computer algebra systems' input
  - other mathematics typesetting languages, such as TEX
  - plain text displays, e.g. VT100 emulators
  - print media, including braille

  It is recognized that conversion to and from other notational systems or media may entail loss of information in the process.
- Allow the passing of information intended for specific renderers and applications.
- Support efficient browsing for lengthy expressions.
- Provide for extensibility.
- Be well suited to template and other mathematics editing techniques.
- Be human legible, and simple for software to generate and process.

No matter how successfully MathML might achieve its goals as a markup language, it is clear that MathML will only be useful if it is implemented well. To this end, the W3C Math working group has identified a short list of additional implementation goals. These goals attempt to describe concisely the minimal functionality MathML rendering and processing software should try to provide.

- MathML equations in HTML pages should render properly in popular Web browsers, in accordance with reader and author viewing preferences, and at the highest quality possible given the capabilities of the platform.
- HTML documents containing MathML equations should print properly and at high-quality printer resolutions.
- MathML equations in Web pages should be able to react to mouse gestures, and coordinate communication with other applications through the browser.
- Equation editors and converters should be developed to facilitate the creation of Web pages containing MathML equations.

These goals can probably be adequately addressed in the near term by using embedded elements such as Java applets, plug-ins and ActiveX controls to render MathML. However, the extent to which these goals are ultimately met depends on the cooperation and support

of browser vendors, and other software developers. The W3C Math working group will continue to work with the working groups for the Document Object Model (DOM) and the Extensible Style Language (XSL) to ensure that the needs of the scientific community will be met in the future.

1.3 The Role of MathML on the Web

1.3.1 Layered Design of Mathematical Web Services

The design goals of MathML require a system for encoding mathematical material for the Web which is flexible and extensible, suitable for interaction with external software, and capable of producing high-quality rendering in several media. Any markup language that encodes enough information to do all these tasks well will of necessity involve some complexity.

At the same time, it is important for many groups, such as students, to have simple ways to include mathematics in Web pages by hand. Similarly, other groups, such as the TEX community, would be best served by a system which allowed the direct entry of markup languages like TEX in Web pages. In general, specific user groups are better served by more specialized kinds of input and output tailored to their needs. Therefore, the ideal system for communicating mathematics on the Web should provide both specialized services for input and output, and general services for interchange of information and rendering to multiple media.

In practical terms, the observation that mathematics on the Web should provide for both specialized and general need naturally leads to the idea of a layered architecture. One layer consists of powerful, general software tools exchanging, processing and rendering suitably encoded mathematical data. A second layer consists of specialized software tools aimed at specific user groups, and which are capable of easily generating encoded mathematical data which can then be shared with a general audience.

MathML is designed to provide the encoding of mathematical data for the bottom, more general layer in a two-layer architecture. It is intended to encode complex notational and semantic structure in an explicit, regular, and easy to process way for renderers, searching and indexing software, and other mathematical applications.

As a consequence, MathML is not primarily intended for direct use by authors. While MathML is human-readable, in all but the simplest cases it is too verbose and error-prone for hand generation. Instead, it is anticipated that authors will use equation editors, conversion programs, and other specialized software tools to generate MathML. Alternatively, some renderers may convert other kinds of input directly included in Web pages into MathML on the fly, in response to a cut-and-paste operation, for example.

In some ways, MathML is analogous to other low-level, communication formats such as Adobe's PostScript language. You can create a PostScript file in a variety of ways, depending on your needs; experts write and modify them by hand, authors create them with word processors, graphic artists with illustration programs, and so on. Once you have a PostScript file, however, you can share it with a very large audience, since devices which render PostScript, such as printers and screen previewers, are widely available.

Part of the reason for designing MathML as a markup language for a low-level, general, communication layer is to stimulate mathematical Web software development in the layers

above. MathML provides a way of coordinating the development of modular authoring tools and rendering software. By making it easier to develop a functional piece of a larger system, MathML can stimulate a 'critical mass' of software development, greatly to the benefit of potential users of mathematics on the Web.

One can envision a similar situation for mathematical data. Authors are free to create MathML documents using the tools best suited to their needs. For example, a student might prefer to use a menu-driven equation editor that can write out MathML to an HTML file. A researcher might use a computer algebra package that automatically encodes the mathematical content of an expression, so that it can be cut from a Web page and evaluated by a colleague. An academic journal publisher might use a program that converts TEX markup to HTML and MathML. Regardless of the method used to create a Web page containing MathML, once it exists, all the advantages of a powerful and general communication layer become available. A variety of MathML software could all be used with the same document to render it in speech or print, to send it to a computer algebra system, or to manage it as part of a large Web document collection. One may expect that eventually MathML can be integrated into other arenas where mathematical formulas occur, such as spreadsheets, statistical packages and engineering tools.

The W3C Math working group is working with vendors to ensure that a wide variety of MathML software will soon be available, including both rendering and authoring tools. A current list of MathML software is maintained at the World Wide Web Consortium.

1.3.2 Relation to Other Web Technology

The original conception of HTML Math was a simple, straightforward extension to HTML that would be natively implemented in browsers. However, very early on, the explosive growth of the Web made it clear that a general extension mechanism was required, and that mathematics was only one of many kinds of structured data which would have to be integrated into the Web using such a mechanism.

Given that MathML must integrate into the Web as an extension, it is extremely important that MathML and MathML software can interact well with the existing Web environment. In particular, MathML has been designed with three kinds of interaction in mind. First, in order to create mathematical Web content, it is important that existing mathematical markup languages can be converted to MathML, and that existing authoring tools can be modified to generate MathML. Second, it must be possible to embed MathML markup seamlessly in HTML markup in such a way that it will be accessible to future browsers, search engines, and all kinds of Web applications which now manipulate HTML. Finally, it must be possible to render MathML embedded in HTML in today's Web browsers in some fashion, even if it is less than ideal.

1.3.2.1 Existing Mathematical Markup Languages

Perhaps the most important influence on mathematical markup languages of the last two decades is the TEX typesetting system developed by Donald Knuth [Knuth1986]. TEX is a de facto standard in the mathematical research community, and it is pervasive in the scientific community at large. TEX sets a standard for quality of visual rendering, and a great deal of effort has gone into ensuring MathML can provide the same visual rendering quality. Moreover, because of the many legacy documents in TEX, and because of the large authoring community versed in TEX, a priority in the design of MathML was the ability

to convert TEX mathematics input into MathML format. The feasibility of such conversion has been demonstrated by prototype software.

Extensive work on encoding mathematics has also been done in the SGML community, and SGML-based encoding schemes are widely used by commercial publishers. ISO 12083 is an important markup language which contains a DTD fragment primarily intended for describing the visual presentation of mathematical notation. Because ISO 12083 mathematical notation and its derivatives share many presentational aspects with TEX, and because SGML enforces structure and regularity more than TEX, much of the work in ensuring MathML is compatible with TEX also applies well to ISO 12083.

MathML also pays particular attention to compatibility with other mathematical software, and in particular, with computer algebra systems. Many of the presentation elements of MathML are derived in part from the mechanism of typesetting boxes. The MathML content elements are heavily indebted to the OpenMath project and the work by Stilo Technologies on a mathematical DTD fragment. The OpenMath project has close ties to both the SGML and computer algebra communities, and has laid a foundation for an SGML-based means of communication between mathematical software packages, among other things. The feasibility of both generating and interpreting MathML in computer algebra systems has been demonstrated by prototype software.

1.3.2.2 HTML Extension Mechanisms

As noted above, the success of HTML has led to enormous pressure to incorporate a wide variety of data types and software applications into the Web. Each new format or application potentially places new demands on HTML and on browser vendors. For some time, it has been clear that a general extension mechanism is necessary to accommodate new extensions to HTML. We began our work thinking of a plain extension to HTML in the spirit of the first mathematics support suggested for HTML 3.2. But for various reasons, once we got into the details this proved to be not so good an idea. Since work first began on MathML, XML has emerged as the leading candidate for such a general extension mechanism.

XML stands for Extensible Markup Language. It is designed as a simplified version of SGML (Standard Generalized Markup Language), the meta-language used to define the grammar and syntax of HTML. One of the goals of XML is to be suitable for use on the Web, and in the context of this discussion it can be viewed as a general mechanism for extending HTML. As its name implies, extensibility is a key feature of XML; authors are free to declare and use new tags and attributes. At the same time, XML grammar and syntax rules carefully enforces document structure to facilitate automatic processing and maintenance of large document collections.

Though details about how XML markup will ultimately be embedded in HTML remain to be resolved, XML has garnered broad industry support including major browser vendors. Devising a standard way of embedding XML in HTML is also important with the W3C. Furthermore, other applications of XML for all kinds of document publishing and processing promise to become increasingly important. Consequently, both on theoretical and pragmatic grounds, it makes a great deal of sense to specify MathML as an XML application, and we have done so.

1.3.2.3 Browser Extension Mechanisms

While details of a general model for rendering and processing XML extensions to HTML is still being being resolved, broad features of the model are already fairly clear. Formatting Properties developed by the Cascading Style Sheets and Formatting Properties Working Group for CSS and made available through the Document Object Model (DOM) will be applied to MathML elements to obtain some stylistic control over the presentation of MathML. Further development of these Formatting Properties falls within the charter of both the CSS&FP and the XSL working groups. Thus, it may soon be possible to write a style sheet which will largely describe the correct display of MathML.

MathML was designed with the goal of style sheet-based rendering in mind. It is the intention of the W3C Math Working Group to work closely with W3C style sheet activities to ensure both that adequate support for MathML is incorporated into future style sheet mechanisms, and that MathML style sheets are developed. In particular, providing for adequate follow-on activities beyond the scope of the W3C Math working group charter is a high priority.

Until style sheet mechanisms are capable of delivering native browser rendering of MathML, however, it is necessary to extend browser capabilities by using embedded elements to render MathML. It may soon be possible to instruct a browser to use a particular embedded renderer to process embedded XML markup such as MathML, and coordinate the resulting output with the surrounding Web page. Indeed, for specialized processing, such as connecting to a computer algebra system, this capability is likely to remain highly desirable. However, for this kind of interaction to be really satisfactory, it will be necessary to define a document object model rich enough to facilitate complicated interactions between browsers and embedded elements. For this reason, the W3C Math working group is coordinating its efforts closely with the Document Object Model working group.

For processing by embedded elements, and for inter-communication between scientific software generally, a style sheet-based layout model is less than ideal in some ways. It can impose an additional implementation burden in a setting where it may offer few advantages, and it imposes implementation requirements for coordination between browsers and embedded renderers that will likely be unavailable in the immediate future.

For these reasons, the MathML specification defines an attribute-based layout model, which has proven very effective for high-quality rendering of complicated mathematical expressions in several independent implementations. MathML presentation attributes utilize W3C Formatting Properties where possible. Also, MathML elements accept class, style and id attributes to facilitate their use with CSS style sheets. However, at present, there are few settings where CSS machinery is currently available to MathML renderers.

Issue (sheet-use): Now that XSL and CSS are available, the following text should be revised.

When style sheet mechanisms become available to MathML, it is anticipated their use will become the dominant method of stylistic control of MathML presentation meant for use in rendering environments which support those mechanisms.

Chapter 2

MathML Fundamentals

2.1 MathML Overview

This chapter introduces the basic ideas of MathML. The first section describes the overall design of MathML. The second section presents a number of motivating examples, to give the reader something concrete to refer to while reading subsequent chapters of the MathML Specification. The final section describes basic features of the MathML syntax and grammar, which apply to all MathML markup. In particular, section 2.3 should be read before chapter 3, chapter 4 and chapter 5.

A fundamental challenge in defining a mathematics markup language for the Web is reconciling the need to encode both the presentation of a mathematical notation and the content of the mathematical idea or object which it represents.

The relationship between a mathematical notation and a mathematical idea is subtle and deep. On a formal level, the results of mathematical logic raise unsettling questions about the correspondence between symbolic logic systems and the phenomena they model. At a more intuitive level, anyone who uses mathematical notation knows the difference that a good choice of notation can make; the symbolic structure of the notation suggests the logical structure. For example, the Leibniz notation for derivatives 'suggests' the chain rule of calculus through the symbolic cancellation of fractions: $\frac{\partial f}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial f}{\partial t}$.

Mathematicians and teachers understand this very well; part of their expertise lies in choosing notation that emphasizes key aspects of a problem while hiding or diminishing extraneous aspects. It is commonplace in mathematics and science to write one thing when technically something else is meant, because long experience shows this actually communicates the idea better at some higher level.

In many other settings, though, mathematical notation is used to encode the full, precise meaning of a mathematical object. Mathematical notation is capable of prodigious rigor, and when used carefully, it is virtually free of ambiguity. Moreover, it is precisely this lack of ambiguity which makes it possible to describe mathematical objects so that they can be used by software applications such as computer algebra systems and voice renderers. In situations where such inter-application communication is of paramount importance, the nuances of visual presentation generally play a minimal role.

MathML allows authors to encode both the notation which represents a mathematical object and the mathematical structure of the object itself. Moreover, authors can mix both kinds of encoding in order to specify both the presentation and content of a mathematical idea.

The remainder of this section gives a basic overview of how MathML can be used in each of these ways.

2.1.1 Taxonomy of MathML Elements

All MathML elements fall into one of three categories: presentation elements, content elements and interface elements. Each of these categories is described in detail in chapter 3, chapter 4 and chapter 7 respectively.

Presentation elements describe mathematical notation structure. Typical examples are the `mrow`element, which is used to indicate a horizontal row of pieces of expressions, and the `msup`element, which is used to indicate a base and superscript. As a general rule, each presentation element corresponds to a single kind of notational 'schema' such as a row, a superscript, an underscript and so on. Since many notational schemata have a number of frequently occurring variants, most presentation elements accept a number of attributes which can be used to select between variants. For example, the superscript element accepts a 'superscript shift' attribute which specifies the minimum amount the superscript should shift upward.

Content elements describe mathematical objects directly, as opposed to describing the notation which represents them. Typical examples include the `plus`element, which denotes the usual addition operator for real numbers, and the `vector`element, which denotes a vector from linear algebra. Each content element corresponds to some mathematical concept. Some elements represent mathematical objects like vectors, while others represent functions or operations like addition.

Every MathML element but one is either a presentation element or a content element. The `math`element is neither, since its role is to serve as a top-level, interface element. One function of the `math`element is to pass on parameters to a MathML processor that affect an entire expression, such as style preferences. A second function is to communicate parameters to a Web browser about what software to use to render a MathML expression, and how the expression should be integrated into the surrounding HTML page. (As XML support is added to browsers, it may ultimately be necessary to introduce one or two more interface elements, to handle these functions separately. See chapter 7 for details.)

2.1.2 Expression Trees and Token Elements

Presentation and content expressions both share a number of formal properties. In both cases, most expressions naturally decompose into pieces, or sub-expressions. For example, the expression $(a + b)^2$ naturally breaks into a 'base', the $(a + b)$, and a 'script', which is the single character '2' in this case. Furthermore, as this example shows, the sub-expressions may themselves decompose into further sub-expressions, and so on. Of course, the decomposition process eventually terminates with indivisible expressions such as digits, letters, or other symbol characters.

Although this particular example involves mathematical notation, and hence presentation markup, the same observation applies equally well to abstract mathematical objects, and hence to content markup. For example, in a context of content markup our superscript example would typically be denoted by an exponentiation operation that would require two operands: a 'base' and an 'exponent'. This is no coincidence, since as a general rule, mathematical notation closely mirrors the logical structure of the underlying mathematical objects.

The recursive nature of mathematical objects and notation is strongly reflected in MathML markup. Most presentation or content elements contain some number of other MathML elements corresponding to the constituent pieces out of which the original object is recursively built. The original schema is commonly called the parent schema, and the constituent pieces are called child schemata. More generally, MathML expressions can be regarded as trees, where each node corresponds to a MathML element, the branches under a 'parent' node correspond to its 'children', and the leaves in the tree correspond to indivisible notation or content units such as numbers, characters, etc.

Most leaf nodes in a MathML expression tree are either canonically empty elements, or token elements. Canonically empty elements directly represent symbols in MathML, such as the content element `plus` MathML token elements are the only MathML elements permitted to directly contain character data. The character data may consist of ASCII characters and MathML entities, which are escape sequences of the form `&name;` MathML entities typically denote non-ASCII Unicode characters such as `&alpha;`, `&rightarrow;` and `&sum;` A third kind of leaf node permitted in MathML is the `annotation` element, which is used to hold data in a non-MathML format.

The most important presentation token elements are `mi`, `mn` and `mo` for representing identifiers, numbers and operators respectively. Typically a renderer will employ slightly different typesetting styles for each of these kinds of character data: numbers are usually in upright font, identifiers in italics, and operators have extra space around them. In content markup, there are only two tokens, `ci` and `cn` for identifiers and numbers respectively. In content markup, separate elements are provided for commonly used functions and operators. The `fn` element is provided for user-defined extensions to the base set.

In terms of markup, most MathML elements have a start tag and an end tag, which enclose the markup for their contents. In the case of tokens, the content is character data, and in most other cases, the content is the markup for child elements. A third category of elements, called canonically empty elements, don't require any contents, and are marked up using a single tag of the form `<name/>` An example of this kind of markup is `<plus/>` in content markup.

Returning to the example of $(a + b)^2$, we can now see how the principles discussed above play out in practice. One form of presentation markup for this example is:

```
<msup>
  <mfenced>
    <mrow>
      <mi>a</mi>
      <mo>+</mo>
      <mi>b</mi>
    </mrow>
  </mfenced>
  <mn>2</mn>
</msup>
```

The content markup for the same example is:

```
<apply>
  <power/>
  <apply>
```

```
    <plus/>
    <ci>a</ci>
    <ci>b</ci>
  </apply>
  <cn>2</cn>
</apply>
```

While a full discussion of presentation and content markup must wait until chapter 3 and chapter 4, the main features of these sample encodings should now be relatively clear.

2.1.3 Presentation Markup

MathML presentation markup consists of 30 elements which accept over 50 attributes. Most of the elements correspond to layout schemata, which contain other presentation elements. Each layout schema corresponds to a two-dimensional notational device, such as a superscript or subscript, fraction or table. In addition, there are the presentation token elements `mi`, `mn` and `mo` introduced above, as well as several other less commonly used token elements. The remaining few presentation elements are empty elements, and are used mostly in connection with alignment.

The layout schemata fall into several classes. One group of elements is concerned with scripts, and contains elements such as `msub`, `munder`, and `mmultiscripts`. Another group focuses on more general layout and includes `mrow`, `mstyle`, and `mfrac`. A third group deals with tables. The `maction` element is a category by itself, and represents various kinds of actions on notation, such as in an expression which toggles between two pieces of notation.

An important feature of many layout schemata is that the order of child schemata is significant. For example, the first child of an `mfrac` element is the numerator and the second child is the denominator. Since the order of child schemata is not enforced at the XML level by the MathML DTD, the information added by ordering is only available to a MathML processor, as opposed to a generic XML processor. When we want to emphasize that a MathML element such as `mfrac` requires children in a specific order, we will refer to them as arguments, and think of the `mfrac` element as a notational 'constructor'.

2.1.4 Content Markup

Content markup consists of about 100 elements accepting roughly a dozen attributes. The majority of these elements are empty elements corresponding to a wide variety of operators, relations and named functions. Examples of this sort include `partialdiff`, `fleq` and `tan` Others such as `matrix` and `set` are used to encode various mathematical data types, and a third, important category of content elements such as `apply` are used to make new mathematical objects from others.

The `apply` element is perhaps the single most important content element. It is used to apply a function to a collection of arguments. The positions of the child schemata is again significant, with the first child denoting the function to be applied, and the remaining children denoting the arguments of the function, with order preserved. Note that the apply construct always uses prefix notation, like the programming language LISP. In particular, even binary operations like subtraction are marked up by applying a prefix subtraction operator to two arguments. For example, *a* - *b* would be marked up as

```
<apply>
```

```
   <minus/>
   <ci>a</ci>
   <ci>b</ci>
</apply>
```

A number of functions and operations require one or more quantifiers to be well-defined. For example, in addition to an integrand, a definite integral must specify the limits of integration and the bound variable. For this reason, there are several qualifier schemata such as `bvar` and `lowlimit`. They are used with operators such as `diff` and `int`.

The `declare` construct is especially important for content markup that might be evaluated by a computer algebra system. The `declare` element provides a basic assignment mechanism, where a variable can be declared to be of a certain type, with a certain value. Typically, declarations are ignored for visual rendering, and are used when an expression is evaluated.

2.1.5 Mixing Presentation and Content

Different kinds of markup will be most appropriate for different kinds of tasks. Legacy data is probably best translated into pure presentation markup, since semantic information about what the author meant can only be guessed at heuristically. By contrast, some mathematical applications and pedagogically-oriented authoring tools will likely choose to be entirely content-based. However, the majority of applications fall somewhere in between these extremes. For these applications, the most appropriate markup is a mixture of both presentation and content markup.

The rules for mixing presentation and content markup derive from the general principle that mixed content should only be allowed in places where it makes sense. For content markup embedded in presentation markup this basically means that any content fragments should be semantically meaningful, and should not require additional arguments or quantifiers to be fully specified. For presentation markup embedded in content markup, this usually means that presentation markup must be contained in a content token element, so that it will be treated as an indivisible notational unit used as a variable or function name.

Another option is to use a `semantics` element. The `semantics` element is used to bind MathML expressions to various kinds of annotations. One common use for the `semantics` element is to bind a content expression to a presentation expression as a semantic annotation. In this way, an author can specify a non-standard notation to be used when displaying a particular content expression. Another use of the `semantics` element is to bind some other kind of semantic specification, such as an OpenMath expression, to a MathML expression. In this way, the `semantics` element can be used to extend the scope of MathML content markup.

2.2 Some MathML Examples

2.2.1 Presentation Examples

Notation: $x^2 + 4x + 4 = 0$.

Markup:

```
<mrow>
  <mrow>
    <msup>
      <mi>x</mi>
      <mn>2</mn>
    </msup>
    <mo>+</mo>
    <mrow>
      <mn>4</mn>
      <mo>&InvisibleTimes;</mo>
      <mi>x</mi>
    </mrow>
    <mo>+</mo>
    <mn>4</mn>
  </mrow>
  <mo>=</mo>
  <mn>0</mn>
</mrow>
```

Note the use of nested `mrow`elements to denote terms, in this case the left-hand side of the equation functioning as an operand of '='. Marking terms greatly facilitates things like spacing for visual rendering, voice rendering, and line breaking.

Notation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Markup:

```
<mrow>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mrow>
        <mo>-</mo>
        <mi>b</mi>
      </mrow>
      <mo>&PlusMinus;</mo>
      <msqrt>
        <mrow>
          <msup>
            <mi>b</mi>
            <mn>2</mn>
          </msup>
          <mo>-</mo>
          <mrow>
            <mn>4</mn>
            <mo>&InvisibleTimes;</mo>
            <mi>a</mi>
            <mo>&InvisibleTimes;</mo>
            <mi>c</mi>
```

```
        </mrow>
      </mrow>
    </msqrt>
  </mrow>
  <mrow>
    <mn>2</mn>
    <mo>&InvisibleTimes;</mo>
    <mi>a</mi>
  </mrow>
 </mfrac>
</mrow>
```

Notice that the plus/minus sign is given by a special named entity `&PlusMinus;`. MathML provides a very comprehensive list of entity names for mathematical symbols. In addition to the mathematical symbols needed for screen and print rendering, MathML provides symbols to facilitate audio rendering. For audio rendering, it is important to be able to automatically determine whether

```
<mrow>
  <mi>z</mi>
  <mfenced>
    <mrow>
      <mi>x</mi>
      <mo>+</mo>
      <mi>y</mi>
    </mrow>
  </mfenced>
</mrow>
```

should be read as '*z* times the quantity *x* plus *y*' or '*z* of *x* plus *y*'. The entities `&InvisibleTimes;` and `&ApplyFunction;` provide a way for authors to directly encode the distinction for audio renderers. For instance, in the first case `&InvisibleTimes;` should be inserted after the line containing the *z*. MathML also introduces entities like `&dd;` which represents a 'differential d' which renders with slightly different spacing in print, and can be rendered as 'd' or 'with respect to' in speech. Unless content tags, or some other mechanism, are used to eliminate the ambiguity, authors should always use these entities, in order to make their documents more accessible.

Notation: $A = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$.

Markup:

```
<mrow>
  <mi>A</mi>
  <mo>=</mo>
  <mfenced open="[" close="]">
    <mtable>
      <mtr>
         <mtd><mi>x</mi></mtd>
         <mtd><mi>y</mi></mtd>
      </mtr>
      <mtr>
```

```
        <mtd><mi>z</mi></mtd>
        <mtd><mi>w</mi></mtd>
      </mtr>
    </mtable>
  </mfenced>
</mrow>
```

Most elements have a number of attributes that control the details of their screen and print rendering. For example, there are several attributes for the `mfenced` element that control what delimiters should be used at the beginning and the end of the expression. The attributes for operator elements given using `<mo>` are set to default values determined by a dictionary. (For the suggested MathML operator dictionary, see appendix D.)

2.2.2 Content Examples

Notation: $x^2 + 4x + 4 = 0$.

Markup:

```
<apply>
  <eq/>
  <apply>
    <plus/>
    <apply>
      <power/>
      <ci>x</ci>
      <cn>2</cn>
    </apply>
    <apply>
      <times/>
      <cn>4</cn>
      <ci>x</ci>
    </apply>
    <cn>4</cn>
  </apply>
  <cn>0</cn>
</apply>
```

Note that the `apply` element is used for relations, operators and functions.

Notation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Markup:

```
<apply>
  <eq/>
  <ci>x</ci>
  <apply>
    <divide/>
    <apply>
      <fn><mo>&PlusMinus;</mo></fn>
      <apply>
```

```
        <minus/>
        <ci>b</ci>
      </apply>
      <apply>
        <root/>
        <apply>
          <minus/>
          <apply>
            <power/>
            <ci>b</ci>
            <cn>2</cn>
          </apply>
          <apply>
            <times/>
            <cn>4</cn>
            <ci>a</ci>
            <ci>c</ci>
          </apply>
        </apply>
        <cn>2</cn>
      </apply>
    </apply>
    <apply>
      <times/>
      <cn>2</cn>
      <ci>a</ci>
    </apply>
  </apply>
</apply>
```

MathML content markup does not directly contain an element for the ‘plus or minus’ operation. Therefore, we use the `fn`element to declare that we want the presentation markup for this operator to act as a content operator. This is a simple example of how presentation and content markup can be mixed to extend content markup.

Notation: $A = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$.

Markup:

```
<apply>
  <eq/>
  <ci>A</ci>
  <matrix>
    <matrixrow>
      <ci>x</ci>
      <ci>y</ci>
    </matrixrow>
    <matrixrow>
      <ci>z</ci>
      <ci>w</ci>
```

```
    </matrixrow>
  </matrix>
</apply>
```

Note that by default, the rendering of the content element `matrix` includes enclosing parentheses, so we need not directly encode them. This is quite different from the presentation element `mtable` which may or may not refer to a matrix, and hence requires explicit encoding of the parentheses if they are desired.

2.2.3 Mixed Markup Examples

Notation: $\int_0^\infty \frac{\mathrm{d}t}{t}$.

Markup:

```
<semantics>
  <mrow>
    <msubsup>
      <mo>&int;</mo>
      <mn>0</mn>
      <mi>t</mi>
    </msubsup>
    <mfrac>
      <mrow>
        <mo>&dd;</mo>
        <mi>x</mi>
      </mrow>
      <mi>x</mi>
    </mfrac>
  </mrow>
  <annotation-xml encoding="MathML-Content">
    <apply>
      <int/>
      <bvar><ci>x</ci></bvar>
      <lowlimit><cn>0</cn></lowlimit>
      <uplimit><ci>t</ci></uplimit>
      <apply>
        <divide/>
        <cn>1</cn>
        <ci>x</ci>
      </apply>
    </apply>
  </annotation-xml>
</semantics>
```

In this example, we use the `semantics` element to provide a MathML content expression to serve as a ‘semantic annotation’ for a presentation expression. The `semantics` element has as its first child the expression being annotated, and the subsequent children are the annotations. There is no restriction on the kind of annotation that can be attached using the `semantics` element. For example, one might give a TEX encoding, or computer algebra

input in an annotation. The type of annotation is specified by the `encoding` attribute and the `annotation` and `annotation-xml` elements.

Another common use of the `semantics` element arises when one wants to use a content coding, and provide a suggestion for its presentation. In this case, we would have the markup:

```
<semantics>
  <apply>
    <int/>
    <bvar><ci>x</ci></bvar>
    <lowlimit><cn>0</cn></lowlimit>
    <uplimit><ci>t</ci></uplimit>
    <apply>
      <divide/>
      <cn>1</cn>
      <ci>x</ci>
    </apply>
  </apply>
  <annotation-xml encoding="MathML-Presentation">
    <mrow>
      <msubsup>
        <mo>&int;</mo>
        <mn>0</mn>
        <mi>t</mi>
      </msubsup>
      <mfrac>
        <mrow>
          <mo>&dd;</mo>
          <mi>x</mi>
        </mrow>
        <mi>x</mi>
      </mfrac>
    </mrow>
  </annotation-xml>
</semantics>
```

This kind of annotation is useful when something other than the default rendering of the content encoding is desired. For example, by default, some renderers might layout the integrand something like '1/*x* d*x*'. Specifying that the integrand should by preference render as 'd*x*/*x*' instead can be accomplished with the use of a MathML Presentation annotation as shown. Be aware, however, that renderers are not required to take into account information contained in annotations, and what use is made of them, if any, will depend on the renderer.

2.3 MathML Syntax and Grammar

MathML is an application of XML, or Extensible Markup Language [Bray1998], and as such its syntax is governed by the rules of XML syntax, and its grammar is in part specified by a DTD, or Document Type Definition. In other words, the details of using tags, attributes, entity references and so on are defined in the XML language specification, and the details

about MathML element and attribute names, which elements can be nested inside each other, and so on are specified in the MathML DTD.

Issue (rewrite-for-schema): The following needs to be revised pending creation of a schema for MathML.

However, MathML also specifies some syntax and grammar rules in addition to the general rules it inherits as an XML application. These rules allow MathML to encode a great deal more information than would ordinarily be possible with pure XML, without introducing many more elements, and using a substantially more complex DTD. A grammar for content markup expressions is given in appendix B. Of course, one drawback to using MathML specific rules is that they are invisible to generic XML processors and validators.

There are basically two kinds of additional MathML grammar and syntax rules. One kind involves placing additional criteria on attribute values. For example, it is not possible in pure XML to require that an attribute value be a positive integer. The second kind of rule specifies more detailed restrictions on the child elements (for example on ordering) than are given in the DTD. For example, it is not possible in XML to specify that the first child be interpreted one way, and the second in another.

The following sections discuss features both of XML syntax and grammar in general, and of MathML in particular. Throughout the remainder of the MathML specification, we will usually take care to distinguish between usage required by XML syntax and the MathML DTD and usage required by MathML specific rules. However, we will frequently allude to 'MathML errors' without identifying which part of the specification is being violated.

2.3.1 An XML Syntax Primer

Since MathML is an application of XML, the MathML Specification uses the terminology of XML to describe it. Briefly, XML data is composed of Unicode characters (which include ordinary ASCII characters), 'entity references' (informally called 'entities') such as `&rightarrow;` which usually represent 'extended characters', and 'elements' such as `<mi fontstyle="normal"> x </mi>`. Elements enclose other XML data called their 'content' between a 'start tag' (sometimes called a 'begin tag') and an 'end tag', much like in HTML. There are also 'empty elements' such as `<plus/>` whose start tag ends with `/>` to indicate that the element has no content or end tag. The start tag can contain named parameters called 'attributes', such as `fontstyle="normal"` in the example above. For further details on XML, consult the XML specification [Bray1998].

As XML is case-sensitive, MathML element and attribute names are case-sensitive. For reasons of legibility, the MathML defines them almost all in lowercase.

In formal discussions of XML markup a distinction is maintained between an element, such as an `mrow` element, and the tags `<mrow>` and `</mrow>` marking it. What is between the `<mrow>` start tag and the `</mrow>` end tag is the content of the `mrow` element. An 'empty element' such as `none` is defined to have no content and so has a single tag of the form `<none/>`. Usually, the distinction between elements and tags will not be so finely drawn in this specification. For instance, we will sometimes refer to the `<mrow>` and `<none/>` elements, really meaning the elements whose tags these are, in order that references to elements are visually distinguishable from references to attributes. However, the words 'element' and 'tag' themselves will be used strictly in accordance with XML terminology.

2.3.2 Children versus Arguments

Many MathML elements require a specific number of child elements and/or attach additional meanings to children in certain positions. As noted above, these kinds of requirements are MathML specific, and cannot be specified entirely in terms of XML syntax and grammar. When the children of a given MathML element are subject to these kinds of additional conditions, we will often refer to them as arguments instead of merely children in order to emphasize their MathML specific usage. Note that especially in chapter 3 the term 'argument' is usually used in this technical sense, unless otherwise noted, and therefore refers to a child element.

In the detailed discussions of element syntax given with each element throughout the MathML specification, the number of required arguments and their order is implicitly indicated by giving names for the arguments at various positions. This information is also given for presentation elements in the table of argument requirements in section 3.1.3, and for content elements in appendix B.

A few elements have other requirements on the number or type of arguments. These additional requirements are described together with the individual elements.

2.3.3 MathML Attribute Values

According to the XML language specification, attributes given to elements must have one of the forms

```
attribute-name = "value"
```

or

```
attribute-name = 'value'
```

where whitespace around the '=' is optional.

Attribute names are generally shown in a `monospaced` font within descriptive text in this specification, but not within examples.

The attribute value, which in general in MathML can be a string of arbitrary characters, must be surrounded by a pair of either double quotes (") or single quotes ('). The kind of quotes not used to surround the value may be included within it.

MathML uses a more complicated syntax for attribute values than the generic XML syntax required by the MathML DTD. These additional rules are intended for use by MathML applications, and it is a MathML error to violate them, though they are not enforced by XML processing. The MathML syntax of each attribute value is specified in the table of attributes provided with the description of each element it can be used with, using a notation described below. In MathML applications these attribute values should be further processed as follows, unless otherwise specified: whitespace is ignored except to separate letter and/or digit sequences into individual words or numbers; and the same entity references (listed in chapter 6) which can be used within token elements to represent characters can be used to represent those characters in attribute values (whenever those characters would be permitted by that attribute value's syntax).

In particular, the characters ", ', & and < can be included in MathML attribute values (when permitted by the attribute value syntax) using the entity references `&quot;`, `&apos;` (or `&apos;`) and `<` respectively.

The MathML DTD provided in appendix A declares most attribute value types as `CDATA` strings. This permits increased interoperability with existing SGML and XML software and allows extension to the lists of predefined values.

2.3.3.1 Syntax notations used in the MathML specification

To describe the MathML-specific syntax of permissible attribute values, the following conventions and notations are used for most attributes in the present document.

Notation	What it matches
number	decimal integer or rational number (digits with one decimal point), optionally starting with ’-’
unsigned-number	decimal integer or real number, no sign
integer	decimal integer, optionally starting with ’-’
positive-integer	decimal integer, unsigned, not 0
string	arbitrary string (always the entire attribute value)
character	single non-whitespace character, or MathML entity reference; whitespace separation is option
#rgb	RGB color value
#rrggbb	RGB color value
h-unit	unit of horizontal length (allowable units are listed below)
v-unit	unit of vertical length (allowable units are listed below)
css-fontfamily	explained in CSS subsection, below
html-color-name	explained in CSS subsection, below
other italicized words	explained in the text for each attribute
form +	one or more instances of form
form *	zero or more instances of form
f1 f2 ... fn	one instance of each form, in sequence, perhaps separated by whitespace
f1 | f2 | ... | fn	any one of the specified forms
[form]	optional instance of form
(form)	same as form
word in plain text	that word, literally present in attribute value (unless it is obviously part of an explanatory phra
quoted symbol	that symbol, literally present in attribute value (e.g. "+" or ’+’)

Issue (rgb-notation): Do we need to explain what RGB colour notation is?

The order of precedence of the syntax notation operators is, from highest to lowest precedence:

- form + or form *
- f1 f2 ... fn (sequence of forms)
- f1 | f2 | ... | fn (alternative forms)

A string can contain arbitrary characters which are specifiable within XML `CDATA` attribute values; it must use entity references for certain characters, as described earlier. It can contain XML-format entity or character references for any of the characters listed in chapter 6. No syntax rule in MathML includes string as only part of an attribute value, only as the entire value.

Issue (character): This needs to be revised for the introduction of the `mchar` element.

A character consists of a single non-whitespace character or entity reference.

As a simple example, the permissible values of boolean attributes are specified as `true | false`, meaning that the entire attribute value should be either `true` or `false`

Adjacent keywords and/or numbers must be separated by whitespace in the actual attribute values, except for unit identifiers (symbolized by `h-unit` or `v-unit` syntax symbols) following numbers. Whitespace is not otherwise required, but is permitted between any of the tokens listed above, except (for compatibility with CSS1) immediately before unit identifiers, between the '-' signs and digits of negative numbers, or between # and `rgb` or `rrggbb`

Numeric attribute values for dimensions that should depend upon the current font can be given in font-related units, or in named absolute units (described in a separate subsection below). Horizontal dimensions are conventionally given in em's, and vertical dimensions in ex's, by immediately following a number by one of the unit identifiers `em` or `ex`. For example, the horizontal spacing around an operator such as '+' is conventionally given in `em`s, though other units can be used. Using font-related units is usually preferable to using absolute units, since it allows renderings to grow or shrink proportionately to the current font size.

For most numeric attributes, only those in a subset of the expressible values are sensible; values outside this subset are not errors, unless otherwise specified, but rather are rounded up or down (at the discretion of the renderer) to the closest value within the allowed subset. The set of allowed values may depend on the renderer, and is not specified by MathML.

If a numeric value within an attribute value syntax description is declared to allow a minus sign ('-'), e.g. `number` or `integer`, it is not a syntax error when one is provided in cases where a negative value is not sensible. Instead, the value should be handled by the processing application as described in the preceding paragraph. An explicit plus sign ('+') is not allowed as part of a numeric value except when it is specifically listed in the syntax (as a quoted '+' or "+"), and its presence can change the meaning of the attribute value (as documented with each attribute which permits it).

Issue (html-color): The phrase `html-color-name` is used but never explained.

The symbols `h-unit`, `v-unit`, `css-fontfamily`, and `html-color-name` are explained in the following subsections.

2.3.3.2 Attributes with units

Some attributes accept horizontal or vertical lengths as numbers followed by a 'unit identifier' (often just called a 'unit'). The syntax symbols `h-unit` and `v-unit` refer to a unit for horizontal or vertical length, respectively. The possible units and the lengths they refer to are shown in the table below; they are the same for horizontal and vertical lengths, but the syntax symbols are distinguished in attribute syntaxes as a reminder of the direction they are each used in.

The unit identifiers and meanings are taken from CSS1. (However, the syntax of numbers followed by unit identifiers in MathML is not identical to the syntax of length values with units in CSS style sheets, since numbers in CSS can't end with decimal points, and are allowed to start with '+' signs.)

The possible horizontal or vertical units in MathML are:

The typesetting units `em` and `ex` are defined in appendix F, and discussed further under 'Additional notes' below.

`%` is a 'relative unit'; when an attribute value is given as `n%` (for any numeric value `n`), the value being specified is the default value for the property being controlled multiplied by `n` divided by 100. The default value (or the way in which it is obtained, when it is not

Unit identifier	Unit description
em	em (font-relative unit traditionally used for horizontal lengths)
ex	ex (font-relative unit traditionally used for vertical lengths)
px	pixels, or pixel size of the current display
in	inches (1 inch = 2.54 centimeters)
cm	centimeters
mm	millimeters
pt	points (1 point = 1/72 inch)
pc	picas (1 pica = 12 points)
%	percentage of default value

constant) is listed in the table of attributes for each element, and its meaning is described in the subsequent documentation about that attribute. (The `mpadded` element has its own syntax for `%` and does not allow it as a unit identifier.)

For consistency with CSS, length units in MathML are rarely optional. When they are, the unit symbol is enclosed in square brackets in the attribute syntax, following the number it applies to, e.g. `number [ h-unit ]`. The meaning of specifying no unit is given in the documentation for each attribute; in general it is that the number given is a multiplier for the default value of the attribute. (In such cases, specifying the number `nnn` without a unit is equivalent to specifying the number `nnn` times 100 followed by `%`. For example, `<mo maxsize="2"> ( </mo>` is equivalent to `<mo maxsize="200%"> ( </mo>`.)

As a special exception (also consistent with CSS), a numeric value equal to 0 need not be followed by a unit identifier even if the syntax specified here requires one. In such cases, the unit identifier (or lack of one) would not matter, since 0 times any unit is 0.

For most attributes, the typical unit which would be used to describe them in typesetting is the same as the one used in that attribute's default value in this specification; when a specific default value is not given, the typical unit is usually mentioned in the syntax table or in the documentation for that attribute. The typical unit is usually `em` or `ex`. However, any unit can be used, unless otherwise specified for a specific attribute.

Additional notes about units

Note that some attributes, e.g. `framespacing` on `<mtable>`, can contain more than one numeric value, each followed by its own unit.

It is conventional to use the font-relative unit `ex` mainly for vertical lengths, and `em` mainly for horizontal lengths, but this is not required. These units are relative to the font and fontsize which would be used for rendering the element in whose attribute value they are specified, which means they should be interpreted after attributes such as `fontfamily` and `fontsize` are processed, if those occur on the same element, since changing the current font or fontsize can change the length of these units.

The definition of the length of each unit (but not the MathML syntax for length values) is as specified in CSS1, except that if a font provides specific values for em and/or ex which differ from the values defined by CSS1 (the font size and 'x'-height respectively), those values should be used.

2.3.3.3 CSS-compatible attributes

Several MathML attributes, listed below, correspond closely with text rendering properties defined by Cascading Style Sheets, Level 1 (CSS1).

The names and acceptable values of these attributes have been aligned with the CSS1 recommendation where possible. In general, the MathML syntax for each attribute is intended to be a subset of the CSS syntax for the corresponding property. Differences at the detail level, where they exist, are explained with the documentation about each attribute, in the sections of this specification listed in the table.

The syntax of certain attributes is partially specified, in the tables of attribute syntax in this specification, using one of the symbols `css-fontfamily` or `html-color-name`, as shown in the following table. These symbols refer to syntaxes from other W3C Recommendations, and are explained in the sections of this specification referred to in the table.

MathML attribute	CSS property	syntax symbol	MathML elements	refer to
fontsize	font-size	-	presentation tokens; `mstyle`	section 3.2.1
fontweight	font-weight	-	presentation tokens; `mstyle`	section 3.2.1
fontstyle	font-style	-	presentation tokens; `mstyle`	section 3.2.1
fontfamily	font-family	css-fontfamily	presentation tokens; `mstyle`	section 3.2.1
color	color	html-color-name	presentation tokens; `mstyle`	section 3.3.4
background	background	html-color-name	`mstyle`	section 3.3.4

See also section 2.3.4 below for a discussion of the `class`, `style` and `id` attributes for use with style sheets.

Order of processing attributes versus style sheets

CSS or analogous style sheets specify changes to rendering properties of selected MathML elements (selecting the elements in various ways). Either the properties listed above, or any other MathML rendering attributes or properties supported by a style sheet mechanism, can be affected, in principle for any element. Since rendering properties can also be changed by attributes on an element, or automatically (which can happen to `fontsize`, as explained in the discussion on `scriptlevel` in section 3.3.4), it is necessary to specify the relative order in which changes from various sources occur. In the case of 'absolute' changes, i.e. setting a new property value independent of the old value (as opposed to 'relative' changes, such as increments or multiplications by a factor), the absolute change performed last will be the only absolute change which is effective, so the sources of changes which should have the highest priority must be processed last.

In the case of CSS1, the order of processing of changes from various sources which affect one MathML element's rendering properties should be as follows:

(first changes; lowest priority)

- automatic changes to properties or attributes based on the type of the parent element, and this element's position in the parent, as for the changes to `fontsize` in relation to `scriptlevel` mentioned above; such changes will usually be implemented by the parent element itself before it passes a set of rendering properties to this element
- style sheet from reader: styles which are not declared 'important'
- explicit attribute settings on this MathML element

- style sheet from author: styles which are not declared ‘important’
- style sheet from reader: styles which are declared ‘important’
- style sheet from author: styles which are declared ‘important’

(last changes; highest priority)

Note that the order of the changes derived from CSS style sheets is specified by CSS itself. The following rationale is related only to the issue of where in this pre-existing order the changes caused by explicit MathML attribute settings should be inserted.

Rationale: MathML rendering attributes are analogous to HTML rendering attributes such as `align`, which the CSS1 section on cascading order specifies should be processed with the same priority. Furthermore, this choice of priority permits readers, by declaring certain CSS styles as ‘important’, to decide which of their style preferences should override explicit attribute settings in MathML. Since MathML expressions, whether composed of ‘presentation’ or ‘content’ elements, are primarily intended to convey meaning, with their ‘graphic design’ (if any) intended mainly to aid in that purpose but not to be essential in it, it is likely that readers will often want their own style preferences to have priority; the main exception will be when a rendering attribute is intended to alter the meaning conveyed by an expression, which is generally discouraged in the presentation attributes of MathML.

2.3.3.4 Default values of attributes

Default values for MathML attributes are in general given along with the detailed descriptions of specific elements in the text. Default values shown in plain text, in the tables of attributes for an element, are literal (unless they are obviously explanatory phrases), but when italicized are descriptions of how default values can be computed.

Default values described as inherited are taken from the rendering environment, as described under `mstyle` or in some cases (described individually) from the values of other attributes of surrounding elements, or from certain parts of those values. The value used will always be one which could have been specified explicitly, had it been known; it will never depend on the content or attributes of the same element, only on its environment. (What it means when used may, however, depend on those.)

Default values described as automatic should be computed by a MathML renderer in a way which will produce a high-quality rendering; how to do this is not usually specified by MathML. The value computed will always be one which could have been specified explicitly, had it been known, but it will usually depend on the element content and/or the rendering environment.

Other italicized descriptions of default values which appear in the tables of attributes are explained for each attribute individually.

The single or double quotes which are required around attribute values in an XML start tag are not shown in the tables of attribute value syntax for each element, but are shown around example attribute values in the text.

Note that, in general, there is no value which can be given explicitly for a MathML attribute which will simulate the effect of not specifying the attribute at all, for attributes which are inherited or automatic. Giving the words ‘inherited’ or ‘automatic’ explicitly will not work, and is not generally allowed. Furthermore, even for presentation attributes for which a specific default value is documented here, the `mstyle` element (section 3.3.4) can be used to change this for the elements it contains. Therefore, the MathML DTD declares

most presentation attribute default values as `#IMPLIED`, which prevents XML preprocessors from adding them with any specific default value.

2.3.3.5 Attribute values in the MathML DTD

In an XML DTD, allowed attribute values can be declared as general strings, or they can be constrained in various ways, either by enumerating the possible values, or by declaring them to be certain special data types. The choice of an XML attribute type affects the extent to which validity checks can be performed using a DTD.

The MathML DTD specifies formal XML attribute types for all MathML attributes, including enumerations of legitimate values in some cases. In general, however, the MathML DTD is relatively permissive, frequently declaring attribute values as strings; this is done to provide for interoperability with SGML parsers while allowing multiple attributes on one MathML element to accept the same values (such as `true` and `false`), and also to allow extension to the lists of predefined values.

At the same time, even though an attribute value may be declared as a string in the DTD, only certain values are legitimate in MathML, as described above and in the rest of this specification. For example, many attributes expect numerical values. In the sections which follow, the allowed attribute values are described for each element. To determine when these constraints are actually enforced in the MathML DTD, consult appendix A. However, lack of enforcement of a requirement in the DTD does not imply that the requirement is not part of the MathML language itself, or that it will not be enforced by a particular MathML renderer. (See section 7.2.2 for a description of how MathML renderers should respond to MathML errors.)

Furthermore, the MathML DTD is provided for convenience; although it is intended to be fully compatible with the text of the specification, the text should be taken as definitive if there is a contradiction. (Any contradictions which may exist between various chapters of the text should be resolved by favoring chapter 6 first, then chapter 3, chapter 4, then section 2.3, and then other parts of the text.)

2.3.4 Attributes Shared by all MathML Elements

In order to facilitate compatibility with Cascading Style Sheets, Level 1 (CSS1), all MathML elements accept `class`, `style` and `id` attributes in addition to the attributes described specifically for each element. MathML renderers not supporting CSS may ignore these attributes. (MathML specifies these attribute values as general strings, even if style-sheet mechanisms have more restrictive syntaxes for them. That is, any value for them is valid in MathML.)

Renderers supporting CSS (or analogous style sheet mechanisms) may use these attributes to help determine which MathML elements should be subject to which style sheet-induced changes to various rendering properties. The properties that can be affected, and how these changes affect them, are discussed in section 2.3.3.3 above.

Every MathML element also accepts the attribute `other` (section 7.2.3) for passing non-standard attributes without violating the MathML DTD. MathML renderers are only required to process this attribute if they respond to any attributes which are not standard in MathML.

See also section 3.2.1 for a list of MathML attributes which can be used on most presentation token elements.

2.3.5 Collapsing Whitespace in Input

MathML ignores whitespace occurring outside token elements. Non-whitespace characters are not allowed there. Whitespace occurring within the content of token elements is 'trimmed' from the ends (i.e. all whitespace at the beginning and end of the content is removed), and 'collapsed' internally (i.e. each sequence of 1 or more whitespace characters is replaced with one blank character).

In MathML, as in XML, 'whitespace' means blanks, tabs, newlines, or carriage returns, i.e. characters with hexadecimal Unicode codes `U+0020`, `U+0009`, `U+000a`, or `U+000d`, respectively.

For example, `<mo> ( </mo>` is equivalent to `<mo>(</mo>`, and

```
<mtext>
  Theorem
  1:
</mtext>
```

is equivalent to `<mtext>Theorem 1:</mtext>`

Authors wishing to encode whitespace characters at the start or end of the content of a token, or in sequences other than a single blank, without having them ignored, must use ` ` or other 'whitespace' non-marking entities as described in section 6.1.4. For example, compare

```
<mtext>
 Theorem
  1:
</mtext>
```

with

```
<mtext>
 Theorem&NewLine;  1:
</mtext>
```

When the first example is rendered, there is no whitespace before 'Theorem', one blank between 'Theorem' and '1:', and no whitespace after '1:'. In the second example, a single blank is rendered before 'Theorem', a new line is placed after 'Theorem', two blanks are rendered before '1:', and there is no whitespace after the '1:'.

Note that the `xml:space` attribute does not apply in this situation since XML processors pass whitespace in tokens to a MathML processor; it is the MathML processing rules which specify that whitespace is trimmed and collapsed.

For whitespace occurring outside the content of the token elements `mi`, `mn`, `mo`, `ms`, `mtext`, `ci`, `cn` and `annotation`, an `mspace` element should be used, as opposed to an `mtext` element containing only 'whitespace' entities.

Chapter 3

Presentation Markup

3.1 Introduction

This chapter specifies the ‘presentation’ elements of MathML, which can be used to describe the layout structure of mathematical notation.

3.1.1 What Presentation Elements Represent

Presentation elements correspond to the ‘constructors’ of traditional mathematical notation - that is, to the basic kinds of symbols and expression-building structures out of which any particular piece of traditional mathematical notation is built. Because of the importance of traditional visual notation, the descriptions of the notational constructs the elements represent are usually given here in visual terms. However, the elements are medium-independent in the sense that they have been designed to contain enough information for good spoken renderings as well. Some attributes of these elements may make sense only for visual media, but most attributes can be treated in an analogous way in audio as well (for example, by a correspondence between time duration and horizontal extent).

MathML presentation elements only suggest (i.e. do not require) specific ways of rendering in order to allow for medium-dependent rendering and for individual preferences of style. This specification describes suggested visual rendering rules in some detail, but a particular MathML renderer is free to use its own rules as long as its renderings are intelligible.

The presentation elements are meant to express the syntactic structure of mathematical notation in much the same way as titles, sections, and paragraphs capture the higher level syntactic structure of a textual document. Because of this, for example, a single row of identifiers and operators, such as ‘*x* + *a* / *b*’, will often be represented not just by one `mrow`element (which renders as a horizontal row of its arguments), but by multiple nested `mrow`elements corresponding to the nested sub-expressions of which one mathematical expression is composed - in this case,

```
<mrow>
  <mi> x </mi>
  <mo> + </mo>
  <mrow>
     <mi> a </mi>
     <mo> / </mo>
     <mi> b </mi>
```

```
  </mrow>
</mrow>
```

Similarly, superscripts are attached not just to the preceding character, but to the full expression constituting their base. This structure allows for better-quality rendering of mathematics, especially when details of the rendering environment such as display widths are not known to the document author; it also greatly eases automatic interpretation of the mathematical structures being represented.

Certain extended characters, represented by entity references, are used to name operators or identifiers that in traditional notation render the same as other symbols, such as `&DifferentialD;`, `&ExponentialE;`, or `&ImaginaryI;`, or operators that usually render invisibly, such as `&InvisibleTimes;`, `&ApplyFunction;`, or `&InvisibleComma;`. These are distinct notational symbols or objects, as evidenced by their distinct spoken renderings and in some cases by their effects on linebreaking and spacing in visual rendering, and as such should be represented by the appropriate specific entity references. For example, the expression represented visually as '$f(x)$' would usually be spoken in English as 'f of x' rather than just '$f\ x$'; this is expressible in MathML by the use of the `&ApplyFunction;` operator after the 'f', which (in this case) can be aurally rendered as 'of'.

The complete list of MathML entities is described in chapter 6.

3.1.2 Terminology Used In This Chapter

It is strongly recommended that, before reading the present chapter, one read section 2.3 on MathML syntax and grammar, which contains important information on MathML notations and conventions. In particular, in this chapter it is assumed that the reader has an understanding of basic XML terminology described in section 2.3.1, and the attribute value notations and conventions described in section 2.3.3.

The remainder of this section introduces MathML-specific terminology and conventions used in this chapter.

3.1.2.1 Types of presentation elements

The presentation elements are divided into two classes. Token elements represent individual symbols, names, numbers, labels, etcetera. In general, tokens can have only characters and `mchar` elements as content. The only exceptions are the vertical alignment element `malignmark`, and entity references. (Note, however, that entity references are deprecated in favor of the `mchar` element in MathML 2.0 .) Layout schemata build expressions out of parts, and can have only elements as content (except for whitespace, which they ignore). There are also a few empty elements used only in conjunction with certain layout schemata.

All individual 'symbols' in a mathematical expression should be represented by MathML token elements. The primary MathML token element types are identifiers (e.g. variables or function names), numbers, and operators (including fences, such as parentheses, and separators, such as commas). There are also token elements for representing text or whitespace that has more aesthetic than mathematical significance, and for representing 'string literals' for compatibility with computer algebra systems. Note that although a token element represents a single meaningful 'symbol' (name, number, label, mathematical symbol, etcetera), such symbols may be comprised of more than one character. For example `sin` and `24` are represented by the single tokens `<mi>sin</mi>` and `<mn>24</mn>` respectively.

In traditional mathematical notation, expressions are recursively constructed out of smaller expressions, and ultimately out of single symbols, with the parts grouped and positioned using one of a small set of notational structures, which can be thought of as 'expression constructors'. In MathML, expressions are constructed in the same way, with the layout schemata playing the role of the expression constructors. The layout schemata specify the way in which sub-expressions are built into larger expressions. The terminology derives from the fact that each layout schema corresponds to a different way of 'laying out' its sub-expressions to form a larger expression in traditional mathematical typesetting.

3.1.2.2 Terminology for other classes of elements and their relationships

The terminology used in this chapter for special classes of elements, and for relationships between elements, is as follows: The presentation elements are the MathML elements defined in this chapter. These elements are listed in section 3.1.5. The content elements are the MathML elements defined in chapter 4. The content elements are listed in section 4.4.

A MathML expression is a single instance of any of the presentation elements with the exception of the empty elements `none` or `mprescripts`, or is a single instance of any of the content elements which are allowed as content of presentation elements (listed in section 5.2.4). The intuition behind the definition of an expression is that it is an element with an unambigous rendering without some larger, enclosing construct. A sub-expression of an expression *E* is any MathML expression that is part of the content of *E*, whether directly or indirectly, i.e. whether it is a 'child' of *E* or not.

Since layout schemata attach special meaning to the number and/or positions of their children, a child of a layout schema is also called an argument of that element. As a consequence of the above definitions, the content of a layout schema consists exactly of a sequence of zero or more nonoverlapping elements that are its arguments.

3.1.3 Required Arguments

Many of the elements described herein require a specific number of arguments (always 1, 2, or 3). In the detailed descriptions of element syntax given below, the number of required arguments is implicitly indicated by giving names for the arguments at various positions. A few elements have additional requirements on the number or type of arguments, which are described with the individual element. For example, some elements accept sequences of zero or more arguments - that is, they are allowed to occur with no arguments at all.

Note that MathML elements encoding rendered space do count as arguments of the elements they appear in. See section 3.2.6 for a discussion of the proper use of such space-like elements.

3.1.3.1 Inferred *mrow*s

The elements listed in the following table as requiring 1* argument (`msqrt`, `mstyle`, `merror`, `mpadded`, `mphantom`, and `mtd`) actually accept any number of arguments. However, if the number of arguments is 0, or is more than 1, they treat their contents as a single inferred `mrow` formed from all their arguments.

For example,

```
<mtd>
</mtd>
```

is treated as if it were

```
<mtd>
  <mrow>
  </mrow>
</mtd>
```

and

```
<msqrt>
   <mo> - </mo>
   <mn> 1 </mn>
</msqrt>
```

is treated as if it were

```
<msqrt>
   <mrow>
      <mo> - </mo>
      <mn> 1 </mn>
   </mrow>
</msqrt>
```

This feature allows MathML data not to contain (and its authors to leave out) many `mrow` elements that would otherwise be necessary.

In the descriptions in this chapter of the above-listed elements' rendering behaviors, their content can be assumed to consist of exactly one expression, which may be an `mrow` element formed from their arguments in this manner. However, their argument counts are shown in the following table as 1*, since they are most naturally understood as acting on a single expression.

3.1.3.2 Table of argument requirements

For convenience, here is a table of each element's argument count requirements, and the roles of individual arguments when these are distinguished. An argument count of 1* indicates an inferred `mrow` as described above.

3.1.4 Elements with Special Behaviors

Certain MathML presentation elements exhibit special behaviors in certain contexts. Such special behaviors are discussed in the detailed element descriptions below. However, for convenience, some of the most important classes of special behavior are listed here.

Certain elements are considered space-like; these are defined in section 3.2.6. This definition affects some of the suggested rendering rules for `mo` elements (section 3.2.4).

Certain elements, e.g. `msup`, are able to embellish operators that are their first argument. These elements are listed in section 3.2.4, which precisely defines an 'embellished operator' and explains how this affects the suggested rendering rules for stretchy operators.

Certain elements treat their arguments as the arguments of an 'inferred `mrow`' if they are not given exactly one argument, as explained in section 3.1.3.

Element	Required argument count	Argument roles (when these differ by position)
mrow	0 or more	
mfrac	2	numerator denominator
msqrt	1*	
mroot	2	base index
mstyle	1*	
merror	1*	
mpadded	1*	
mphantom	1*	
mfenced	0 or more	
msub	2	base subscript
msup	2	base superscript
msubsup	3	base subscript superscript
munder	2	base underscript
mover	2	base overscript
munderover	3	base underscript overscript
mmultiscripts	1 or more	base (subscript superscript)* [<mprescripts/> presubscript presupe
mtable	0 or more rows	0 or more mtr elements
mtr	0 or more table elements	0 or more mtd elements
mtd	1*	
maction	1 or more	depend on actiontype attribute

In MathML 1.x, the mtable element could infer mtr elements around its arguments, and the mtr element could infer mtd elements. In MathML 2.0, mtr and mtd elements must be explicit. However, for backward compatibility renderers may wish to continue supporting inferred mtr and mtd elements.

3.1.5 Summary of Presentation Elements

3.1.5.1 Token Elements

mi	identifier
mn	number
mo	operator, fence, or separator
mtext	text
mspace	space
ms	string literal
mchar	referring to non-ASCII characters
mglyph	adding new character glyphs to MathML

3.1.5.2 General Layout Schemata

3.1.5.3 Script and Limit Schemata

3.1.5.4 Tables and Matrices

3.1.5.5 Enlivening Expressions

3.2 Token Elements

Token elements can contain any sequence of zero or more characters, or extended characters represented by entity references. In particular, tokens with empty content are allowed,

`mrow`	group any number of sub-expressions horizontally
`mfrac`	form a fraction from two sub-expressions
`msqrt`	form a square root sign (radical without an index)
`mroot`	form a radical with specified index
`mstyle`	style change
`merror`	enclose a syntax error message from a preprocessor
`mpadded`	adjust space around content
`mphantom`	make content invisible but preserve its size
`mfenced`	surround content with a pair of fences

`msub`	attach a subscript to a base
`msup`	attach a superscript to a base
`msubsup`	attach a subscript-superscript pair to a base
`munder`	attach an underscript to a base
`mover`	attach an overscript to a base
`munderover`	attach an underscript-overscript pair to a base
`mmultiscripts`	attach prescripts and tensor indices to a base

`mtable`	table or matrix
`mtr`	row in a table or matrix
`mtd`	one entry in a table or matrix
`malignmark` and `malignmark`	alignment markers

`maction`	bind actions to a sub-expression

and should typically render invisibly, with no width except for the normal extra spacing for that kind of token element. The allowed set of entity references for extended characters is given in chapter 6.

In MathML, characters and MathML entity references are only allowed to occur as part of the content of a token element. The only exception is whitespace between elements, which is ignored.

The `malignmark` element (see section 3.5.5) is the only element allowed in the content of tokens. It marks a place that can be vertically aligned with other objects, as explained in that section.

3.2.1 Attributes common to token elements

Several attributes related to text formatting are provided on all presentation token elements except `mspace`, `mchar` and `mglyph`, and on no other elements except `mstyle`. These are:

Name	values	default
fontsize	number v-unit	inherited
fontweight	normal | bold	inherited
fontstyle	normal | italic	normal (except on `<mi>`)
fontfamily	string | css-fontfamily	inherited
color	#rgb | #rrggbb | html-color-name	inherited

(See section 2.3.3 for terminology and notation used in attribute value descriptions.)

Token elements (other than `mspace`) should be rendered as their content (i.e. in the visual case, as a closely-spaced horizontal row of standard glyphs for the characters in their content) using the attributes listed above, with surrounding spacing modified by rules or attributes specific to each type of token element. Some of the individual attributes are further discussed below.

Recall that all MathML elements, including tokens, accept `class`, `style` and `id` attributes for compatibility with style sheet mechanisms, as described in section 2.3.4. In general, the font properties controlled by the attributes listed above are better handled using CSS or XSL style sheets depending on the context.

MathML expressions are often embedded in a textual data format such as HTML, and their renderings are likewise embedded in a rendering of the surrounding text. The renderer of the surrounding text (e.g. a browser) should provide the MathML renderer with information about the rendering environment, including attributes of the surrounding text such as its font size, so that the MathML can be rendered in a compatible style. For this reason, most attribute values affecting text rendering are inherited from the rendering environment, as shown in the 'default' column in the table above. (Note that it is also important for the rendering environment to provide the renderer with additional information, such as the baseline position of surrounding text, which is not specified by any MathML attributes.)

The exception to the general pattern of inheritance is the `fontstyle` attribute, whose default value is `normal` (non-slanted) for most tokens, but for `mi` depends on the content in a way described in the section about `mi`, section 3.2.2. Note that `fontstyle` is not inherited in MathML, even though the corresponding CSS1 property 'font-style' is inherited in CSS.

The `fontsize` attribute specifies the desired font size. `v-unit` represents a unit of vertical length (see section 2.3.3.3). The most common unit for specifying font sizes in typesetting is `pt` (points).

If the requested size of the current font is not available, the renderer should approximate it in the manner likely to lead to the most intelligible, highest quality rendering.

Many MathML elements automatically change `fontsize` in some of their children; see the discussion of `scriptlevel` in the section on `mstyle`, section 3.3.4.

The value of the `fontfamily` attribute should be the name of a font that may be available to a MathML renderer, or information that permits the renderer to select a font in some manner; acceptable values and their meanings are dependent on the specific renderer and rendering environment in use, and are not specified by MathML (but see the note about `css-fontfamily` below). (Note that the renderer's mechanism for finding fonts by name may be case-sensitive.)

If the value of `fontfamily` is not recognized by a particular MathML renderer, this should never be interpreted as a MathML error; rather, the renderer should either use a font that it considers to be a suitable substitute for the requested font, or ignore the attribute and act as if no value had been given.

Note that any use of the `fontfamily` attribute is unlikely to be portable across all MathML renderers. In particular, it should never be used to try to achieve the effect of a reference to an extended character (for example, by using a reference to a character in some symbol font that maps ordinary characters to glyphs for extended characters). As a corollary to this principle, MathML renderers should attempt to always produce intelligible renderings for the extended characters listed in chapter 6, even when these characters are not available in the font family indicated. Such a rendering is always possible - as a last resort, a character

can be rendered to appear as an XML-style entity reference using one of the entity names given for the same character in chapter 6.

The symbol `css-fontfamily` refers to a legal value for the `font-family` property in CSS1, which is a comma-separated list of alternative font family names or generic font types in order of preference, as documented in more detail in CSS1. MathML renderers are encouraged to make use of the CSS syntax for specifying fonts when this is practical in their rendering environment, even if they do not otherwise support CSS. (See also the subsection CSS-compatible attributes within section 2.3.3.3.

The syntax and meaning of the `color` attribute are as described for the same attribute of `<mstyle>` (section 3.3.4).

3.2.2 Identifier (`mi`)

3.2.2.1 Description

An `mi` element represents a symbolic name or arbitrary text that should be rendered as an identifier. Identifiers can include variables, function names, and symbolic constants.

Not all 'mathematical identifiers' are represented by `mi` elements - for example, subscripted or primed variables should be represented using `msub` or `msup` respectively. Conversely, arbitrary text playing the role of a 'term' (such as an ellipsis in a summed series) can be represented using an `mi` element, as shown in an example in section 3.2.5.4.

It should be stressed that `mi` is a presentation element, and as such, it only indicates that its content should be rendered as an identifier. In the majority of cases, the contents of an `mi` will actually represent a mathematical identifier such as a variable or function name. However, as the preceding paragraph indicates, the correspondence between notations that should render like identifiers and notations that are actually intended to represent mathematical identifiers is not perfect. For an element whose semantics is guaranteed to be that of an identifier, see the description of `ci` in chapter 4.

3.2.2.2 Attributes

`mi` elements accept the attributes listed in section 3.2.1, but in one case with a different default value:

Name	values	default
fontstyle	normal | italic	(depends on content; described below)

A typical graphical renderer would render an `mi` element as the characters in its content, with no extra spacing around the characters (except spacing associated with neighboring elements). The default `fontstyle` would (typically) be `normal` (non-slanted) unless the content is a single character, in which case it would be `italic`. Note that this rule for `fontstyle` is specific to `mi` elements; the default value for the `fontstyle` attribute of other MathML token elements is `normal`.

3.2.2.3 Examples

```
<mi> x </mi>
<mi> D </mi>
```

```
<mi> sin </mi>
<mi></mi>
```

An `mi` element with no content is allowed; `<mi></mi>` might, for example, be used by an 'expression editor' to represent a location in a MathML expression which requires a 'term' (according to conventional syntax for mathematics) but does not yet contain one.

Identifiers include function names such as 'sin'. Expressions such as 'sin *x*' should be written using the `&ApplyFunction;` operator (which also has the short name `&af;`) as shown below; see also the discussion of invisible operators in section 3.2.4.

```
<mrow>
   <mi> sin </mi>
   <mo> &ApplyFunction; </mo>
   <mi> x </mi>
</mrow>
```

Miscellaneous text that should be treated as a 'term' can also be represented by an `mi` element, as in:

```
<mrow>
   <mn> 1 </mn>
   <mo> + </mo>
   <mi> ... </mi>
   <mo> + </mo>
   <mi> n </mi>
</mrow>
```

When an `mi` is used in such exceptional situations, explicitly setting the `fontstyle` attribute may give better results than the default behavior of some renderers.

The names of symbolic constants should be represented as `mi` elements:

```
<mi> &pi; </mi>
<mi> &ImaginaryI; </mi>
<mi> &ExponentialE; </mi>
```

Use of special entity references for such constants can simplify the interpretation of MathML presentation elements. See chapter 6 for a complete list of character entity references in MathML.

3.2.3 Number (`mn`)

3.2.3.1 Description

An `mn` element represents a 'numeric literal' or other data that should be rendered as a numeric literal. Generally speaking, a numeric literal is a sequence of digits, perhaps including a decimal point, representing an unsigned integer or real number.

The concept of a mathematical 'number' depends on the context, and is not well-defined in the abstract. As a consequence, not all mathematical numbers should be represented using `mn`; examples of mathematical numbers that should be represented differently are shown below, and include negative numbers, complex numbers, ratios of numbers shown as fractions, and names of numeric constants.

Conversely, since mn is a presentation element, there are a few situations where it may desirable to include arbitrary text in the content of an mn that should merely render as a numeric literal, even though that content may not be unambiguously interpretable as a number according to any particular standard encoding of numbers as character sequences. As a general rule, however, the mn element should be reserved for situations where its content is actually intended to represent a numeric quantity in some fashion. For an element whose semantics are guaranteed to be that of a particular kind of mathematical number, see the description of cn in chapter 4.

3.2.3.2 Attributes

mn elements accept the attributes listed in section 3.2.1.

A typical graphical renderer would render an mn element as the characters of its content, with no extra spacing around them (except spacing from neighboring elements such as mo). Unlike mi, mn elements are (typically) rendered in an unslanted font by default, regardless of their content.

3.2.3.3 Examples

```
<mn> 2 </mn>
<mn> 0.123 </mn>
<mn> 1,000,000 </mn>
<mn> 2.1e10 </mn>
<mn> 0xFFEF </mn>
<mn> MCMLXIX </mn>
<mn> twenty one </mn>
```

3.2.3.4 Numbers that should not be written using *mn* alone

Many mathematical numbers should be represented using presentation elements other than mn alone; this includes negative numbers, complex numbers, ratios of numbers shown as fractions, and names of numeric constants. Examples of MathML representations of such numbers include:

```
<mrow> <mo> - </mo> <mn> 1 </mn> </mrow>
<mrow>
   <mn> 2 </mn>
   <mo> + </mo>
   <mrow>
      <mn> 3 </mn>
      <mo> &InvisibleTimes; </mo>
      <mi> &ImaginaryI; </mi>
   </mrow>
</mrow>
<mfrac> <mn> 1 </mn> <mn> 2 </mn> </mfrac>
<mi> &pi; </mi>
<mi> &ExponentialE; </mi>
```

3.2.4 Operator, Fence, Separator or Accent (mo)

3.2.4.1 Description

An mo element represents an operator or anything that should be rendered as an operator. In general, the notational conventions for mathematical operators are quite complicated, and therefore MathML provides a relatively sophisticated mechanism for specifying the rendering behavior of an mo element. As a consequence, in MathML the list of things that should 'render as an operator' includes a number of notations that are not mathematical operators in the ordinary sense. Besides ordinary operators with infix, prefix, or postfix forms, these include fence characters such as braces, parentheses, and 'absolute value' bars, separators such as comma and semicolon, and mathematical accents such as a bar or tilde over a symbol.

The term 'operator' as used in the present chapter means any symbol or notation that should render as an operator, and that is therefore representable by an mo element. That is, the term 'operator' includes any ordinary operator, fence, separator, or accent unless otherwise specified or clear from the context.

All such symbols are represented in MathML with mo elements since they are subject to essentially the same rendering attributes and rules; subtle distinctions in the rendering of these classes of symbols, when they exist, are supported using the boolean attributes fence, separator and accent, which can be used to distinguish these cases.

A key feature of the mo element is that its default attribute values are set on a case-by-case basis from an 'operator dictionary' as explained below. In particular, default values for fence, separator and accent can usually be found in the operator dictionary and therefore need not be specified on each mo element.

Note that some mathematical operators are represented not by mo elements alone, but by mo elements 'embellished' with (for example) surrounding superscripts; this is further described below. Conversely, as presentation elements, mo elements can contain arbitrary text, even when that text has no standard interpretation as an operator; for an example, see the discussion 'Mixing text and mathematics' in section 3.2.5. See also chapter 4 for definitions of MathML content elements that are guaranteed to have the semantics of specific mathematical operators.

3.2.4.2 Attributes

mo elements accept the attributes listed in section 3.2.1, and the additional attributes listed here. Most attributes get their default values from the section 3.2.4.7, as described later in this section. When a dictionary entry is not found for a given mo element, the default value shown here in parentheses is used.

h-unit represents a unit of horizontal length, and v-unit represents a unit of vertical length (see section 2.3.3.2). namedspace is one of veryverythinmathspace, verythinmathspace, thinmathspace, mediummathspace, thickmathspace, verythickmathspace, or veryverythickmathspace. These values are settable by the mstyle element which is discussed in section 3.3.4. The default values of veryverythinmathspace, ..., veryverythickmathspace are 1/18em...7/18em, respectively.

If no unit is given with maxsize or minsize, the number is a multiplier of the normal size of the operator in the direction (or directions) in which it stretches. These attributes are further explained below.

Name	values	default
form	prefix | infix | postfix	set by position of operator in an `mrow` (rule given b
fence	true | false	set by dictionary (false)
separator	true | false	set by dictionary (false)
lspace	number h-unit | namedspace	set by dictionary (thickmathspace)
rspace	number h-unit | namedspace	set by dictionary (thickmathspace)
stretchy	true | false	set by dictionary (false)
symmetric	true | false	set by dictionary (true)
maxsize	number [v-unit | h-unit] | namedspace | infinity	set by dictionary (infinity)
minsize	number [v-unit | h-unit] | namedspace	set by dictionary (1)
largeop	true | false	set by dictionary (false)
movablelimits	true | false	set by dictionary (false)
accent	true | false	set by dictionary (false)

Typical graphical renderers show all `mo` elements as the characters of their content, with additional spacing around the element determined from the attributes listed above. Detailed rules for determining operator spacing in visual renderings are described in a subsection below. As always, MathML does not require a specific rendering, and these rules are provided as suggestions for the convenience of implementors.

Renderers without access to complete fonts for the MathML character set may choose not to render an `mo` element as precisely the characters in its content in some cases. For example, `<mo> &le; </mo>` might be rendered as `<=` to a terminal. However, as a general rule, renderers should attempt to render the content of an `mo` element as literally as possible. That is, `<mo> &le </mo>` and `<mo> <= </mo>` should render differently. (The first one should render as a single extended character representing a less-than-or-equal-to sign, and the second one as the two-character sequence `<=`.)

3.2.4.3 Examples with ordinary operators

```
<mo> + </mo>
<mo> &lt; </mo>
<mo> &le; </mo>
<mo> &lt;= </mo>
<mo> ++ </mo>
<mo> &sum; </mo>
<mo> .NOT. </mo>
<mo> and </mo>
<mo> &InvisibleTimes; </mo>
```

3.2.4.4 Examples with fences and separators

Note that the `mo` elements in these examples don't need explicit `fence` or `separator` attributes, since these can be found using the operator dictionary as described below. Some of these examples could also be encoded using the `mfenced` element described in section 3.3.8.

(*a*+*b*)

```
<mrow>
```

```
   <mo> ( </mo>
   <mrow>
      <mi> a </mi>
      <mo> + </mo>
      <mi> b </mi>
   </mrow>
   <mo> ) </mo>
</mrow>
```

[0,1)

```
<mrow>
   <mo> [ </mo>
   <mrow>
      <mn> 0 </mn>
      <mo> , </mo>
      <mn> 1 </mn>
   </mrow>
   <mo> ) </mo>
</mrow>
```

$f(x,y)$

```
<mrow>
   <mi> f </mi>
   <mo> &ApplyFunction; </mo>
   <mrow>
      <mo> ( </mo>
      <mrow>
         <mi> x </mi>
         <mo> , </mo>
         <mi> y </mi>
      </mrow>
      <mo> ) </mo>
   </mrow>
</mrow>
```

3.2.4.5 Invisible operators

Certain operators that are 'invisible' in traditional mathematical notation should be represented using specific entity references within `mo` elements, rather than simply by nothing. The entity references used for these 'invisible operators' are:

Full name	Short name	Examples of use
`&InvisibleTimes;`	`&it;`	xy
`&ApplyFunction;`	`&af;`	$f(x) \sin x$
`&InvisibleComma;`	`&ic;`	m_{12}

The MathML representations of the examples in the above table are:

```
<mrow>
   <mi> x </mi>
   <mo> &InvisibleTimes; </mo>
   <mi> y </mi>
</mrow>
<mrow>
   <mi> f </mi>
   <mo> &ApplyFunction; </mo>
   <mrow>
      <mo> ( </mo>
      <mi> x </mi>
      <mo> ) </mo>
   </mrow>
</mrow>
<mrow>
   <mi> sin </mi>
   <mo> &ApplyFunction; </mo>
   <mi> x </mi>
</mrow>
<msub>
   <mi> m </mi>
   <mrow>
      <mn> 1 </mn>
      <mo> &InvisibleComma; </mo>
      <mn> 2 </mn>
   </mrow>
</msub>
```

The reasons for using specific `mo` elements for invisible operators include:

- such operators should often have specific effects on visual rendering (particularly spacing and linebreaking rules) that are not the same as either the lack of any operator, or spacing represented by `<mspace/>` or `mtext` elements;
- these operators should often have specific audio renderings different than that of the lack of any operator;
- automatic semantic interpretation of MathML presentation elements is made easier by the explicit specification of such operators.

For example, an audio renderer might render $f(x)$ (represented as in the above examples) by speaking 'f of x', but use the word 'times' in its rendering of xy. Although its rendering must still be different depending on the structure of neighboring elements (sometimes leaving out 'of' or 'times' entirely), its task is made much easier by the use of a different `mo` element for each invisible operator.

3.2.4.6 Entity references for other special operators

MathML also includes `&DifferentialD;` for use in an `mo` element representing the differential operator symbol usually denoted by 'd'. The reasons for explicitly using this special entity are similar to those for using the special entities for invisible operators described in the preceding section.

3.2.4.7 Detailed rendering rules for *mo* elements

Typical visual rendering behaviors for mo elements are more complex than for the other MathML token elements, so the rules for rendering them are described in this separate subsection.

Note that, like all rendering rules in MathML, these rules are suggestions rather than requirements. Furthermore, no attempt is made to specify the rendering completely; rather, enough information is given to make the intended effect of the various rendering attributes as clear as possible.

The operator dictionary

Many mathematical symbols, such as an integral sign, a plus sign, or a parenthesis, have a well-established, predictable, traditional notational usage. Typically, this usage amounts to certain default attribute values for mo elements with specific contents and a specific form attribute. Since these defaults vary from symbol to symbol, MathML anticipates that renderers will have an 'operator dictionary' of default attributes for mo elements (see appendix D) indexed by each mo element's content and form attribute. If an mo element is not listed in the dictionary, the default values shown in parentheses in the table of attributes for mo should be used, since these values are typically acceptable for a generic operator.

Some operators are 'overloaded', in the sense that they can occur in more than one form (prefix, infix, or postfix), with possibly different rendering properties for each form. For example, '+' can be either a prefix or an infix operator. Typically, a visual renderer would add space around both sides of an infix operator, while only on the left of a prefix operator. The form attribute allows specification of which form to use, in case more than one form is possible according to the operator dictionary and the default value described below is not suitable.

Default value of the *form* attribute

The form attribute does not usually have to be specified explicitly, since there are effective heuristic rules for inferring the value of the form attribute from the context. If it is not specified, and there is more than one possible form in the dictionary for an mo element with given content, the renderer should choose which form to use as follows (but see the exception for embellished operators, described later):

- If the operator is the first argument in an mrow of length (i.e. number of arguments) greater than one (ignoring all space-like arguments (see section 3.2.6) in the determination of both the length and the first argument), the prefix form is used;
- if it is the last argument in an mrow of length greater than one (ignoring all space-like arguments), the postfix form is used;
- in all other cases, including when the operator is not part of an mrow, the infix form is used.

Note that these rules make reference to the mrow in which the mo element lies. In some situations, this mrow might be an inferred mrow implicitly present around the arguments of an element such as msqrt or mtd

Opening (left) fences should have form="prefix", and closing (right) fences should have form="postfix"; separators are usually 'infix', but not always, depending on their surroundings. As with ordinary operators, these values do not usually need to be specified explicitly.

If the operator does not occur in the dictionary with the specified form, the renderer should use one of the forms that is available there, in the order of preference: infix, postfix, prefix; if no forms are available for the given `mo` element content, the renderer should use the defaults given in parentheses in the table of attributes for `mo`.

Exception for embellished operators

There is one exception to the above rules for choosing an `mo` element's default `form` attribute. An `mo` element that is 'embellished' by one or more nested subscripts, superscripts, surrounding text or whitespace, or style changes behaves differently. It is the embellished operator as a whole (this is defined precisely, below) whose position in an `mrow` is examined by the above rules and whose surrounding spacing is affected by its form, not the `mo` element at its core; however, the attributes influencing this surrounding spacing are taken from the `mo` element at the core (or from that element's dictionary entry).

For example, the '$+_4$' in $a+_4 b$ should be considered an infix operator as a whole, due to its position in the middle of an `mrow`, but its rendering attributes should be taken from the `mo` element representing the '+', or when those are not specified explicitly, from the operator dictionary entry for `<mo form="infix"> + </mo>`. The precise definition of an 'embellished operator' is:

- an `mo` element;
- or one of the elements `msub`, `msup`, `msubsup`, `munder`, `mover`, `munderover`, `mmultiscripts`, `mfrac`, or `semantics` (section 4.2.6), whose first argument exists and is an embellished operator;
- or one of the elements `mstyle`, `mphantom`, or `mpadded`, such that an `mrow` containing the same arguments would be an embellished operator;
- or an `maction` element whose selected sub-expression exists and is an embellished operator;
- or an `mrow` whose arguments consist (in any order) of one embellished operator and zero or more space-like elements.

Note that this definition permits nested embellishment only when there are no intervening enclosing elements not in the above list.

The above rules for choosing operator forms and defining embellished operators are chosen so that in all ordinary cases it will not be necessary for the author to specify a `form` attribute.

Rationale for definition of embellished operators

The following notes are included as a rationale for certain aspects of the above definitions, but should not be important for most users of MathML.

An `mfrac` is included as an 'embellisher' because of the common notation for a differential operator:

```
<mfrac>
   <mo> &DifferentialD; </mo>
   <mrow>
      <mo> &DifferentialD; </mo>
      <mi> x </mi>
   </mrow>
</mfrac>
```

Since the definition of embellished operator affects the use of the attributes related to stretching, it is important that it includes embellished fences as well as ordinary operators; thus it applies to any `mo` element.

Note that an `mrow` containing a single argument is an embellished operator if and only if its argument is an embellished operator. This is because an `mrow` with a single argument must be equivalent in all respects to that argument alone (as discussed in section 3.3.1). This means that an `mo` element that is the sole argument of an `mrow` will determine its default `form` attribute based on that `mrow`'s position in a surrounding, perhaps inferred, `mrow` (if there is one), rather than based on its own position in the `mrow` it is the sole argument of.

Note that the above definition defines every `mo` element to be 'embellished' - that is, 'embellished operator' can be considered (and implemented in renderers) as a special class of MathML expressions, of which `mo` is a specific case.

Spacing around an operator

The amount of space added around an operator (or embellished operator), when it occurs in an `mrow`, can be directly specified by the `lspace` and `rspace` attributes. These values are in ems if no units are given. By convention, operators that tend to bind tightly to their arguments have smaller values for spacing than operators that tend to bind less tightly. This convention should be followed in the operator dictionary included with a MathML renderer. In TEX, these values can only be one of three values; typically they are 3/18em, 4/18em, and 5/18em. MathML does not impose this limit.

Some renderers may choose to use no space around most operators appearing within subscripts or superscripts, as is done in TEX.

Non-graphical renderers should treat spacing attributes, and other rendering attributes described here, in analogous ways for their rendering medium.

3.2.4.8 Stretching of operators, fences and accents

Four attributes govern whether and how an operator (perhaps embellished) stretches so that it matches the size of other elements: `stretchy`, `symmetric`, `maxsize`, and `minsize`. If an operator has the attribute `stretchy="true"`, then it (that is, each character in its content) obeys the stretching rules listed below, given the constraints imposed by the fonts and font rendering system. In practice, typical renderers will only be able to stretch a small set of characters, and quite possibly will only be able to generate a discrete set of character sizes.

There is no provision in MathML for specifying in which direction (horizontal or vertical) to stretch a specific character or operator; rather, when `stretchy="true"` it should be stretched in each direction for which stretching is possible. It is up to the renderer to know in which directions it is able to stretch each character. (Most characters can be stretched in at most one direction by typical renderers, but some renderers may be able to stretch certain characters, such as diagonal arrows, in both directions independently.)

The `minsize` and `maxsize` attributes limit the amount of stretching (in either direction). These two attributes are given as multipliers of the operator's normal size in the direction or directions of stretching, or as absolute sizes using units. For example, if a character has `maxsize="3"`, then it can grow to be no more than three times its normal (unstretched) size.

The `symmetric` attribute governs whether the height and depth above and below the axis of the character are forced to be equal (by forcing both height and depth to become the

maximum of the two). An example of a situation where one might set `symmetric=false` arises with parentheses around a matrix not aligned on the axis, which frequently occurs when multiplying non-square matrices. In this case, one wants the parentheses to stretch to cover the matrix, whereas stretching the parentheses symmetrically would cause them to protrude beyond one edge of the matrix. The `symmetric` attribute only applies to characters that stretch vertically (otherwise it is ignored).

If a stretchy `mo` element is embellished (as defined earlier in this section), the `mo` element at its core is stretched to a size based on the context of the embellished operator as a whole, i.e. to the same size as if the embellishments were not present. For example, the parentheses in the following example (which would typically be set to be stretchy by the operator dictionary) will be stretched to the same size as each other, and the same size they would have if they were not underlined and overlined, and furthermore will cover the same vertical interval:

```
<mrow>
   <munder>
      <mo> ( </mo>
      <mo> &UnderBar; </mo>
   </munder>
   <mfrac>
      <mi> a </mi>
      <mi> b </mi>
   </mfrac>
   <mover>
      <mo> ) </mo>
      <mo> &OverBar; </mo>
   </mover>
</mrow>
```

Note that this means that the stretching rules given below must refer to the context of the embellished operator as a whole, not just to the `mo` element itself.

Example of stretchy attributes

This shows one way to set the maximum size of a parenthesis so that it does not grow, even though its default value is `stretchy=true`

```
<mrow>
   <mo maxsize="1"> ( </mo>
   <mfrac>
      <mi> a </mi> <mi> b </mi>
   </mfrac>
   <mo maxsize="1"> ) </mo>
</mrow>
```

The above should render as $(\frac{a}{b})$ as opposed to the default rendering $\left(\frac{a}{b}\right)$.

Note that each parenthesis is sized independently; if only one of them had `maxsize="1"`, they would render with different sizes.

Vertical Stretching Rules

- If a stretchy operator is a direct sub-expression of an `mrow` element, or is the sole direct sub-expression of an `mtd` element in some row of a table, then it should stretch to cover the height and depth (above and below the `axis`) of the non-stretchy direct sub-expressions in the `mrow` element or table row, unless stretching is constrained by `minsize` or `maxsize` attributes.
- In the case of an embellished stretchy operator, the preceding rule applies to the stretchy operator at its core.
- If `symmetric=true`, then the maximum of the height and depth is used to determine the size, before application of the `minsize` or `maxsize` attributes.
- The preceding rules also apply in situations where the `mrow` element is inferred.

Most common opening and closing fences are defined in the operator dictionary to stretch by default; and they stretch vertically. Also, operators such as `&sum;` `&int;`, /, and vertical arrows stretch vertically by default.

In the case of a stretchy operator in a table cell (i.e. within an `mtd` element), the above rules assume each cell of the table row containing the stretchy operator covers exactly one row. (Equivalently, the value of the `rowspan` attribute is assumed to be 1 for all the table cells in the table row, including the cell containing the operator.) When this is not the case, the operator should only be stretched vertically to cover those table cells that are entirely within the set of table rows that the operator's cell covers. Table cells that extend into rows not covered by the stretchy operator's table cell should be ignored. See section 3.5.4.2 for details about the the `rowspan` attribute.

Horizontal Stretching Rules

- If a stretchy operator, or an embellished stretchy operator, is a direct sub-expression of an `munder`, `mover`, or `munderover` element, or if it is the sole direct sub-expression of an `mtd` element in some column of a table (see `mtable`), then it, or the `mo` element at its core, should stretch to cover the width of the other direct sub-expressions in the given element (or in the same table column), given the constraints mentioned above.
- If a stretchy operator is a direct sub-expression of an `munder`, `mover`, or `munderover` element, or if it is the sole direct sub-expression of an `mtd` element in some column of a table, then it should stretch to cover the width of the other direct sub-expressions in the given element (or in the same table column), given the constraints mentioned above.
- In the case of an embellished stretchy operator, the preceding rule applies to the stretchy operator at its core.

By default, most horizontal arrows and some accents stretch horizontally.

In the case of a stretchy operator in a table cell (i.e. within an `mtd` element), the above rules assume each cell of the table column containing the stretchy operator covers exactly one column. (Equivalently, the value of the `columnspan` attribute is assumed to be 1 for all the table cells in the table row, including the cell containing the operator.) When this is not the case, the operator should only be stretched horizontally to cover those table cells that are entirely within the set of table columns that the operator's cell covers. Table cells that extend into columns not covered by the stretchy operator's table cell should be ignored. See section 3.5.4.2 for details about the the `rowspan` attribute.

The rules for horizontal stretching include `mtd` elements to allow arrows to stretch for use in commutative diagrams laid out using `mtable` The rules for the horizontal stretchiness include scripts to make examples such as the following work:

```
<mrow>
   <mi> x </mi>
   <munder>
      <mo> &RightArrow; </mo>
      <mtext> maps to </mtext>
   </munder>
   <mi> y </mi>
</mrow>
```

This displays as $x \underset{\text{maps to}}{\longrightarrow} y$.

Rules Common to both Vertical and Horizontal Stretching

If a stretchy operator is not required to stretch (i.e. if it is not in one of the locations mentioned above, or if there are no other expressions whose size it should stretch to match), then it has the standard (unstretched) size determined by the font and current fontsize.

If a stretchy operator is required to stretch, but all other expressions in the containing element or object (as described above) are also stretchy, all elements that can stretch should grow to the maximum of the normal unstretched sizes of all elements in the containing object, if they can grow that large. If the value of `minsize` or `maxsize` prevents this then that (min or max) size is used.

For example, in an `mrow` containing nothing but vertically stretchy operators, each of the operators should stretch to the maximum of all of their normal unstretched sizes, provided no other attributes are set that override this behavior. Of course, limitations in fonts or font rendering may result in the final, stretched sizes being only approximately the same.

3.2.4.9 Other attributes of *mo*

The `largeop` attribute specifies whether the operator should be drawn larger than normal if `displaystyle="true"` in the current rendering environment. This roughly corresponds to TEX's `\displaystyle` style setting. MathML uses two attributes, `displaystyle` and `scriptlevel`, to control orthogonal presentation features that TEX encodes into one 'style' attribute with values `\displaystyle`, `\textstyle`, `\scriptstyle`, and `\scriptscriptstyle`. These attributes are discussed further in section 3.3.4 describing the `mstyle` element. Note that these attributes can be specified directly on an `mstyle` element's begin tag, but not on most other elements. Examples of large operators include `&int;` and `&prod;`

The `movablelimits` attribute specifies whether underscripts and overscripts attached to this `mo` element should be drawn as subscripts and superscripts when `displaystyle="false"`. `movablelimits="false"` means that underscripts and overscripts should never be drawn as subscripts and superscripts. In general, `displaystyle` is `true` for displayed mathematics and `false` for inline mathematics. Also, `displaystyle` is `false` by default within tables, scripts and fractions, and a few other exceptional situations detailed in section 3.3.4. Thus, operators with `movablelimits="true"` will display with limits (i.e. underscripts and overscripts) in displayed mathematics, and with subscripts and superscripts in inline mathematics, tables, scripts and so on. Examples of operators that typically have `movablelimits="true"` are `sum`, `prod`, and `lim`.

The `accent` attribute determines whether this operator should be treated by default as an accent (diacritical mark) when used as an underscript or overscript; see `munder`, `mover`, and `munderover` (section 3.4.4, section 3.4.5 and section 3.4.6).

The `separator` attribute may affect automatic linebreaking in renderers that position ordinary infix operators at the beginnings of broken lines rather than at the ends (that is, which avoid linebreaking just after such operators), since linebreaking should be avoided just before separators, but is acceptable just after them.

The `fence` attribute has no effect in the suggested visual rendering rules given here; it is not needed for properly rendering traditional notation using these rules. It is provided so that specific MathML renderers, especially non-visual renderers, have the option of using this information.

3.2.5 Text (`mtext`)

3.2.5.1 Description

An `mtext` element is used to represent arbitrary text that should be rendered as itself. In general, the `mtext` element is intended to denote commentary text that is not central to the mathematical meaning or notational structure of the expression it is contained in.

Note that some text with a clearly defined notational role might be more appropriately marked up using `mi` or `mo`; this is discussed further below.

An `mtext` element can be used to contain 'renderable whitespace', i.e. invisible characters that are intended to alter the positioning of surrounding elements. In non-graphical media, such characters are intended to have an analogous effect, such as introducing positive or negative time delays or affecting rhythm in an audio renderer. This is not related to any whitespace in the source MathML consisting of blanks, newlines, tabs, or carriage returns; whitespace present directly in the source is trimmed and collapsed, as described in section 2.3.5. Whitespace that is intended to be rendered as part of an element's content must be represented by entity references (unless it consists only of single blanks between non-whitespace characters).

Renderable whitespace can have a positive or negative width, as in ` ` and `&NegativeThinSpace;`, or zero width, as in `&ZeroWidthSpace;`. The complete list of such characters is given in chapter 6. Note that there is no formal distinction in MathML between renderable whitespace characters and any other class of characters, in `mtext` or in any other element.

Renderable whitespace can also include characters that affect alignment or linebreaking. Some of these characters are:

Entity name	Purpose (rough description)
NewLine	start a new line and do not indent
IndentingNewLine	start a new line and do indent
NoBreak	do not allow a linebreak here
GoodBreak	if a linebreak is needed on the line, here is a good spot
BadBreak	if a linebreak is needed on the line, try to avoid breaking here

For the complete list of MathML entities, consult chapter 6.

3.2.5.2 Attributes

`mtext` elements accept the attributes listed in section 3.2.1.

See also the warnings about the legal grouping of 'space-like elements' in section 3.2.6, and about the use of such elements for 'tweaking' or conveying meaning in section 3.3.6.

3.2.5.3 Examples

```
<mtext> Theorem 1: </mtext>
<mtext>   </mtext>
<mtext>    </mtext>
<mtext> /* a comment */ </mtext>
```

3.2.5.4 Mixing text and mathematics

In some cases, text embedded in mathematics could be more appropriately represented using `mo` or `mi` elements. For example, the expression 'there exists $\delta > 0$ such that $f(x)$ <1' is equivalent to $\exists \delta > 0 \ni f(x) < 1$ and could be represented as:

```
<mrow>
   <mo> there exists </mo>
   <mrow>
      <mrow>
         <mi> &delta; </mi>
         <mo> &gt; </mo>
         <mn> 0 </mn>
      </mrow>
      <mo> such that </mo>
      <mrow>
         <mrow>
            <mi> f </mi>
            <mo> &ApplyFunction; </mo>
            <mrow>
               <mo> ( </mo>
               <mi> x </mi>
               <mo> ) </mo>
            </mrow>
         </mrow>
         <mo> &lt; </mo>
         <mn> 1 </mn>
      </mrow>
   </mrow>
</mrow>
```

An example involving an `mi` element is: $x+x^2+\cdots+x^n$. In this example, ellipsis should be represented using an `mi` element, since it takes the place of a term in the sum (see section 3.2.2, `mi`).

On the other hand, expository text within MathML is best represented with an `mtext` element. An example of this is:

Theorem 1: if $x > 1$, then $x^2 > x$.

However, when MathML is embedded in HTML, the example is probably best rendered with only the two inequalities represented as MathML at all, letting the text be part of the surrounding HTML.

Another factor to consider in deciding how to mark up text is the effect on rendering. Text enclosed in an `mo` element is unlikely to be found in a renderer's operator dictionary, so it will be rendered with the format and spacing appropriate for an 'unrecognized operator', which may or may not be better than the format and spacing for 'text' obtained by using an `mtext` element. An ellipsis entity in an `mi` element is apt to be spaced more appropriately for taking the place of a term within a series than if it appeared in an `mtext` element.

3.2.6 Space (`mspace`)

3.2.6.1 Description

An `mspace` empty element represents a blank space of any desired size, as set by its attributes. The default value for each attribute is `0em` or `0ex`, so it will not be useful without some attributes specified.

3.2.6.2 Attributes

Name	values	default
width	number h-unit | namedspace	0em
height	number v-unit	0ex
depth	number v-unit	0ex

`h-unit` and `v-unit` represent units of horizontal or vertical length, respectively (see section 2.3.3.2).

Note the warning about the legal grouping of 'space-like elements' given below, and the warning about the use of such elements for 'tweaking' or conveying meaning in section 3.3.6. See also the other elements that can render as whitespace, namely `mtext`, `mphantom`, and `maligngroup`.

3.2.6.3 Definition of space-like elements

A number of MathML presentation elements are 'space-like' in the sense that they typically render as whitespace, and do not affect the mathematical meaning of the expressions in which they appear. As a consequence, these elements often function in somewhat exceptional ways in other MathML expressions. For example, space-like elements are handled specially in the suggested rendering rules for `mo` given in section 3.2.4. The following MathML elements are defined to be 'space-like':

- an `mtext`, `mspace`, `maligngroup`, or `malignmark` element;
- an `mstyle`, `emphantom`, or `mpadded` element, all of whose direct sub-expressions are space-like;
- an `maction` element whose selected sub-expression exists and is space-like;
- an `mrow` all of whose direct sub-expressions are space-like.

Note that an `mphantom` is not automatically defined to be space-like, unless its content is space-like. This is because operator spacing is affected by whether adjacent elements are space-like. Since the `mphantom` element is primarily intended as an aid in aligning expressions, operators adjacent to an `mphantom` should behave as if they were adjacent to the contents of the `mphantom`, rather than to an equivalently sized area of whitespace.

3.2.6.4 Legal grouping of space-like elements

Authors who insert space-like elements or `mphantom` elements into an existing MathML expression should note that such elements are counted as arguments, in elements that require a specific number of arguments, or that interpret different argument positions differently.

Therefore, space-like elements inserted into such a MathML element should be grouped with a neighboring argument of that element by introducing an `mrow` for that purpose. For example, to allow for vertical alignment on the right edge of the base of a superscript, the expression

```
<msup> <mi> x </mi> <malignmark edge="right"/> <mn> 2 </mn> </msup>
```

is illegal, because `msup` must have exactly 2 arguments; the correct expression would be:

```
<msup>
   <mrow>
      <mi> x </mi>
      <malignmark edge="right"/>
   </mrow>
   <mn> 2 </mn>
</msup>
```

See also the warning about ‘tweaking’ in section 3.3.6.

3.2.7 String Literal (`ms`)

3.2.7.1 Description

The `ms` element is used to represent ‘string literals’ in expressions meant to be interpreted by computer algebra systems or other systems containing ‘programming languages’. By default, string literals are displayed surrounded by double quotes. As explained in section 3.2.5, ordinary text embedded in a mathematical expression should be marked up with `mtext`, or in some cases `mo` or `mi`, but never with `ms`.

Note that the string literals encoded by `ms` are ‘Unicode strings’ rather than ‘ASCII strings’. In practice, non-ASCII characters will typically be represented by entity references. For example, `<ms>&amp;</ms>` represents a string literal containing a single character, &, and `<ms>&amp;amp;</ms>` represents a string literal containing 5 characters, the first one of which is &. (In fact, MathML string literals are even more general than Unicode string literals, since not all MathML entity references necessarily refer to existing Unicode characters, as discussed in chapter 6.)

Like all token elements, `ms` does trim and collapse whitespace in its content according to the rules of section 2.3.5, so whitespace intended to remain in the content should be encoded as described in that section.

3.2.7.2 Attributes

`ms` elements accept the attributes listed in section 3.2.1, and additionally:

Name	values	default
lquote	string	`&quot;`
rquote	string	`&quot;`

In visual renderers, the content of an `ms` element is typically rendered with no extra spacing added around the string, and a quote character at the beginning and the end of the string. By default, the left and right quote characters are both the standard double quote character `&quot;` However, these characters can be changed with the `lquote` and `rquote` attributes respectively.

The content of `ms` elements should be rendered with visible 'escaping' of certain characters in the content, including at least 'double quote' itself, and preferably whitespace other than individual blanks. The intent is for the viewer to see that the expression is a string literal, and to see exactly which characters form its content. For example, `<ms>double quote is "</ms>` might be rendered as 'double quote is \"'.

3.2.8 Refering to non-ASCII characters (`mchar`)

3.2.8.1 Description

The `mchar` element is used to reference characters. This provides an alternative to using entity references. Character entities are deprecated for MathML 2.0 because they are not a part of the current proposal for schemas, and documents containing entities are not well-formed MathML in the absence of the MathML DTD.

Numeric character references (e.g. `&#1234;`) are not deprecated because they do not have the problems listed above.

`mchar` is valid content in any MathML token element listed in section 3.1.5 (`mi`, etc.) or section 4.2.2 (`ci`, etc.) unless otherwise restricted by an attribute (e.g. `base`=2 to `<cn>`).

3.2.8.2 Attributes of *`mchar`*

Name	values	default
name	string	required

The `name` attribute must be one of the names specified in chapter 6. It is an error to use a name that is not in that list.

Issue (specific-xref): The cross-reference above should be made more specific.

3.2.8.3 Examples

In MathML 1.x expressions involving entity references such as <mi> α1 </mi> were common. In MathML 2.0, the equivalent construction using `mchar` is preferred:

```
<mi> <mchar name='alpha'/>1 </mi>
```

3.2.9 Adding new character glyphs to MathML (`mglyph`)

3.2.9.1 Description

Unicode defines a large number of characters used in mathematics, and in most all cases, glyphs representing these characters are widely available in a variety of fonts. Although these characters should meet almost all users needs, MathML recognizes that Mathematics is not static and that new characters are added when convenient. Characters that become well accepted will likely be eventually incorporated by the Unicode Consortium or other standards bodies, but that is often a lengthy process. In the mean time, a mechanism is necessary for accessing glyphs from non-standard fonts representing these characters.

The `mglyph` element is the means by which users can directly access glyphs for characters that are not defined by Unicode. Similarly, the `mglyph` element can also be used to select glyph variants for existing Unicode characters, as might be desirable when a glyph variant has begun to differentiate itself as a new character by taking on a distinguished mathematical meaning.

The `mglyph` element names a specific character glyph, and is valid inside any MathML leaf content listed in section 3.1.5 (`mi`, etc.) or section 4.2.2 (`ci`, etc.) unless otherwise restricted by an attribute (e.g. `base`=2 to `<cn>`). In order for a visually-oriented renderer to render the character, the renderer must be told what font to use and what index within that font to use.

3.2.9.2 Attributes

Name	values	default
alt	string	required
fontfamily	string | css-fontfamily	required
index	integer	required

The `alt` attribute provides an alternate name for the glyph. If the specified font can't be found, the renderer may use this name in a warning message or some unknown glyph notation. The name might also be used by an audio renderer or symbol processing system and should be chosen to be descriptive. The `fontfamily` and `index` uniquely identify the `mglyph`; two `mglyph`s with the same values for `fontfamily` and `index` should be considered identical by applications that must determine whether two characters/glyphs are identical. The `alt` attribute should not be part of the identity test.

The `fontfamily` and `index` attributes name a font and position within that font. All font properties apart from `fontfamily` are inherited. Variants of the font (e.g., bold) that may be inherited may be ignored if the variant of the font is not present.

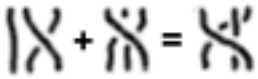
Authors should be aware that rendering requires the fonts referenced by `mglyph`, which the MathML renderer may not have access to or may be not be supported by the system on which the renderer runs. For these reasons, authors are encouraged to use `mglyph` only when absolutely necessary, and not for stylistic purposes.

3.2.9.3 Example

The following example illustrates how a researcher might use the `mglyph` construct with an experimental font to work with braid group notation.

```
<mrow>
  <mi><mglyph fontfamily="my-braid-font" index="2" alt="23braid"></mi>
  <mo>+</mo>
  <mi><mglyph fontfamily="my-braid-font" index="5" alt="132braid"></mi>
  <mo>=</mo>
  <mi><mglyph fontfamily="my-braid-font" index="3" alt="13braid"></mi>
```

This might render as:



3.3 General Layout Schemata

Besides tokens there are several families of MathML presentation elements. One family of elements deals with various ‘scripting’ notations, such as subscript and superscript. Another family is concerned with matrices and tables. The remainder of the elements, discussed in this section, describe other basic notations such as fractions and radicals, or deal with general functions such as setting style properties and error handling.

3.3.1 Horizontally Group Sub-Expressions (`mrow`)

3.3.1.1 Description

An `mrow` element is used to group together any number of sub-expressions, usually consisting of one or more `mo` elements acting as ‘operators’ on one or more other expressions that are their ‘operands’.

Several elements automatically treat their arguments as if they were contained in an `mrow` element. See the discussion of inferred `mrow`s in section 3.1.3. See also `mfenced` (section 3.3.8), which can effectively form an `mrow` containing its arguments separated by commas.

3.3.1.2 Attributes

None (except the attributes allowed for all MathML elements, listed in section 2.3.4).

`mrow` elements are typically rendered visually as a horizontal row of their arguments, left to right in the order in which the arguments occur, or audibly as a sequence of renderings of the arguments. The description in section 3.2.4 of suggested rendering rules for `mo` elements assumes that all horizontal spacing between operators and their operands is added by the rendering of `mo` elements (or, more generally, embellished operators), not by the rendering of the `mrow`s they are contained in.

MathML is designed to allow renderers to automatically linebreak expressions (that is, to break excessively long expressions into several lines), without requiring authors to specify explicitly how this should be done. This is because linebreaking positions can’t be chosen well without knowing the width of the display device and the current font size, which for many uses of MathML will not be known except by the renderer at the time of each rendering.

Determining good positions for linebreaks is complex, and rules for this are not described here; whether and how it is done is up to each MathML renderer. Typically, linebreaking will involve selection of 'good' points for insertion of linebreaks between successive arguments of `mrow` elements.

Although MathML does not require linebreaking or specify a particular linebreaking algorithm, it has several features designed to allow such algorithms to produce good results. These include the use of special entities for certain operators, including invisible operators (see section 3.2.4), or for providing hints related to linebreaking when necessary (see section 3.2.5), and the ability to use nested `mrow`s to describe sub-expression structure (see below).

mrow of one argument

MathML renderers are required to treat an `mrow` element containing exactly one argument as equivalent in all ways to the single argument occurring alone, provided there are no attributes on the `mrow` element's begin tag. If there are attributes on the `mrow` element's begin tag, no requirement of equivalence is imposed. This equivalence condition is intended to simplify the implementation of MathML-generating software such as template-based authoring tools. It directly affects the definitions of embellished operator and space-like element and the rules for determining the default value of the `form` attribute of an `mo` element; see sections section 3.2.4 and section 3.2.6. See also the discussion of equivalence of MathML expressions in chapter 7.

3.3.1.3 Proper grouping of sub-expressions using *mrow*

Sub-Expressions should be grouped by the document author in the same way as they are grouped in the mathematical interpretation of the expression; that is, according to the underlying 'syntax tree' of the expression. Specifically, operators and their mathematical arguments should occur in a single `mrow`; more than one operator should occur directly in one `mrow` only when they can be considered (in a syntactic sense) to act together on the interleaved arguments, e.g. for a single parenthesized term and its parentheses, for chains of relational operators, or for sequences of terms separated by `+` and `-`. A precise rule is given below.

Proper grouping has several purposes: it improves display by possibly affecting spacing; it allows for more intelligent linebreaking and indentation; and it simplifies possible semantic interpretation of presentation elements by computer algebra systems, and audio renderers.

Although improper grouping will sometimes result in suboptimal renderings, and will often make interpretation other than pure visual rendering difficult or impossible, any grouping of expressions using `mrow`s is allowed in MathML syntax; that is, renderers should not assume the rules for proper grouping will be followed.

Precise rule for proper grouping

A precise rule for when and how to nest sub-expressions using `mrow`s is especially desirable when generating MathML automatically by conversion from other formats for displayed mathematics, such as TEX, which don't always specify how sub-expressions nest. When a precise rule for grouping is desired, the following rule should be used:

Two adjacent operators (i.e. `mo` elements, possibly embellished), possibly separated by operands (i.e. anything other than operators), should occur in the same `mrow` only when

the left operator has an infix or prefix form (perhaps inferred), the right operator has an infix or postfix form, and the operators are listed in the same group of entries in the operator dictionary provided in appendix D. In all other cases, nested `mrow`s should be used.

When forming a nested `mrow`(during generation of MathML) that includes just one of two successive operators with the forms mentioned above (which mean that either operator could in principle act on the intervening operand or operands), it is necessary to decide which operator acts on those operands directly (or would do so, if they were present). Ideally, this should be determined from the original expression; for example, in conversion from an operator-precedence-based format, it would be the operator with the higher precedence. If this cannot be determined directly from the original expression, the operator that occurs later in the suggested operator dictionary (appendix D) can be assumed to have a higher precedence for this purpose.

Note that the above rule has no effect on whether any MathML expression is valid, only on the recommended way of generating MathML from other formats for displayed mathematics or directly from written notation.

(Some of the terminology used in stating the above rule in defined in section 3.2.4.)

3.3.1.4 Examples

As an example, $2x+y+x$ should be written as:

```
<mrow>
  <mrow>
     <mn> 2 </mn>
     <mo> &InvisibleTimes; </mo>
     <mi> x </mi>
  </mrow>
  <mo> + </mo>
  <mi> y </mi>
  <mo> - </mo>
  <mi> z </mi>
</mrow>
```

The proper encoding of (x, y) furnishes a less obvious example of nesting `mrow`s:

```
<mrow>
   <mo> ( </mo>
   <mrow>
      <mi> x </mi>
      <mo> , </mo>
      <mi> y </mi>
   </mrow>
   <mo> ) </mo>
</mrow>
```

In this case, a nested `mrow`is required inside the parentheses, since parentheses and commas, thought of as fence and separator ‘operators’, do not act together on their arguments.

3.3.2 Fractions (`mfrac`)

3.3.2.1 Description

The `mfrac` element is used for fractions. It can also be used to mark up fraction-like objects such as binomial coefficients and Legendre symbols. The syntax for `mfrac` is

```
<mfrac>{numerator}{denominator} </mfrac>
```

3.3.2.2 Attributes of *`mfrac`*

Name	values	default
linethickness	number [v-unit] | thin | medium | thick	1 (rule thickness)
numalign	left | center | right	center
denomalign	left | center | right	center
beveled	true | false	false

The `linethickness` attribute indicates the thickness of the horizontal 'fraction bar', or 'rule', typically used to render fractions. A fraction with `linethickness="0"` renders without the bar, and might be used within binomial coefficients. A `linethickness` greater than one might be used with nested fractions. These cases are shown below:

$$\frac{\binom{a}{b} \quad \frac{a}{b}}{\frac{c}{d}}$$

In general, the value of `linethickness` can be a number, as a multiplier of the default thickness of the fraction bar (the default thickness is not specified by MathML), or a number with a unit of vertical length (see section 2.3.3.2), or one of the keywords `medium` (same as 1), `thin` (thinner than 1, otherwise up to the renderer), or `thick` (thicker than 1, otherwise up to the renderer).

The `numalign` and `denomalign` attributes control the horizontal alignment of the numerator and denominator respectively. Typically, numerators and denominators are centered, but a very long numerator or denominator might be displayed on several lines and a left alignment might be more appropriate for displaying them.

The `beveled` attribute determines whether the fraction is displayed with the numerator above the denominator separated by a horizontal line or whether a diagonal line is used to separate a slightly raised numerator from a slightly lowered denominator. The later form corresponds to the attribute value being `true` and provides for a more compact form for simple numerator and denominators. An example illustrating the beveled form is show below:

$$\frac{1}{x^3+\frac{x}{3}} = 1 \Big/ x^3+\frac{x}{3}$$

The `mfrac` element sets `displaystyle` to `false`, or if it was already false increments `scriptlevel` by 1, within numerator and denominator. These attributes are inherited by every element from its rendering environment, but can be set explicitly only on the `mstyle` element. (See section 3.3.4.)

3.3.2.3 Examples

The examples shown above can be represented in MathML as:

```
<mrow>
   <mo> ( </mo>
   <mfrac linethickness="0">
      <mi> a </mi>
      <mi> b </mi>
   </mfrac>
   <mo> ) </mo>
</mrow>
<mfrac linethickness="2">
   <mfrac>
      <mi> a </mi>
      <mi> b </mi>
   </mfrac>
   <mfrac>
      <mi> c </mi>
      <mi> d </mi>
   </mfrac>
</mfrac>


<mfrac>
   <mn> 1 </mn>
   <mrow>
      <msup>
         <mi> x </mi>
         <mn> 3 </mn>
       </msup>
       <mo> + </mo>
       <mfrac>
         <mi> x </mi>
         <mn> 3 </mn>
       </mfrac>
   </mrow>
</mfrac>
<mo> = </mo>
<mfrac beveled="true">
   <mn> 1 </mn>
   <mrow>
      <msup>
         <mi> x </mi>
         <mn> 3 </mn>
       </msup>
       <mo> + </mo>
       <mfrac>
         <mi> x </mi>
         <mn> 3 </mn>
       </mfrac>
   </mrow>
</mfrac>
```

A more generic example is:

```
<mfrac>
   <mrow>
      <mn> 1 </mn>
      <mo> + </mo>
      <msqrt>
         <mn> 5 </mn>
      </msqrt>
   </mrow>
   <mn> 2 </mn>
</mfrac>
```

3.3.3 Radicals (`msqrt`, `mroot`)

3.3.3.1 Description

These elements construct radicals. The `msqrt` element is used for square roots, while the `mroot` element is used to draw radicals with indices, e.g. a cube root. The syntax for these elements is:

```
<msqrt>{base} </msqrt>
<mroot>{base}{index} </mroot>
```

The `mroot` element requires exactly 2 arguments. However, `msqrt` accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' containing its arguments, as described in section 3.1.3.

3.3.3.2 Attributes

None (except the attributes allowed for all MathML elements, listed in section 2.3.4).

The `mroot` element increments `scriptlevel` by 2, and sets `displaystyle` to `false`, within index, but leaves both attributes unchanged within base. The `msqrt` element leaves both attributes unchanged within all its arguments. These attributes are inherited by every element from its rendering environment, but can be set explicitly only on `mstyle` (See section 3.3.4.)

3.3.4 Style Change (`mstyle`)

3.3.4.1 Description

The `mstyle` element is used to make style changes that affect the rendering of its contents. `mstyle` can be given any attribute accepted by any MathML presentation element provided that the attribute value is inherited, computed or has a default value; presentation element attributes whose values are required are not accepted by the `mstyle` element. In addition `mstyle` can also be given certain special attributes listed below.

The `mstyle` element accepts any number of arguments. If this number is not 1, its contents are treated as a single 'inferred `mrow`' formed from all its arguments, as described in section 3.1.3.

Loosely speaking, the effect of the `mstyle` element is to change the default value of an attribute for the elements it contains. Style changes work in one of several ways, depending on the way in which default values are specified for an attribute. The cases are:

- Some attributes, such as `displaystyle` or `scriptlevel` (explained below), are inherited from the surrounding context when they are not explicitly set. Specifying such an attribute on an `mstyle` element sets the value that will be inherited by its child elements. Unless a child element overrides this inherited value, it will pass it on to its children, and they will pass it to their children, and so on. But if a child element does override it, either by an explicit attribute setting or automatically (as is common for `scriptlevel`), the new (overriding) value will be passed on to that element's children, and then to their children, etc, until it is again overridden.
- Other attributes, such as `linethickness` on `mfrac`, have default values that are not normally inherited. That is, if the `linethickness` attribute is not set on the begin tag of an `mfrac` element, it will normally use the default value of `1`, even if it was contained in a larger `mfrac` element that set this attribute to a different value. For attributes like this, specifying a value with an `mstyle` element has the effect of changing the default value for all elements within its scope. The net effect is that setting the attribute value with `mstyle` propagates the change to all the elements it contains directly or indirectly, except for the individual elements on which the value is overridden. Unlike in the case of inherited attributes, elements that explicitly override this attribute have no effect on this attribute's value in their children.
- Another group of attributes, such as `stretchy` and `form`, are computed from operator dictionary information, position in the enclosing `mrow`, and other similar data. For these attributes, a value specified by an enclosing `mstyle` overrides the value that would normally be computed.

Note that attribute values inherited from an `mstyle` in any manner affect a given element in the `mstyle`'s content only if that attribute is not given a value in that element's begin tag. On any element for which the attribute is set explicitly, the value specified on the begin tag overrides the inherited value. The only exception to this rule is when the value given on the begin tag is documented as specifying an incremental change to the value inherited from that element's context or rendering environment.

Note also that the difference between inherited and non-inherited attributes set by `mstyle`, explained above, only matters when the attribute is set on some element within the `mstyle`'s contents that has children also setting it. Thus it never matters for attributes, such as `color`, which can only be set on token elements (or on `mstyle` itself).

There is one exceptional element, `mpadded`, whose attributes cannot be set with `mstyle`. When the attributes `width`, `height` and `depth` are specified on an `mstyle` element, they apply only to the `mspace` element. Similarly, when `lspace` is set with `mstyle`, it applies only to the `mo` element.

3.3.4.2 Attributes

As stated above, `mstyle` accepts all attributes of all MathML presentation elements which do not have required values. That is, all attributes which have an explicit default value or a default value which is inherited or computed are accepted by the `mstyle` element. Additionally, `mstyle` can be given the following special attributes that are implicitly inherited by every MathML element as part of its rendering environment:

Name	values	default
scriptlevel	[’+’ | ’-’] unsigned-integer	inherited
displaystyle	true | false	inherited
scriptsizemultiplier	number	0.71
scriptminsize	number v-unit	8pt
color	#rgb | #rrggbb | html-color-name	inherited
background	#rgb | #rrggbb | transparent | html-color-name	transparent
veryverythinmathspace	number h-unit	0.0555556em
verythinmathspace	number h-unit	0.111111em
thinmathspace	number h-unit	0.166667em
mediummathspace	number h-unit	0.222222em
thickmathspace	number h-unit	0.277778em
verythickmathspace	number h-unit	0.333333em
veryverythickmathspace	number h-unit	0.388889em

scriptlevel and *displaystyle*

MathML uses two attributes, `displaystyle` and `scriptlevel`, to control orthogonal presentation features that TEX encodes into one `style` attribute with values \displaystyle, \textstyle, \scriptstyle, and \scriptscriptstyle. The corresponding values of `displaystyle` and `scriptlevel` for those TEX styles would be `true` and `0`, `false` and `0`, `false` and `1`, and `false` and `2`, respectively.

The main effect of the `displaystyle` attribute is that it determines the effect of other attributes such as the `largeop` and `movablescripts` attributes of `mo`. The main effect of the `scriptlevel` attribute is to control the font size. Typically, the higher the `scriptlevel`, the smaller the font size. (Non-visual renderers can respond to the font size in an analogous way for their medium.) More sophisticated renderers may also choose to use these attributes in other ways, such as rendering expressions with `displaystyle="false"` in a more vertically compressed manner.

These attributes are given initial values for the outermost expression of an instance of MathML based on its rendering environment. A short list of layout schemata described below modify these values for some of their sub-expressions. Otherwise, values are determined by inheritance whenever they are not directly specified on a given element’s start tag.

For an instance of MathML embedded in a textual data format (such as HTML) in ‘display’ mode, i.e. in place of a paragraph, `displaystyle="true"` and `scriptlevel="0"` for the outermost expression of the embedded MathML; if the MathML is embedded in ‘inline’ mode, i.e. in place of a character, `displaystyle="false"` and `scriptlevel="0"` for the outermost expression. See chapter 7 for further discussion of the distinction between ‘display’ and ‘inline’ embedding of MathML and how this can be specified in particular instances. In general, a MathML renderer may determine these initial values in whatever manner is appropriate for the location and context of the specific instance of MathML it is rendering, or if it has no way to determine this, based on the way it is most likely to be used; as a last resort it is suggested that it use the most generic values `displaystyle="true"` and `scriptlevel="0"`.

The MathML layout schemata that typically display some of their arguments in smaller type or with less vertical spacing, namely the elements for scripts, fractions, radicals, and tables or matrices, set `displaystyle` to `false`, and in some cases increase `scriptlevel`

for those arguments. The new values are inherited by all sub-expressions within those arguments, unless they are overridden.

The specific rules by which each element modifies `displaystyle` and/or `scriptlevel` are given in the specification for each element that does so; the complete list of elements that modify either attribute are: the 'scripting' elements `msub`, `msup`, `msubsup`, `munder`, `mover`, `munderover`, and `mmultiscripts`, and the elements `mfrac`, `mroot`, and `mtable`.

When `mstyle` is given a `scriptlevel` attribute with no sign, it sets the value of `scriptlevel` within its contents to the value given, which must be a nonnegative integer. When the attribute value consists of a sign followed by an integer, the value of `scriptlevel` is incremented (for '+') or decremented (for '-') by the amount given. The incremental syntax for this attribute is an exception to the general rules for setting inherited attributes using `mstyle`, and is not allowed by any other attribute on `mstyle`.

Whenever the `scriptlevel` is changed, either automatically or by being explicitly incremented, decremented, or set, the current font size is multiplied by the value of `scriptsizemultiplier` to the power of the change in `scriptlevel`. For example, if `scriptlevel` is increased by 2, the font size is multiplied by `scriptsizemultiplier` twice in succession; if `scriptlevel` is explicitly set to 2 when it had been 3, the font size is divided by `scriptsizemultiplier`.

The default value of `scriptsizemultiplier` is less than one (in fact, it is approximately the square root of 1/2), resulting in a smaller font size with increasing `scriptlevel`. To prevent scripts from becoming unreadably small, the font size is never allowed to go below the value of `scriptminsize` as a result of a change to `scriptlevel`, though it can be set to a lower value using the `fontsize` attribute (section 3.2.1) on `mstyle` or on token elements. If a change to `scriptlevel` would cause the font size to become lower than `scriptminsize` using the above formula, the font size is instead set equal to `scriptminsize` within the sub-expression for which `scriptlevel` was changed.

In the syntax for `scriptminsize`, `v-unit` represents a unit of vertical length (as described in section 2.3.3.2). The most common unit for specifying font sizes in typesetting is `pt` (points).

Explicit changes to the `fontsize` attribute have no effect on the value of `scriptlevel`.

Further details on *`scriptlevel`* for renderers

For MathML renderers that support CSS1 style sheets, or some other analogous style sheet mechanism, absolute or relative changes to `fontsize` (or other attributes) may occur implicitly on any element in response to a style sheet. Changes to `fontsize` of this kind also have no effect on `scriptlevel`. A style sheet-induced change to `fontsize` overrides `scriptminsize` in the same way as for an explicit change to `fontsize` in the element's begin tag (discussed above), whether it is specified in the style sheet as an absolute or a relative change. (However, any subsequent `scriptlevel`-induced change to `fontsize` will still be affected by it.) As is required for inherited attributes in CSS1, the style sheet-modified `fontsize` is inherited by child elements.

If the same element is subject to both a style sheet-induced and an automatic (`scriptlevel` related) change to its own `fontsize`, the `scriptlevel`-related change is done first - in fact, in the simplest implementation of the element-specific rules for `scriptlevel`, this change would be done by the element's parent as part of producing the rendering properties it passes to the given element, since it is the parent element that knows whether `scriptlevel` should be changed for each of its child elements.

If the element's own `fontsize` is changed by a style sheet and it also changes `scriptlevel` (and thus `fontsize`) for one of its children, the style sheet-induced change is done first, followed by the change inherited by that child. If more than one child's `scriptlevel` is changed, the change inherited by each child has no effect on the other children. (As a mnemonic rule that applies to a 'parse tree' of elements and their children, style sheet-induced changes to `fontsize` can be associated to nodes of the tree, i.e. to MathML elements, and `scriptlevel`-related changes can be associated to the edges between parent and child elements; then the order of the associated changes corresponds to the order of nodes and edges in each path down the tree.) For general information on the relative order of processing of properties set by style sheets versus by attributes, see the appropriate subsection of CSS-compatible attributes in section 2.3.3.3.

If `scriptlevel` is changed incrementally by an `mstyle` element that also sets certain other attributes, the overall effect of the changes may depend on the order in which they are processed. In such cases, the attributes in the following list should be processed in the following order, regardless of the order in which they occur in the XML-format attribute list of the `mstyle` begin tag: `scriptsizemultiplier`, `scriptminsize`, `scriptlevel`, `fontsize`.

Note that `scriptlevel` can, in principle, attain any integral value by being decremented sufficiently, even though it can only be explicitly set to nonnegative values. Negative values of `scriptlevel` generated in this way are legal and should work as described, generating font sizes larger than those of the surrounding expression. Since `scriptlevel` is initially 0 and never decreases automatically, it will always be nonnegative unless it is decremented past 0 using `mstyle`.

Explicit decrements of `scriptlevel` after the font size has been limited by `scriptminsize` as described above would produce undesirable results. This might occur, for example, in a representation of a continued fraction, in which the scriptlevel was decremented for part of the denominator back to its value for the fraction as a whole, if the continued fraction itself was located in a place that had a high `scriptlevel`. To prevent this problem, MathML renderers should, when decrementing `scriptlevel`, use as the initial font size the value the font size would have had if it had never been limited by `scriptminsize`. They should not, however, ignore the effects of explicit settings of `fontsize`, even to values below `scriptminsize`.

Since MathML renderers may be unable to make use of arbitrary font sizes with good results, they may wish to modify the mapping from scriptlevel to fontsize to produce better renderings in their judgement. In particular, if fontsizes have to be rounded to available values, or limited to values within a range, the details of how this is done are up to the renderer. Renderers should, however, ensure that a series of incremental changes to `scriptlevel` resulting in its return to the same value for some sub-expression that it had in a surrounding expression results in the same fontsize for that sub-expression as for the surrounding expression.

Color and background attributes

The `color` attribute controls the color in which the content of tokens is rendered. Additionally, when inherited from `mstyle` or from a MathML expression's rendering environment, it controls the color of all other drawing by MathML elements, including the lines or radical signs that can be drawn by `mfrac`, `mtable`, or `msqrt`.

Note that the `background` attribute, though not inherited, has the default value ‘transparent’ (as in CSS1), which effectively allows an element’s parent to control its background.

The values of `color` and `background` can be specified as a string consisting of ’#’ followed without intervening whitespace by either 1-digit or 2-digit hexadecimal values for the red, green, and blue components, respectively, of the desired color, with the same number of digits used for each component (or as the keyword ‘transparent’ for `background`). The hexadecimal digits are not case-sensitive. The possible 1-digit values range from 0 (component not present) to F (component fully present), and the possible 2-digit values range from 00 (component not present) to FF (component fully present), with the 1-digit value *x* being equivalent to the 2-digit value xx (rather than x0). % *x*0 would be a more strictly correct notation, but renders terribly in some browsers.

These attributes can also be specified as an `html-color-name`, which is defined in the following subsection.

CSS compatibility of color attributes

The color syntax described above is a subset of the syntax of the `color` and `background-color` properties of CSS1. (The `background-color` syntax is in turn a subset of the full CSS1 `background` property syntax, which also permits specification of (for example) background images with optional repeats. The more general attribute name `background` is used in MathML to facilitate possible extensions to the attribute’s scope in future versions of MathML.)

Color values on either attribute can also be specified as an `html-color-name`, that is, as one of the color-name keywords defined in [HTML4.0]. The list of allowed color names includes most of the commonest English color words, though not `orange`, `brown`, or `pink`, and also includes a number of less-common color words; see the reference for the complete list and the equivalent RGB values. Note that the color name keywords are not case-sensitive, unlike most keywords in MathML attribute values. (The same color name keywords are defined for the CSS1 `color` property, but with unspecified RGB values. See also section 2.3.3.3.)

Precise background region not specified

The suggested MathML visual rendering rules do not define the precise extent of the region whose background is affected by using the `background` attribute on `mstyle`, except that, when `mstyle`’s content does not have negative dimensions and its drawing region is not overlapped by other drawing due to surrounding negative spacing, this region should lie behind all the drawing done to render the content of the `mstyle` but should not lie behind any of the drawing done to render surrounding expressions. The effect of overlap of drawing regions caused by negative spacing on the extent of the region affected by the `background` attribute is not defined by these rules.

Meaning of named mathspaces

The spacing between operators is often one of a small number of potential values. MathML names these values and allows their values to be changed. Because the default values for spacing around operators that are given in the operator dictionary appendix D are defined using these named spaces, changing their values will produce tighter or looser spacing. These values can be used anywhere a `h-unit` or `v-unit` unit is allowed section 2.3.3.2.

The predefined `namedspace` values are: `veryverythinmathspace`, `verythinmathspace`, `thinmathspace`, `mediummathspace`, `thickmathspace`, `verythickmathspace`, and `veryverythickmathspace`. The default values of `veryverythinmathspace`, ..., `veryverythickmathspace` are 1/18em...7/18em, respectively.

3.3.4.3 Examples

The example of limiting the stretchiness of a parenthesis shown in the section on <mo>,

```
<mrow>
   <mo maxsize="1"> ( </mo>
   <mfrac> <mi> a </mi> <mi> b </mi> </mfrac>
   <mo maxsize="1"> ) </mo>
</mrow>
```

can be rewritten using `mstyle` as:

```
<mstyle maxsize="1">
   <mrow>
      <mo> ( </mo>
      <mfrac> <mi> a </mi> <mi> b </mi> </mfrac>
      <mo> ) </mo>
   </mrow>
</mstyle>
```

3.3.5 Error Message (`merror`)

3.3.5.1 Description

The `merror` element displays its contents as an 'error message'. This might be done, for example, by displaying the contents in red, flashing the contents, or changing the background color. The contents can be any expression or expression sequence.

`merror` accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' as described in section 3.1.3.

The intent of this element is to provide a standard way for programs that generate MathML from other input to report syntax errors in their input. Since it is anticipated that preprocessors that parse input syntaxes designed for easy hand entry will be developed to generate MathML, it is important that they have the ability to indicate that a syntax error occurred at a certain point. See section 7.2.2.

The suggested use of `merror` for reporting syntax errors is for a preprocessor to replace the erroneous part of its input with an `merror` element containing a description of the error, while processing the surrounding expressions normally as far as possible. By this means, the error message will be rendered where the erroneous input would have appeared, had it been correct; this makes it easier for an author to determine from the rendered output what portion of the input was in error.

No specific error message format is suggested here, but as with error messages from any program, the format should be designed to make as clear as possible (to a human viewer of the rendered error message) what was wrong with the input and how it can be fixed. If the erroneous input contains correctly formatted subsections, it may be useful for these

to be preprocessed normally and included in the error message (within the contents of the `merror` element), taking advantage of the ability of `merror` to contain arbitrary MathML expressions rather than only text.

3.3.5.2 Attributes

None (except the attributes allowed for all MathML elements, listed in section 2.3.4).

3.3.5.3 Example

If a MathML syntax-checking preprocessor received the input

```
<mfraction>
   <mrow> <mn> 1 </mn> <mo> + </mo> <msqrt> <mn> 5 </mn> </msqrt> </mrow>
   <mn> 2 </mn>
</mfraction>
```

which contains the non-MathML element `mfraction` (presumably in place of the MathML element `mfrac`), it might generate the error message

```
<merror>
   <mtext> Unrecognized element: mfraction;
          arguments were:  </mtext>
   <mrow> <mn> 1 </mn> <mo> + </mo> <msqrt> <mn> 5 </mn> </msqrt> </mrow>
   <mtext>  and  </mtext>
   <mn> 2 </mn>
</merror>
```

Note that the preprocessor's input is not, in this case, valid MathML, but the error message it outputs is valid MathML.

3.3.6 Adjust Space Around Content (`mpadded`)

3.3.6.1 Description

An `mpadded` element renders the same as its content, but with its overall size and other dimensions (such as baseline position) modified according to its attributes. The `mpadded` element does not rescale (stretch or shrink) its content; its only effect is to modify the apparent size and position of the 'bounding box' around its content, so as to affect the relative position of the content with respect to the surrounding elements. The name of the element reflects the use of `mpadded` to effectively add 'padding', or extra space, around its content. If the 'padding' is negative, it is possible for the content of `mpadded` to be rendered outside the `mpadded` element's bounding box; see below for warnings about several potential pitfalls of this effect.

The `mpadded` element accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' as described in section 3.1.3.

It is suggested that audio renderers add (or shorten) time delays based on the attributes representing horizontal space (`width` and `lspace`).

Name	values	default
width	[+ | -] unsigned-number (% [pseudo-unit] | pseudo-unit | h-unit | namedspace)	same as content
lspace	[+ | -] unsigned-number (% [pseudo-unit] | pseudo-unit | h-unit)	0
height	[+ | -] unsigned-number (% [pseudo-unit] | pseudo-unit | v-unit)	same as content
depth	[+ | -] unsigned-number (% [pseudo-unit] | pseudo-unit | v-unit)	same as content

3.3.6.2 Attributes

(The pseudo-unit syntax symbol is described below.)

These attributes modify the dimensions of the 'bounding box' of the `mpadded` element. The dimensions (which have the same names as the attributes) are defined in the next subsection. Depending on the format of the attribute value, a dimension may be set to a new value, or to an incremented or decremented version of the content's corresponding dimension. Values may be specified as multiples or percentages of any of the dimensions of the normal rendering of the element's content (using so-called 'pseudo-units'), or they can be set directly using standard units section 2.3.3.2.

If an attribute value begins with a `+` or `-` sign, it specifies an increment or decrement of the corresponding dimension by the following length value (interpreted as explained below). Otherwise, the corresponding dimension is set directly to the following length value. Note that the `+` and `-` do not mean that the following value is positive or negative, even when an explicit length unit (h-unit or v-unit) is given. In particular, these attributes cannot directly set a dimension to a negative value.

Length values (after the optional sign, which is not part of the length value) can be specified in several formats. Each format begins with an unsigned-number, which may be followed by a `%` sign and an optional 'pseudo-unit' (denoted by pseudo-unit in the attribute syntaxes above), by a pseudo-unit alone, or by one of the length units (denoted by h-unit or v-unit) specified in section 2.3.3.2, not including `%`. The possible pseudo-units are the keywords `width`, `lspace`, `height`, and `depth`; they each represent the length of the same-named dimension of the `mpadded` element's content (not of the `mpadded` element itself). The lengths represented by h-unit or v-unit are described in section 2.3.3.2.

In any of these formats, the length value specified is the product of the specified number the length represented by the unit or pseudo-unit, and multiplied by 0.01 if `%` is given. If no pseudo-unit is given after `%`, the one with the same name as the attribute being specified is assumed.

Some examples of attribute formats using pseudo-units (explicit or default) are as follows: `depth="100% height"` and `depth="1.0 height"` both set the depth of the `mpadded` element to the height of its content. `depth="105%"` sets the depth to 1.05 times the content's depth, and either `depth="+100%"` or `depth="200%"` sets the depth to twice the content's depth.

Dimensions that would be positive if the content was rendered normally cannot be made negative using `mpadded`; a positive dimension is set to 0 if it would otherwise become negative. Dimensions that are initially 0 can be made negative, but this should generally be avoided. See the warnings below on the use of negative spacing for 'tweaking' or conveying meaning.

The rules given above imply that all of the following attribute settings have the same effect, which is to leave the content's dimensions unchanged:

```
<mpadded width="+0em"> ... </mpadded>
<mpadded width="+0%"> ... </mpadded>
<mpadded width="-0em"> ... </mpadded>
<mpadded width="- 0 height"> ... </mpadded>
<mpadded width="100%"> ... </mpadded>
<mpadded width="100% width"> ... </mpadded>
<mpadded width="1 width"> ... </mpadded>
<mpadded width="1.0 width"> ... </mpadded>
<mpadded> ... </mpadded>
```

3.3.6.3 Meanings of dimension attributes

See appendix F for further information about some of the typesetting terms used here.

The `width` attribute refers to the overall horizontal width of a bounding box. By default (i.e. when `lspace` is not modified), the bounding box of the content of an `mpadded` element should be rendered flush with the left edge of the `mpadded` element's bounding box. Thus, increasing `width` alone effectively adds space on the right edge of the box.

The `lspace` attribute refers to the amount of space between the left edge of a bounding box and where the rendering of its contents' bounding box actually begins. Unlike the other dimensions, `lspace` does not correspond to a real property of a bounding box, but exists only transiently during the computations done by each instance of `mpadded`. It is provided so that there is a way to add space on the left edge of a bounding box.

The rationale behind using `width` and `lspace` to control horizontal padding instead of more symmetric attributes, such as a hypothetical `rspace` and `lspace`, is that it is desirable to have a 'width' pseudo unit, in part because 'width' is an actual property of a bounding box.

The `height` attribute refers to the amount of vertical space between the baseline (the line along the bottom of most letter glyphs in normal text rendering) and the top of the bounding box.

The `depth` attribute refers to the amount of vertical space between the bottom of the bounding box and the baseline.

MathML renderers should ensure that, except for the effects of the attributes, relative spacing between the contents of `mpadded` and surrounding MathML elements is not modified by replacing an `mpadded` element with an `mrow` element with the same content. This holds even if linebreaking occurs within the `mpadded` element. However, if an `mpadded` element with non-default attribute values is subjected to linebreaking, MathML does not define how its attributes or rendering interact with the linebreaking algorithm.

3.3.6.4 Warning: nonportability of 'tweaking'

A likely temptation for the use of the `mpadded` and `mspace` elements (and perhaps also `mphantom` and `mtext`) will be for an author to improve the spacing generated by a specific renderer by slightly modifying it in specific expressions, i.e. to 'tweak' the rendering.

Authors are strongly warned that different MathML renderers may use different spacing rules for computing the relative positions of rendered symbols in expressions that have no explicit modifications to their spacing; if renderer B improves upon renderer A's spacing

rules, explicit spacing added to improve the output quality of renderer A may produce very poor results in renderer B, very likely worse than without any ‘tweaking’ at all.

Even when a specific choice of renderer can be assumed, its spacing rules may be improved in successive versions, so that the effect of tweaking in a given MathML document may grow worse with time. Also, when style sheet mechanisms are extended to MathML, even one version of a renderer may use different spacing rules for users with different style sheets.

Therefore, it is suggested that MathML markup never use `mpadded` or `mspace` elements to tweak the rendering of specific expressions, unless the MathML is generated solely to be viewed using one specific version of one MathML renderer, using one specific style sheet (if style sheets are available in that renderer).

In cases where the temptation to improve spacing proves too strong, careful use of `mpadded`, `mphantom`, or the alignment elements (section 3.5.5) may give more portable results than the direct insertion of extra space using `mspace` or `mtext`. Advice given to the implementors of MathML renderers might be still more productive, in the long run.

3.3.6.5 Warning: spacing should not be used to convey meaning

MathML elements that permit ‘negative spacing’, namely `mspace`, `mpadded`, and `mtext`, could in theory be used to simulate new notations or ‘overstruck’ characters by the visual overlap of the renderings of more than one MathML sub-expression.

This practice is strongly discouraged in all situations, for the following reasons:

- it will give different results in different MathML renderers (so the warning about ‘tweaking’ applies);
- it is likely to appear much worse than a more standard construct supported by good renderers;
- such expressions are almost certain to be uninterpretable by audio renderers, computer algebra systems, text searches for standard symbols, or other processors of MathML input.

More generally, any construct that uses spacing to convey mathematical meaning, rather than simply as an aid to viewing expression structure, is discouraged. That is, the constructs that are discouraged are those that would be interpreted differently by a human viewer of rendered MathML if all explicit spacing was removed.

If such constructs are used in spite of this warning, they should be enclosed in a `semantics` element that also provides an additional MathML expression that can be interpreted in a standard way.

For example, the MathML expression

```
<mrow>
   <mpadded width="0"> <mi> C </mi> </mpadded>
   <mspace width="0.3em"/>
   <mtext> | </mtext>
</mrow>
```

forms an overstruck symbol in violation of the policy stated above; it might be intended to represent the set of complex numbers for a MathML renderer that lacks support for the standard symbol used for this purpose. This kind of construct should always be avoided in

MathML, for the reasons stated above; indeed, it should never be necessary for standard symbols, since a MathML renderer with no better method of rendering them is free to use overstriking internally, so that it can still support general MathML input.

However, if for whatever reason such a construct is used in MathML, it should always be enclosed in a `semantics` element such as

```
<semantics>
   <mrow>
      <mpadded width="0"> <mi> C </mi> </mpadded>
      <mspace width="0.3em"/>
      <mtext> | </mtext>
   </mrow>
   <annotation-xml encoding="MathML-Presentation">
      <mi> &Copf; </mi>
   </annotation-xml>
</semantics>
```

which provides an alternative, standard encoding for the desired symbol, which is much more easily interpreted than the construct using negative spacing. (The alternative encoding in this example uses MathML presentation elements; the content elements described in chapter 4 should also be considered.)

(The above warning also applies to most uses of rendering attributes to alter the meaning conveyed by an expression, with the exception of attributes on `mi` (such as `fontweight`) used to distinguish one variable from another.)

3.3.7 Making Content Invisible (`mphantom`)

3.3.7.1 Description

The `mphantom` element renders invisibly, but with the same size and other dimensions, including baseline position, that its contents would have if they were rendered normally. `mphantom` can be used to align parts of an expression by invisibly duplicating sub-expressions.

The `mphantom` element accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' formed from all its arguments, as described in section 3.1.3.

It is suggested that audio renderers render `mphantom` elements in an analogous way for their medium, by rendering them as silence of the same duration as the normal rendering of their contents.

3.3.7.2 Attributes

None (except the attributes allowed for all MathML elements, listed in section 2.3.4).

Note that it is possible to wrap both an `mphantom` and an `mpadded` element around one MathML expression, as in `<mphantom><mpadded attribute-settings> ... </mpadded></mphantom>` to change its size and make it invisible at the same time.

MathML renderers should ensure that the relative spacing between the contents of an `mphantom` element and the surrounding MathML elements is the same as it would be if the

mphantom element were replaced by an mrow element with the same content. This holds even if linebreaking occurs within the mphantom element.

For the above reason, mphantom is not considered space-like (section 3.2.6) unless its content is space-like, since the suggested rendering rules for operators are affected by whether nearby elements are space-like. Even so, the warning about the legal grouping of space-like elements may apply to uses of mphantom.

There is one situation where the preceding rule for rendering an mphantom may not give the desired effect. When an mphantom is wrapped around a subsequence of the arguments of an mrow, the default determination of the form attribute for an mo element within the subsequence can change. (See the default value of the form attribute described in section 3.2.4.) It may be necessary to add an explicit form attribute to such an mo in these cases. This is illustrated in the following example.

3.3.7.3 Examples

In this example, mphantom is used to ensure alignment of corresponding parts of the numerator and denominator of a fraction:

```
<mfrac>
   <mrow>
      <mi> x </mi>
      <mo> + </mo>
      <mi> y </mi>
      <mo> + </mo>
      <mi> z </mi>
   </mrow>
   <mrow>
      <mi> x </mi>
      <mphantom>
         <mo form="infix"> + </mo>
         <mi> y </mi>
      </mphantom>
      <mo> + </mo>
      <mi> z </mi>
   </mrow>
</mfrac>
```

This would render as something like

$$\frac{x+y+z}{x\quad +z}$$

rather than as

$$\frac{x+y+z}{x+z}$$

The explicit attribute setting form="infix" on the mo element inside the mphantom sets the form attribute to what it would have been in the absence of the surrounding mphantom. This is necessary since otherwise, the + sign would be interpreted as a prefix operator, which might have slightly different spacing.

Alternatively, this problem could be avoided without any explicit attribute settings, by wrapping each of the arguments <mo>+</mo> and <mi>y</mi> in its own mphantom element, i.e.

```
<mfrac>
   <mrow>
      <mi> x </mi>
      <mo> + </mo>
      <mi> y </mi>
      <mo> + </mo>
      <mi> z </mi>
   </mrow>
   <mrow>
      <mi> x </mi>
      <mphantom>
         <mo> + </mo>
      </mphantom>
      <mphantom>
         <mi> y </mi>
      </mphantom>
      <mo> + </mo>
      <mi> z </mi>
   </mrow>
</mfrac>
```

3.3.8 Content Inside Pair of Fences (`mfenced`)

3.3.8.1 Description

The `mfenced` element provides a convenient form in which to express common constructs involving fences (i.e. braces, brackets, and parentheses), possibly including separators (such as comma) between the arguments.

For example, `<mfenced> <mi>x</mi> </mfenced>` renders as '(x)' and is equivalent to

```
<mrow> <mo> ( </mo> <mi>x</mi> <mo> ) </mo> </mrow>
```

and `<mfenced> <mi>x</mi> <mi>y</mi> </mfenced>` renders as '(x, y)' and is equivalent to

```
<mrow>
   <mo> ( </mo>
   <mrow> <mi>x</mi> <mo>,</mo> <mi>y</mi> </mrow>
   <mo> ) </mo>
</mrow>
```

Individual fences or separators are represented using `mo` elements, as described in section 3.2.4. Thus, any `mfenced` element is completely equivalent to an expanded form described below; either form can be used in MathML, at the convenience of an author or of a MathML-generating program. A MathML renderer is required to render either of these forms in exactly the same way.

In general, an `mfenced` element can contain zero or more arguments, and will enclose them between fences in an `mrow`; if there is more than one argument, it will insert separators between adjacent arguments, using an additional nested `mrow` around the arguments and

separators for proper grouping (section 3.3.1). The general expanded form is shown below. The fences and separators will be parentheses and comma by default, but can be changed using attributes, as shown in the following table.

3.3.8.2 Attributes

Name	values	default
open	string	(
close	string	)
separators	character *	,

A generic `mfenced` element, with all attributes explicit, looks as follows:

```
<mfenced open="opening-fence"
        close="closing-fence"
        separators="sep#1 sep#2 ... sep#(n-1)" >
   arg#1
   ...
   arg#n
</mfenced>
```

The `opening-fence` and `closing-fence` are arbitrary strings. (Since they are used as the content of `mo` elements, any whitespace they contain will be trimmed and collapsed as described in section 2.3.5.)

The value of `separators` is a sequence of zero or more separator characters (or entity references), optionally separated by whitespace. Each `sep#i` consists of exactly one character or entity reference. Thus, `separators=",;"` is equivalent to `separators=" , ; "`.

The general `mfenced` element shown above is equivalent to the following expanded form:

```
<mrow>
   <mo fence="true"> opening-fence </mo>
   <mrow>
      arg#1
      <mo separator="true"> sep#1 </mo>
      ...
      <mo separator="true"> sep#(n-1) </mo>
      arg#n
   </mrow>
   <mo fence="true"> closing-fence </mo>
</mrow>
```

Each argument except the last is followed by a separator. The inner `mrow` is added for proper grouping, as described in section 3.3.1.

When there is only one argument, the above form has no separators; since `<mrow> arg#1 </mrow>` is equivalent to `arg#1` (as described in section 3.3.1), this case is also equivalent to:

```
<mrow>
   <mo fence="true"> opening-fence </mo>
```

```
   arg#1
   <mo fence="true"> closing-fence </mo>
</mrow>
```

If there are too many separator characters, the extra ones are ignored. If separator characters are given, but there are too few, the last one is repeated as necessary. Thus, the default value of `separators="`,`"` is equivalent to `separators="`,,`"`, `separators="`,,,`"`, etcetera. If there are no separator characters provided but some are needed, for example if `separators="``"` or `""` and there is more than one argument, then no separator elements are inserted at all - that is, the elements `<mo separator="true"> sep#i </mo>` are left out entirely. Note that this is different from inserting separators consisting of `mo` elements with empty content.

Finally, for the case with no arguments, i.e.

```
<mfenced open="opening-fence"
        close="closing-fence"
        separators="anything" >
</mfenced>
```

the equivalent expanded form is defined to include just the fences within an `mrow`

```
<mrow>
   <mo fence="true"> opening-fence </mo>
   <mo fence="true"> closing-fence </mo>
</mrow>
```

Note that not all ‘fenced expressions’ can be encoded by an `mfenced` element. Such exceptional expressions include those with an ‘embellished’ separator or fence or one enclosed in an `mstyle` element, a missing or extra separator or fence, or a separator with multiple content characters. In these cases, it is necessary to encode the expression using an appropriately modified version of an expanded form. As discussed above, it is always permissible to use the expanded form directly, even when it is not necessary. In particular, authors cannot be guaranteed that MathML preprocessors won’t replace occurrences of `mfenced` with equivalent expanded forms.

Note that the equivalent expanded forms shown above include attributes on the `mo` elements that identify them as fences or separators. Since the most common choices of fences and separators already occur in the operator dictionary with those attributes, authors would not normally need to specify those attributes explicitly when using the expanded form directly. Also, the rules for the default `form` attribute (section 3.2.4) cause the opening and closing fences to be effectively given the values `form="prefix"` and `form="postfix"` respectively, and the separators to be given the value `form="infix"`.

Note that it would be incorrect to use `mfenced` with a separator of, for instance, ‘+’, as an abbreviation for an expression using ‘+’ as an ordinary operator, e.g.

```
<mrow>
  <mi>x</mi> <mo>+</mo> <mi>y</mi> <mo>+</mo> <mi>z</mi>
</mrow>
```

This is because the + signs would be treated as separators, not infix operators. That is, it would render as if they were marked up as `<mo separator="true">+</mo>`, which might therefore render inappropriately.

3.3.8.3 Examples

(*a*+*b*)

```
<mfenced>
   <mrow>
      <mi> a </mi>
      <mo> + </mo>
      <mi> b </mi>
   </mrow>
</mfenced>
```

Note that the above `mrow` is necessary so that the `mfenced` has just one argument. Without it, this would render incorrectly as '(*a*, +, *b*)'.

[0,1)

```
<mfenced open="[">
   <mn> 0 </mn>
   <mn> 1 </mn>
</mfenced>
```

$f(x,y)$

```
<mrow>
   <mi> f </mi>
   <mo> &ApplyFunction; </mo>
   <mfenced>
      <mi> x </mi>
      <mi> y </mi>
   </mfenced>
</mrow>
```

3.3.9 Enclose Content Inside Notation (`menclose`)

3.3.9.1 Description

The `menclose` element renders its content inside the enclosing notation specified by its `notation` attribute. `menclose` accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' containing its arguments, as described in section 3.1.3.

3.3.9.2 Attributes

Name	values	default
notation	longdiv | actuarial | radical	longdiv

With `notation` has the value `longdiv`, the contents are drawn enclosed by a long division symbol. A complete example of long division is accomplished by also using `mtable` and `malign`. When `notation` is specified as `actuarial`, the contents are drawn enclosed by an actuarial symbol. The case of `notation=radical` is equivalent to the `msqrt` schema.

3.3.9.3 Examples

The following markup might be used to encode an elementary US-style long division problem.

```
<mtable columnspacing='0' rowspacing='0'>
<mtr>
  <mtd></mtd>
  <mtd columnalign='right'><mn>10</mn></mtd>
</mtr>
<mtr>
  <mtd columnalign='right'><mn>131</mn></mtd>
  <mtd columnalign='right'>
    <menclose notation='longdiv'><mn>1413</mn></menclose>
  </mtd>
</mtr>
<mtr>
  <mtd></mtd>
  <mtd columnalign='right'>
    <mrow>
    <munder>
      <mn>131</mn>
      <mo>&UnderBar;</mo>
    </munder>
    <mphantom><mn>3</mn></mphantom>
    </mrow>
  </mtd>
</mtr>
<mtr>
  <mtd></mtd>
  <mtd columnalign='right'><mn>103</mn></mtd>
</mtr>
</mtable>
```

This might be rendered roughly as:

$$\begin{array}{rr} & 10 \\ \hline 131 &)1413 \\ & \underline{131} \\ & 103 \end{array}$$

An example of using `menclose` for actuarial notation is

```
<msub>
  <mi>a</mi>
  <mrow>
    <menclose notation='actuarial'>
      <mi>n</mi>
    </menclose>
    <mo>&it;</mo>
    <mi>i</mi>
  </mrow>
```

```
</msub>
```

which renders roughly as

$$\frac{a}{n}\Big|_{i}$$

3.4 Script and Limit Schemata

The elements described in this section position one or more scripts around a base. Attaching various kinds of scripts and embellishments to symbols is a very common notational device in mathematics. For purely visual layout, a single general-purpose element could suffice for positioning scripts and embellishments in any of the traditional script locations around a given base. However, in order to capture the abstract structure of common notation better, MathML provides several more specialized scripting elements.

In addition to sub/superscript elements, MathML has overscript andunderscript elements that place scripts above and below the base. These elements can be used to place limits on large operators, or for placing accents and lines above or below the base. The rules for rendering accents differ from those for overscripts and underscripts, and this difference can be controlled with the `accent`and `accentunder`attributes, as described in the appropriate sections below.

Rendering of scripts is affected by the `scriptlevel`and `displaystyle`attributes, which are part of the environment inherited by the rendering process of every MathML expression, and are described under `mstyle`(section 3.3.4). These attributes cannot be given explicitly on a scripting element, but can be specified on the start tag of a surrounding `mstyle`element if desired.

MathML also provides an element for attachment of tensor indices. Tensor indices are distinct from ordinary subscripts and superscripts in that they must align in vertical columns. Tensor indices can also occur in prescript positions.

Because presentation elements should be used to describe the abstract notational structure of expressions, it is important that the base expression in all ‘scripting’ elements (i.e. the first argument expression) should be the entire expression that is being scripted, not just the rightmost character. For example, $(x+y)^2$ should be written as:

```
<msup>
   <mrow>
      <mo> ( </mo>
      <mrow>
         <mi> x </mi>
         <mo> + </mo>
         <mi> y </mi>
      </mrow>
      <mo> ) </mo>
   </mrow>
   <mn> 2 </mn>
</msup>
```

3.4.1 Subscript (`msub`)

3.4.1.1 Description

The syntax for the `msub` element is:

```
<msub>{base}{subscript} </msub>
```

3.4.1.2 Attributes

Name	values	default
subscriptshift	number v-unit	automatic (typical unit is ex)

The `subscriptshift` attribute specifies the minimum amount to shift the baseline of subscript down.

v-unit represents a unit of vertical length (see section 2.3.3.2).

The `msub` element increments `scriptlevel` by 1, and sets `displaystyle` to `false`, within subscript, but leaves both attributes unchanged within base. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.2 Superscript (`msup`)

3.4.2.1 Description

The syntax for the `msup` element is:

```
<msub>{base}{superscript} </msub>
```

3.4.2.2 Attributes

Name	values	default
superscriptshift	number v-unit	automatic (typical unit is ex)

The `superscriptshift` attribute specifies the minimum amount to shift the baseline of superscript up.

v-unit represents a unit of vertical length (see section 2.3.3.2).

The `msup` element increments `scriptlevel` by 1, and sets `displaystyle` to `false`, within superscript, but leaves both attributes unchanged within base. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.3 Subscript-superscript Pair (`msubsup`)

3.4.3.1 Description

The `msubsup` element is used so that the subscript and superscript are both tight against the base, i.e. vertically aligned as in the second case shown here: ${x_1}^2$ versus x_1^2.

The syntax for the `msubsup` element is:

```
<msubsup>{base}{subscript}{superscript} </msubsup>
```

3.4.3.2 Attributes

Name	values	default
subscriptshift	number v-unit	automatic (typical unit is ex)
superscriptshift	number v-unit	automatic (typical unit is ex)

The `subscriptshift` attribute specifies the minimum amount to shift the baseline of subscript down. The `superscriptshift` attribute specifies the minimum amount to shift the baseline of superscript up.

v-unit represents a unit of vertical length (see section 2.3.3.2).

The `msubsup` element increments `scriptlevel` by 1, and sets `displaystyle` to `false`, within subscript and superscript, but leaves both attributes unchanged within base. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.3.3 Examples

The `msubsup` is most commonly used for adding sub/superscript pairs to identifiers as illustrated above. However, another important use is placing limits on certain large operators whose limits are traditionally displayed in the script positions even when rendered in display style. The most common of these is the integral. For example,

$$\int_0^1 e^x \, dx$$

would be represented as

```
<mrow>
   <msubsup>
      <mo> &int; </mo>
      <mn> 0 </mn>
      <mn> 1 </mn>
   </msubsup>
   <mrow>
      <msup>
         <mi> &ExponentialE; </mi>
         <mi> x </mi>
      </msup>
      <mo> &InvisibleTimes; </mo>
      <mrow>
         <mo> &DifferentialD; </mo>
         <mi> x </mi>
      </mrow>
   </mrow>
</mrow>
```

3.4.4 Underscript (`munder`)

3.4.4.1 Description

The syntax for the `munder` element is:

```
<munder>{base}{underscript} </munder>
```

3.4.4.2 Attributes

Name	values	default
accentunder	true | false	automatic

The `accentunder` attribute controls whether underscript is drawn as an 'accent' or as a limit. The main difference between an accent and a limit is that the limit is reduced in size whereas an accent is the same size as the base. A second difference is that the accent is drawn closer to the base.

The default value of `accentunder` is false, unless underscript is an `mo` element or an embellished operator (see section 3.2.4). If underscript is an `mo` element, the value of its `accent` attribute is used as the default value of `accentunder`. If underscript is an embellished operator, the `accent` attribute of the `mo` element at its core is used as the default value. As with all attributes, an explicitly given value overrides the default.

Here is an example (accent versus underscript): $\underbrace{x+y+z}$ versus $\underbrace{x+y+z}$. The MathML representation for this example is shown below.

If the base is an operator with `movablelimits`=`true` (or an embellished operator whose `mo` element core has `movablelimits`=`true`), and `displaystyle`=`false`, then underscript is drawn in a subscript position. In this case, the `accentunder` attribute is ignored. This is often used for limits on symbols such as `&sum;`

Within underscript, `munder` always sets `displaystyle` to `false`, but increments `scriptlevel` by 1 only when `accentunder` is `false`. Within base, it always leaves both attributes unchanged. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.4.3 Examples

The MathML representation for the example shown above is:

```
<mrow>
   <munder accentunder="true">
      <mrow>
         <mi> x </mi>
         <mo> + </mo>
         <mi> y </mi>
         <mo> + </mo>
         <mi> z </mi>
      </mrow>
      <mo> &UnderBrace; </mo>
   </munder>
   <mtext> versus </mtext>
   <munder accentunder="false">
      <mrow>
         <mi> x </mi>
```

```
        <mo> + </mo>
        <mi> y </mi>
        <mo> + </mo>
        <mi> z </mi>
      </mrow>
      <mo> &UnderBrace; </mo>
   </munder>
</mrow>
```

3.4.5 Overscript (`mover`)

3.4.5.1 Description

The syntax for the `mover` element is:

```
<mover>{base}{overscript} </mover>
```

3.4.5.2 Attributes

Name	values	default
accent	true | false	automatic

The `accent` attribute controls whether overscript is drawn as an 'accent' (diacritical mark) or as a limit. The main difference between an accent and a limit is that the limit is reduced in size whereas an accent is the same size as the base. A second difference is that the accent is drawn closer to the base. This is shown below (accent versus limit): $\hat{x}$ versus $\hat{x}$.

These differences also apply to 'mathematical accents' such as bars over expressions: $\overbrace{x+y+z}$ versus $\overset{\frown}{x+y+z}$. The MathML representation for each of these examples is shown below.

The default value of accent is false, unless overscript is an `mo` element or an embellished operator (see section 3.2.4). If overscript is an `mo` element, the value of its `accent` attribute is used as the default value of `accent` for `mover`. If overscript is an embellished operator, the `accent` attribute of the `mo` element at its core is used as the default value.

If the base is an operator with `movablelimits="true"` (or an embellished operator whose `mo` element core has `movablelimits="true"`), and `displaystyle="false"`, then overscript is drawn in a superscript position. In this case, the `accent` attribute is ignored. This is often used for limits on symbols such as `&sum;`

Within overscript, `mover` always sets `displaystyle` to `false`, but increments `scriptlevel` by 1 only when `accent` is `false`. Within base, it always leaves both attributes unchanged. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.5.3 Examples

The MathML representation for the examples shown above is:

```
<mrow>
   <mover accent="true">
      <mi> x </mi>
      <mo> &Hat; </mo>
   </mover>
   <mtext> versus </mtext>
   <mover accent="false">
      <mi> x </mi>
      <mo> &Hat; </mo>
   </mover>
</mrow>

<mrow>
   <mover accent="true">
      <mrow>
         <mi> x </mi>
         <mo> + </mo>
         <mi> y </mi>
         <mo> + </mo>
         <mi> z </mi>
      </mrow>
      <mo> &OverBar; </mo>
   </mover>
   <mtext> versus </mtext>
   <mover accent="false">
      <mrow>
         <mi> x </mi>
         <mo> + </mo>
         <mi> y </mi>
         <mo> + </mo>
         <mi> z </mi>
      </mrow>
      <mo> &OverBar; </mo>
   </mover>
</mrow>
```

3.4.6 Underscript-overscript Pair (`munderover`)

3.4.6.1 Description

The syntax for the `munderover` element is:

```
<munderover>{base}{underscript}{overscript} </munderover>
```

3.4.6.2 Attributes

The `munderover` element is used so that the underscript and overscript are vertically spaced equally in relation to the base and so that they follow the slant of the base as in the second

Name	values	default
accent	true | false	automatic
accentunder	true | false	automatic

expression shown below:

$$\int_0^\infty$$

versus

$$\int_0^\infty$$

The MathML representation for this example is shown below.

The difference in the vertical spacing is too small to be noticed on a low resolution display at a normal font size, but is noticeable on a higher resolution device such as a printer and when using large font sizes. In addition to the visual differences, attaching both the underscript and overscript to the same base more accurately reflects the semantics of the expression.

The `accent` and `accentunder` attributes have the same effect as the attributes with the same names on `mover` (section 3.4.5) and `munder` (section 3.4.4), respectively. Their default values are also computed in the same manner as described for those elements, with the default value of `accent` depending on overscript and the default value of `accentunder` depending on underscript.

If the base is an operator with `movablelimits`=`true` (or an embellished operator whose `mo` element core has `movablelimits`=`true`), and `displaystyle`=`false`, then `underscript` and overscript are drawn in a subscript and superscript position, respectively. In this case, the `accent` and `accentunder` attributes are ignored. This is often used for limits on symbols such as `&sum;`

Within underscript, `munderover` always sets `displaystyle` to `false`, but increments `scriptlevel` by 1 only when `accentunder` is `false`. Within overscript, `munderover` always sets `displaystyle` to `false`, but increments `scriptlevel` by 1 only when `accent` is `false`. Within base, it always leaves both attributes unchanged. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4).

3.4.6.3 Examples

The MathML representation for the example shown above with the left expression made using separate `munder` and `mover` elements, and the right one using an `munderover` element, is:

```
<mrow>
   <mover>
      <munder>
         <mo> &int; </mo>
         <mn> 0 </mn>
      </munder>
```

```
      <mi> &infin; </mi>
   </mover>
   <mtext> versus </mtext>
   <munderover>
      <mo> &int; </mo>
      <mn> 0 </mn>
      <mi> &infin; </mi>
   </munderover>
</mrow>
```

3.4.7 Prescripts and Tensor Indices (`mmultiscripts`)

3.4.7.1 Description

The syntax for the `mmultiscripts` element is:

```
<mmultiscripts>
    {base}
     ({subscript superscript})*
     [ <mprescripts/> {presubscript presuperscript})* ]
</mmultiscripts>
```

Presubscripts and tensor notations are represented by a single element, `mmultiscripts`. This element allows the representation of any number of vertically-aligned pairs of subscripts and superscripts, attached to one base expression. It supports both postscripts (to the right of the base in visual notation) and prescripts (to the left of the base in visual notation). Missing scripts can be represented by the empty element `none`.

The prescripts are optional, and when present are given after the postscripts, because prescripts are relatively rare compared to tensor notation.

The argument sequence consists of the base followed by zero or more pairs of vertically-aligned subscripts and superscripts (in that order) that represent all of the postscripts. This list is optionally followed by an empty element `mprescripts` and a list of zero or more pairs of vertically-aligned presubscripts and presuperscripts that represent all of the prescripts. The pair lists for postscripts and prescripts are given in a left-to-right order. If no subscript or superscript should be rendered in a given position, then the empty element `none` should be used in that position.

The base, subscripts, superscripts, the optional separator element `mprescripts`, the presubscripts, and the presuperscripts, are all direct sub-expressions of the `mmultiscripts` element, i.e. they are all at the same level of the expression tree. Whether a script argument is a subscript or a superscript, or whether it is a presubscript or a presuperscript is determined by whether it occurs in an even-numbered or odd-numbered argument position, respectively, ignoring the empty element `mprescripts` itself when determining the position. The first argument, the base, is considered to be in position 1. The total number of arguments must be odd, if `mprescripts` is not given, or even, if it is.

The empty elements `mprescripts` and `none` are only allowed as direct sub-expressions of `mmultiscripts`.

3.4.7.2 Attributes

Same as the attributes of `msubsup`.

The `mmultiscripts` element increments `scriptlevel` by 1, and sets `displaystyle` to `false`, within each of its arguments except base, but leaves both attributes unchanged within base. (These attributes are inherited by every element through its rendering environment, but can be set explicitly only on `mstyle`; see section 3.3.4.)

3.4.7.3 Examples

Two examples of the use of `mmultiscripts`:

${}_0F_1(;a;z)$.

```
<mrow>
   <mmultiscripts>
      <mi> F </mi>
      <mn> 1 </mn>
      <none/>
      <mprescripts/>
      <mn> 0 </mn>
      <none/>
   </mmultiscripts>
   <mo> &ApplyFunction; </mo>
   <mrow>
      <mo> ( </mo>
      <mrow>
         <mo> ; </mo>
         <mi> a </mi>
         <mo> ; </mo>
         <mi> z </mi>
      </mrow>
      <mo> ) </mo>
   </mrow>
</mrow>
```

${R_{i\,kl}^{\ j}}$ (where k and l are different indices)

```
<mmultiscripts>
   <mi> R </mi>
   <mi> i </mi>
   <none/>
   <none/>
   <mi> j </mi>
   <mi> k </mi>
   <none/>
   <mi> l </mi>
   <none/>
</mmultiscripts>
```

3.5 Tables and Matrices

Matrices, arrays and other table-like mathematical notation are marked up using `mtable`, `mtr`, `mlabeledtr` and `mtd` elements. These elements are similar to the `TABLE`, `TR` and `TD` el-

ements of HTML, except that they provide specialized attributes for the fine layout control necessary for commutative diagrams, block matrices and so on.

The `mlabeledtr` element represents a labeled row of a table and can be used for numbered equations. `mlabeledtr`'s first child is the label. A label is somewhat special in that it is not considered an expression in the matrix and is not counted when determining the number of columns in that row.

3.5.1 Table or Matrix (`mtable`)

3.5.1.1 Description

A matrix or table is specified using the `mtable` element. Inside of the `mtable` element, only `mtr` or `mlabeledtr` elements may appear.

In MathML 1.x, the `mtable` element could infer `mtr` elements around its arguments, and the `mtr` element could infer `mtd` elements. In other words, if some argument to an `mtable` was not an `mtr` element, a MathML applications was to assume a row with a single column (i.e. the argument was effectively wrapped with an inferred `mtr`). Similarly, if some argument to a (possibly inferred) `mtr` element was not an `mtd` element, that argument was to be treated as a table entry by wrapping it with an inferred `mtd` element.

In MathML 2.0, `mtr` and `mtd` elements are required, and may no longer be inferred. However, for backward compatibility renderers may wish to continue supporting inferred `mtr` and `mtd` elements. In this case, however, renderers should make an effort to notify users that inferred `mtr` and `mtd` elements are not valid in MathML 2.0.

Table rows that have fewer columns than other rows of the same table (whether the other rows precede or follow them) are effectively padded on the right with empty `mtd` elements so that the number of columns in each row equals the maximum number of columns in any row of the table. Note that the use of `mtd` elements with non-default values of the `rowspan` or `columnspan` attributes may affect the number of `mtd` elements that should be given in subsequent `mtr` elements to cover a given number of columns. Note also that the label in an `mtlabeledtr` element is not considered a column in the table.

3.5.1.2 Attributes

Issue (equation-numbering-examples): We need some way to specify the indentation to be used for the table and label. We seem to have concensus that we want to add an attribute: width = 'h-unit' Specifies the desired width of the entire table. When the value is a percentage value, the value is relative available horizontal space (typically the screen or page width). The default value is for the table to be as large as necessary. For tables with labels, one might say "width = '80%'". This would make the table 80% of the window width. If the 'math' tag centered its contents and the window width were 10in, this would result in 1in margins on either side. However, this leaves a lot of detail to work out. Basically, we need to deal with how columns shrink or expand when their 'natural' widths don't gibe with the width attribute.

Note that the default value for each of `rowlines`, `columnlines` and `frame` is the literal string 'none', meaning that the default is to render no lines, rather than that there is no default.

As described in section 2.3.3, the notation `(x | y)+` means one or more occurrences of either `x` or `y`, separated by whitespace. For example, possible values for `columnalign` are

Name	values	default
align	(top | bottom | center | baseline | axis) [rownumber]	axis
rowalign	(top | bottom | center | baseline | axis) +	baseline
columnalign	(left | center | right) +	center
groupalign	group-alignment-list-list	left
alignmentscope	(true | false) +	true
columnwidth	(auto | number h-unit | namedspace | fit) +	auto
rowspacing	(number v-unit) +	1.0ex
columnspacing	(number h-unit | namedspace) +	0.8em
rowlines	(none | solid | dashed) +	none
columnlines	(none | solid | dashed) +	none
frame	none | solid | dashed	none
framespacing	(number h-unit | namedspace) (number v-unit | namedspace)	0.4em 0.5ex
equalrows	true | false	true
equalcolumns	true | false	true
displaystyle	true | false	false
side	left | right | leftoverlap | rightoverlap	right
minlabelspacing	number h-unit	0.8em

`left`, `left left`, and `left right center center`. If there are more entries than are necessary (e.g. more entries than columns for `columnalign`), then only the first entries will be used. If there are fewer entries, then the last entry is repeated as often as necessary. For example, if `columnalign="right center"` and the table has three columns, the first column will be right aligned and the second and third columns will be centered. The label in a `mlabeledtr` is not considered as a column in the table and the attribute values that apply to columns do not apply to labels

The `align` attribute specifies where to align the table with respect to its environment. 'axis' means to align the center of the table on the environment's axis. (The axis of an equation is an alignment line used by typesetters. It is the line on which a minus sign typically lies. The center of the table is the midpoint of the table's vertical extent.) 'center' and 'baseline' both mean to align the center of the table on the environment's baseline. 'top' or 'bottom' aligns the top or bottom of the table on the environment's baseline.

If the `align` attribute value ends with a rownumber between 1 and *n* (for a table with *n* rows), the specified row is aligned in the way described above, rather than the table as a whole; the top (first) row is numbered 1, and the bottom (last) row is numbered *n*. The same is true if the `rownumber` is negative, between -1 and -*n*, except that the bottom row is referred to as -1 and the top row as -*n*. Other values of `rownumber` are illegal.

The `rowalign` attribute specifies how the entries in each row should be aligned. For example, 'top' means that the tops of each entry in each row should be aligned with the tops of the other entries in that row. The `columnalign` attribute specifies how the entries in each column should be aligned.

The `groupalign` and `alignmentscope` attributes are described with the alignment elements, `malignmgroup` and `malignmark`, in section 3.5.5.

The `columnwidth` attribute specifies how wide a column should be. The `auto` value means that the column should be as wide as needed, which is the default. If an explicit value is given, then the column is exactly that wide and the contents of that column are made to fit in that width. The contents are linewrapped or clipped at the discretion of the renderer. If

`fit` is given as a value, the remaining page width after subtracting the widths for columns specified as `auto` and/or specific widths is divided equally among the `fit` columns and this value is used for the column width. If insufficient room remains to hold the contents of the `fit` columns, renderers may linewrap or clip the contents of the `fit` columns.

The `rowspacing` and `columnspacing` attributes specify how much space should be added between each row and column. However, spacing before the first row and after the last row (i.e. at the top and bottom of the table) is given by the second number in the value of the `framespacing` attribute, and spacing before the first column and after the last column (i.e. on the left and on the right of the table) is given by the first number in the value of the `framespacing` attribute.

In those attributes' syntaxes, h-unit or v-unit represents a unit of horizontal or vertical length, respectively (see section 2.3.3.2). The units shown in the attributes' default values (`em` or `ex`) are typically used.

The `rowlines` and `columnlines` attributes specify whether and what kind of lines should be added between each row and column. Lines before the first row or column and after the last row or column are given using the `frame` attribute.

If a frame is desired around the table, the `frame` attribute is used. If the attribute value is not 'none', then `framespacing` is used to add spacing between the lines of the frame and the first and last rows and columns of the table. If `frame`="none", then the `framespacing` attribute is ignored. The `frame` and `framespacing` attributes are not part of the `rowlines`/`columnlines`, `rowspacing`/`columnspacing` options because having them be so would often require that `rowlines` and `columnlines` would need to be fully specified instead of just giving a single value. For example, if a table had five columns and we wanted lines between the columns, but no frame, then we would have to write `columnlines="none solid solid solid solid none"`. By separating the frame from the internal lines, we only need to write `columnlines="solid"`

The `equalrows` attribute forces the rows all to be the same total height when set to `true`. The `equalcolumns` attribute forces the columns all to be the same width when set to `true`.

The `displaystyle` attribute specifies the value of `displaystyle` (described under `mstyle` in section 3.3.4) within each cell (`mtd` element) of the table. Setting `displaystyle=true` can be useful for tables whose elements are whole mathematical expressions; the default value of `false` is appropriate when the table is part of an expression, for example, when it represents a matrix. In either case, `scriptlevel` (section 3.3.4) is not changed for the table cells.

The `side` attribute specifies what side of a table a label for a table row should should be placed. This attribute is intended to be used for labeled expressions. If `left` or `right` is specified, the label is placed on the left or right side of the table row respectively. The other two attribute values are variations on `left` and `right`: if the labeled row fits within the width allowed for the table without the label, but does not fit within the width if the label is included, then the label overlaps the row and is displayed above the row if `rowalign` for that row is `top`, otherwise the label is displayed below the row.

If there are multiple labels in a table, the aligment of the labels within the virtual column that they form is left-aligned for labels on the left side of the table, and right-aligned for labels on the right side of the table. The alignment can be overriden by specifying `columnalignment` for a `mlabeledtr` element.

The `minlabelspacing` attribute specifies the minimum space allowed between a label and the adjacent entry in the row.

3.5.1.3 Examples

A 3 by 3 identity matrix could be represented as follows:

```
<mrow>
   <mo> ( </mo>
   <mtable>
      <mtr> <mn>1</mn> <mn>0</mn> <mn>0</mn> </mtr>
      <mtr> <mn>0</mn> <mn>1</mn> <mn>0</mn> </mtr>
      <mtr> <mn>0</mn> <mn>0</mn> <mn>1</mn> </mtr>
   </mtable>
   <mo> ) </mo>
</mrow>
```

This might be rendered as:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note that the parentheses must be represented explicitly; they are not part of the `mtable` element's rendering. This allows use of other surrounding fences, such as brackets, or none at all.

3.5.2 Row in Table or Matrix (`mtr`)

3.5.2.1 Description

An `mtr` element represents one row in a table or matrix. An `mtr` element is only allowed as a direct sub-expression of an `mtable` element, and specifies that its contents should form one row of the table. Each argument of `mtr` is placed in a different column of the table, starting at the leftmost column.

As described in section 3.5.1, `mtr` elements are effectively padded on the right with `mtd` elements when they are shorter than other rows in a table.

3.5.2.2 Attributes

Name	values	default
rowalign	top | bottom | center | baseline | axis	inherited
columnalign	(left | center | right) +	inherited
groupalign	group-alignment-list-list	inherited

The `rowalign` and `columnalign` attributes allow a specific row to override the alignment specified by the same attributes in the surrounding `mtable` element.

As with `mtable`, if there are more entries than necessary in the value of `columnalign` (i.e. more entries than columns in the row), then the extra entries will be ignored. If there are fewer entries than columns, then the last entry will be repeated as many times as needed.

The `groupalign` attribute is described with the alignment elements, `maligngroup` and `malignmark`, in section 3.5.5.

3.5.3 Labeled Row in Table or Matrix (`mlabeledtr`)

3.5.3.1 Description

An `mlabeledtr` element represents one row in a table that has a label on either the left or right side, as determined by the `side` attribute. The label is the first child of `mlabeledtr`. The rest of the children represent the contents of the row and are identical to those used for `mtr`; all of the children except the first must be `mtd` elements.

An `mlabeledtr` element is only allowed as a direct sub-expression of an `mtable` element. Each argument of `mlabeledtr` except for the first argument (the label) is placed in a different column of the table, starting at the leftmost column.

Note that the label element is not considered to be a cell in the table row. In particular, the label element is not taken into consideration in the table layout for purposes of width and alignment calculations. For example, in the case of an `mlabeledtr` with a label and a single centered `mtd` child, the child is first centered in the enclosing `mtable`, and then the label is placed. Specifically, the child is not centered in the space that remains in the table after placing the label.

3.5.3.2 Attributes

The attributes for `mlabeledtr` are the same as for `mtr`. Unlike the attributes for the `mtable` element, attributes of `mlabeledtr` that apply to column elements also apply to the label. For example, in a one column table,

```
<mlabeledtr rowalign='center baseline'>
```

means that the label is vertically centered on the row, and that the actual entry is baseline aligned.

The default values for two attributes also differ between the `mlabeledtr` and `mtr` elements. For `mlabeledtr`, the default value for `columnwidth` is `fit`, and the default value for `columnalign` is `center`. The rationale is that when the `mlabeledtr` element is used for equation numbering, the most natural default is a centered equation with an equation number displayed flush with the right margin.

3.5.3.3 Equation Numbering

One of the important uses of `mlabeledtr` is for numbered equations. In a `mlabeledtr`, the label represents the equation number and the elements in the row are the equation being numbered. The `side` and `minlabelspacing` attributes of of `mtable` determine the placement of the equation number.

In larger documents with many numbered equations, automatic numbering becomes important. While automatic equation numbering and automatically resolving references to equation numbers is outside the scope of MathML, these problems can be addressed by the use of style sheets or other means. The mlabeledtr construction provides support for both of these functions in a way that is intended to facilitate XSL processing. The `mlabeledtr` element can be used to indicate the presence of a numbered equation, and the first child can be changed to the current equation number, along with incrementing the global equation

number. For cross references, a class id on either the mlabeledtr element or on the first element itself could be used as a target of any link.

```
<mtable>
 <mlabeledtr id='famousequation'>
  <mtext> (2.1) </mtext>
  <mtd>
   <mrow>
     <mi>E</mi>
     <mo>=</mo>
     <mrow>
      <mi>m</mi>
      <mo>&it;</mo>
      <msup>
       <mi>c</mi>
       <mn>2</mn>
      </msup>
     </mrow>
   </mrow>
  <mtd>
 </mlabeltr>
</mtable>
```

This should be rendered as:

$$E = mc^2 \quad (2.1)$$

3.5.4 Entry in Table or Matrix (`mtd`)

3.5.4.1 Description

An `mtd`element represents one entry in a table or matrix. An `mtd`element is only allowed as a direct sub-expression of an `mtr`element.

The `mtd`element accepts any number of arguments; if this number is not 1, its contents are treated as a single 'inferred `mrow`' formed from all its arguments, as described in section 3.1.3.

3.5.4.2 Attributes

Name	values	default
rowspan	number	1
columnspan	number	1
rowalign	top | bottom | center | baseline | axis	inherited
columnalign	left | center | right	inherited
groupalign	group-alignment-list	inherited

The `rowspan`and `columnspan`attributes allow a specific matrix element to be treated as if it occupied the number of rows or columns specified. The interpretation of how this larger

element affects specifying subsequent rows and columns is meant to correspond with the similar attributes for HTML 4.0 tables.

The `rowspan` and `columnspan` attributes can be used around an `mtd` element that represents the label in a `mlabeledtr` element. Also, the label of a `mlabeledtr` element is not considered to be part of a previous `rowspan` and `columnspan`

The `rowalign` and `columnalign` attributes allow a specific matrix element to override the alignment specified by a surrounding `mtable` or `mtr` element.

The `groupalign` attribute is described with the alignment elements, `maligngroup` and `malignmark`, in section 3.5.5.

3.5.5 Alignment Markers

3.5.5.1 Description

These are space-like elements (see section 3.2.6) that can be used to vertically align specified points within a column of MathML expressions, by the automatic insertion of the necessary amount of horizontal space between specified sub-expressions.

The discussion that follows will use the example of a set of simultaneous equations that should be rendered with vertical alignment of the coefficients and variables of each term, by inserting spacing somewhat like that shown here:

```
8.44x + 55  y =  0
3.1 x -  0.7y = -1.1
```

If the example expressions shown above were arranged in a column but not aligned, they would appear as:

```
8.44x + 55y = 0
3.1x - 0.7y = -1.1
```

(For audio renderers, it is suggested that the alignment elements produce the analogous behavior of altering the rhythm of pronunciation so that it is the same for several sub-expressions in a column, by the insertion of the appropriate time delays in place of the extra horizontal spacing described here.)

The expressions whose parts are to be aligned (each equation, in the example above) must be given as the table elements (i.e. as the `mtd` elements) of one column of an `mtable`. To avoid confusion, the term ‘table cell’ rather than ‘table element’ will be used in the remainder of this section.

All interactions between alignment elements are limited to the `mtable` column they arise in. That is, every column of a table specified by an `mtable` element acts as an ‘alignment scope’ that contains within it all alignment effects arising from its contents. It also excludes any interaction between its own alignment elements and the alignment elements inside any nested alignment scopes it might contain.

The reason `mtable` columns are used as alignment scopes is that they are the only general way in MathML to arrange expressions into vertical columns. Future versions of MathML may provide an `malignscope` element that allows an alignment scope to be created around any MathML element, but even then, table columns would still sometimes need to act as alignment scopes, and since they are not elements themselves, but rather are made from corresponding parts of the content of several `mtr` elements, they could not individually be the content of an alignment scope element.

An `mtable` element can be given the attribute `alignmentscope=false` to cause its columns not to act as alignment scopes. This is discussed further at the end of this section. Otherwise, the discussion in this section assumes that this attribute has its default value of `true`.

3.5.5.2 Specifying alignment groups

To cause alignment, it is necessary to specify, within each expression to be aligned, the points to be aligned with corresponding points in other expressions, and the beginning of each alignment group of sub-expressions that can be horizontally shifted as a unit to effect the alignment. Each alignment group must contain one alignment point. It is also necessary to specify which expressions in the column have no alignment groups at all, but are affected only by the ordinary column alignment for that column of the table, i.e. by the `columnalign` attribute, described elsewhere.

The alignment groups start at the locations of invisible `malignmark` elements, which are rendered with zero width when they occur outside of an alignment scope, but within an alignment scope are rendered with just enough horizontal space to cause the desired alignment of the alignment group that follows them. A simple algorithm by which a MathML application can achieve this is given later. In the example above, each equation would have one `malignmark` element before each coefficient, variable, and operator on the left-hand side, one before the = sign, and one before the constant on the right-hand side.

In general, a table cell containing *n* `malignmark` elements contains *n* alignment groups, with the *i*th group consisting of the elements entirely after the *i*th `malignmark` element and before the (*i*+1)-th; no element within the table cell's content should occur entirely before its first `malignmark` element.

Note that the division into alignment groups does not necessarily fit the nested expression structure of the MathML expression containing the groups - that is, it is permissible for one alignment group to consist of the end of one `mrow`, all of another one, and the beginning of a third one, for example. This can be seen in the MathML markup for the present example, given at the end of this section.

The nested expression structure formed by `mrow`s and other layout schemata should reflect the mathematical structure of the expression, not the alignment-group structure, to make possible optimal renderings and better automatic interpretations; see the discussion of proper grouping in section section 3.3.1. Insertion of alignment elements (or other space-like elements) should not alter the correspondence between the structure of a MathML expression and the structure of the mathematical expression it represents.

Although alignment groups need to coincide with the nested expression structure of layout schemata, there are nonetheless restrictions on where an `malignmark` element is allowed within a table cell. The `malignmark` element may only be contained within elements of the following types (which are themselves contained in the table cell):

- an `mrow` element, including an inferred `mrow` such as the one formed by a multi-argument `mtd` element;
- an `mstyle` element;
- an `mphantom` element;
- an `mfenced` element;
- an `maction` element, though only its selected sub-expression is checked;
- a `semantics` element.

These restrictions are intended to ensure that alignment can be unambiguously specifed, while avoiding complexities involving things like overscripts, radical signs and fraction bars. They also ensure that a simple algorithm suffices to accomplish the desired alignment.

Note that some positions for an `malignmark` element, although legal, are not useful, such as for an `malignmark` element to be an argument of an `mfenced` element. When inserting an `malignmark` element before a given element in pre-existing MathML, it will often be necessary, and always acceptable, to form a new `mrow` element to contain just the `malignmark` element and the element it is inserted before. In general, this will be necessary except when the `malignmark` element is inserted directly into an `mrow` or into an element that can form an inferred `mrow` from its contents. See the warning about the legal grouping of 'space-like elements' in section 3.2.6.

For the table cells that are divided into alignment groups, every element in their content must be part of exactly one alignment group, except the elements from the above list that contain `malignmark` elements inside them, and the `malignmark` elements themselves. This means that, within any table cell containing alignment groups, the first complete element must be an `malignmark` element, though this may be preceded by the begin tags of other elements.

This requirement removes a potential confusion about how to align elements before the first `malignmark` element, and makes it easy to identify table cells that are left out of their column's alignment process entirely.

Note that it is not required that the table cells in a column that are divided into alignment groups each contain the same number of groups. If they don't, zero-width alignment groups are effectively added on the right side of each table cell that has fewer groups than other table cells in the same column.

3.5.5.3 Table cells that are not divided into alignment groups

Expressions in a column that are to have no alignment groups should contain no `malignmark` elements. Expressions with no alignment groups are aligned using only the `columnalign` attribute that applies to the table column as a whole, and are not affected by the `groupalign` attribute described below. If such an expression is wider than the column width needed for the table cells containing alignment groups, all the table cells containing alignment groups will be shifted as a unit within the column as described by the `columnalign` attribute for that column. For example, a column heading with no internal alignment could be added to the column of two equations given above by preceding them with another table row containing an `mtext` element for the heading, and using the default `columnalign="center"` for the table, to produce:

```
equations with aligned variables
     8.44x + 55  y =  0
     3.1 x -  0.7y = -1.1
```

or, with a shorter heading,

```
   some equations
8.44x + 55  y =  0
3.1 x -  0.7y = -1.1
```

3.5.5.4 Specifying alignment points using *malignmark*

Each alignment group's alignment point can either be specified by an `malignmark` element anywhere within the alignment group (except within another alignment scope wholly contained inside it), or it is determined automatically from the `groupalign` attribute. The `groupalign` attribute can be specified on the group's preceding `malignngroup` element or on its surrounding `mtd`, `mtr`, or `mtable` elements. In typical cases, using the `groupalign` attribute is sufficient to describe the desired alignment points, so no `malignmark` elements need to be provided.

The `malignmark` element indicates that the alignment point should occur on the right edge of the preceding element, or the left edge of the following element or character, depending on the `edge` attribute of `malignmark`. Note that it may be necessary to introduce an `mrow` to group an `malignmark` element with a neighboring element, in order not to alter the argument count of the containing element. (See the warning about the legal grouping of 'space-like elements' in section 3.2.6).

When an `malignmark` element is provided within an alignment group, it can occur in an arbitrarily deeply nested element within the group, as long as it is not within a nested alignment scope. It is not subject to the same restrictions on location as `malignngroup` elements. However, its immediate surroundings need to be such that the element to its immediate right or left (depending on its `edge` attribute) can be unambiguously identified. If no such element is present, renderers should behave as if a zero-width element had been inserted there.

For the purposes of alignment, an element X is considered to be to the immediate left of an element Y, and Y to the immediate right of X, whenever X and Y are successive arguments of one (possibly inferred) `mrow` element, with X coming before Y. In the case of `mfenced` elements, MathML applications should evaluate this relation as if the `mfenced` element had been replaced by the equivalent expanded form involving `mrow`. Similarly, an `maction` element should be treated as if it were replaced by its currently selected subexpression. In all other cases, no relation of 'to the immediate left or right' is defined for two elements X and Y. However, in the case of content elements interspersed in presentation markup, MathML applications should attempt to evaluate this relation in a sensible way. For example, if a renderer maintains an internal presentation structure for rendering content elements, the relation could be evaluated with respect to that. (See chapter 4 and chapter 5 for further details about mixing presentation and content markup.)

Unlike all other elements in MathML, `malignmark` elements are allowed to occur within the content of token elements, such as `mn`, `mi`, or `mtext`. When this occurs, the character immediately before or after the `malignmark` element will carry the alignment point; in all other cases, the element to its immediate left or right will carry the alignment point. The rationale for this is that it is sometimes desirable to align on the edges of specific characters within multi-character token elements.

If there is more than one `malignmark` element in an alignment group, all but the first one will be ignored. MathML applications may wish to provide a mode in which they will warn about this situation, but it is not an error, and should trigger no warnings by default. (Rationale: it would be inconvenient to have to remove all unnecessary `malignmark` elements from automatically generated data, in certain cases, such as when they are used to specify alignment on 'decimal points' other than the '.' character.)

Name	values	default
edge	left | right	left

3.5.5.5 Attributes

`malignmark` has one attribute, `edge`, which specifies whether the alignment point will be found on the left or right edge of some element or character. The precise location meant by 'left edge' or 'right edge' is discussed below. If `edge`="right", the alignment point is the right edge of the element or character to the immediate left of the `malignmark` element. If `edge`="left", the alignment point is the left edge of the element or character to the immediate right of the `malignmark` element. Note that the attribute refers to the choice of edge rather than to the direction in which to look for the element whose edge will be used.

For `malignmark` elements that occur within the content of MathML token elements, the preceding or following character in the token element's content is used; if there is no such character, a zero-width character is effectively inserted for the purpose of carrying the alignment point on its edge. For all other `malignmark` elements, the preceding or following element is used; if there is no such element, a zero-width element is effectively inserted to carry the alignment point.

The precise definition of the 'left edge' or 'right edge' of a character or glyph (e.g. whether it should coincide with an edge of the character's bounding box) is not specified by MathML, but is at the discretion of the renderer; the renderer is allowed to let the edge position depend on the character's context as well as on the character itself.

For proper alignment of columns of numbers (using `groupalign` values of `left`, `right`, or `decimalpoint`), it is likely to be desirable for the effective width (i.e. the distance between the left and right edges) of decimal digits to be constant, even if their bounding box widths are not constant (e.g. if '1' is narrower than other digits). For other characters, such as letters and operators, it may be desirable for the aligned edges to coincide with the bounding box.

The 'left edge' of a MathML element or alignment group refers to the left edge of the leftmost glyph drawn to render the element or group, except that explicit space represented by `mspace` or `mtext` elements should also count as 'glyphs' in this context, as should glyphs that would be drawn if not for `mphantom` elements around them. The 'right edge' of an element or alignment group is defined similarly.

3.5.5.6 Attributes

Name	values	default
groupalign	left | center | right | decimalpoint	inherited

`maligngroup` has one attribute, `groupalign`, which is used to determine the position of its group's alignment point when no `malignmark` element is present. The following discussion assumes that no `malignmark` element is found within a group.

In the example given at the beginning of this section, there is one column of 2 table cells, with 7 alignment groups in each table cell; thus there are 7 columns of alignment groups, with 2 groups, one above the other, in each column. These columns of alignment groups should be given the 7 `groupalign` values 'decimalpoint left left decimalpoint left left decimalpoint', in that order. How to specify this list of values for a table cell or table column as

a whole, using attributes on elements surrounding the `maligngroup` element is described later.

If `groupalign` is 'left', 'right', or 'center', the alignment point is defined to be at the group's left edge, at its right edge, or halfway between these edges, respectively. The meanings of 'left edge' and 'right edge' are as discussed above in relation to `malignmark`.

If `groupalign` is 'decimalpoint', the alignment point is the right edge of the last character before the decimal point. The decimal point is the first '.' character (ASCII 0x2e) in the first `mn` element found along the alignment group's baseline. More precisely, the alignment group is scanned recursively, depth-first, for the first `mn` element, descending into all arguments of each element of the types `mrow` (including inferred `mrow`s), `mstyle`, `mpadded`, `mphantom`, `mfenced`, or `msqrt`, descending into only the first argument of each 'scripting' element (`msub`, `msup`, `msubsup`, `munder`, `mover`, `munderover`, `mmultiscripts`), or of each `mroot` or `semantics` element, descending into only the selected sub-expression of each `maction` element, and skipping the content of all other elements. The first `mn` so found always contains the alignment point, which is the right edge of the last character before the first decimal point in the content of the `mn` element. If there is no decimal point in the `mn` element, the alignment point is the right edge of the last character in the content. If the decimal point is the first character of the `mn` element's content, the right edge of a zero-width character inserted before the decimal point is used. If no `mn` element is found, the right edge of the entire alignment group is used (as for `groupalign="right"`).

In order to permit alignment on decimal points in `cn` elements, a MathML application can convert a content expression into a presentation expression that renders the same way before searching for decimal points as described above.

If characters other than '.' should be used as 'decimal points' for alignment, they should be preceded by `malignmark` elements within the `mn` token's content itself.

For any of the `groupalign` values, if an explicit `malignmark` element is present anywhere within the group, the position it specifies (described earlier) overrides the automatic determination of alignment point from the `groupalign` value.

3.5.5.7 Inheritance of *groupalign* values

It is not usually necessary to put a `groupalign` attribute on every `maligngroup` element. Since this attribute is usually the same for every group in a column of alignment groups to be aligned, it can be inherited from an attribute on the `mtable` that was used to set up the alignment scope as a whole, or from the `mtr` or `mtd` elements surrounding the alignment group. It is inherited via an 'inheritance path' that proceeds from `mtable` through successively contained `mtr`, `mtd`, and `maligngroup` elements. There is exactly one element of each of these kinds in this path from an `mtable` to any alignment group inside it. In general, the value of `groupalign` will be inherited by any given alignment group from the innermost element that surrounds the alignment group and provides an explicit setting for this attribute.

Note, however, that each `mtd` element needs, in general, a list of `groupalign` values, one for each `maligngroup` element inside it, rather than just a single value. Furthermore, an `mtr` or `mtable` element needs, in general, a list of lists of `groupalign` values, since it spans multiple `mtable` columns, each potentially acting as an alignment scope. Such lists of group-alignment values are specified using the following syntax rules:

```
group-alignment           := left | right | center | decimalpoint
group-alignment-list      := group-alignment +
group-alignment-list-list := ( ’{’ group-alignment-list ’}’ ) +
```

As described in section 2.3.3, | separates alternatives; + represents optional repetition (i.e. 1 or more copies of what precedes it), with extra values ignored and the last value repeated if necessary to cover additional table columns or alignment group columns; ’’ and ’’ represent literal braces; and (and) are used for grouping, but do not literally appear in the attribute value.

The permissible values of the groupalign attribute of the elements that have this attribute are specified using the above syntax definitions as follows:

Element type	groupalign attribute syntax	default value
mtable	group-alignment-list-list	left
mtr	group-alignment-list-list	inherited from mtable attribute
mtd	group-alignment-list	inherited from within mtr attribute
maligngroup	group-alignment	inherited from within mtd attribute

In the example near the beginning of this section, the group alignment values could be specified on every mtd element using groupalign= ‘decimalpoint left left decimalpoint left left decimalpoint’, or on every mtr element using groupalign= ‘decimalpoint left left decimalpoint left left decimalpoint’, or (most conveniently) on the mtable as a whole using groupalign= ‘decimalpoint left left decimalpoint left left decimalpoint’, which provides a single braced list of group-alignment values for the single column of expressions to be aligned.

3.5.5.8 MathML representation of an alignment example

The above rules are sufficient to explain the MathML representation of the example given near the start of this section. To repeat the example, the desired rendering is:

```
   8.44x + 55  y =  0
   3.1 x -  0.7y = -1.1
```

One way to represent that in MathML is:

```
<mtable groupalign="decimalpoint left left decimalpoint left left decimalpoint">
  <mtd>
    <mrow>
      <mrow>
        <maligngroup/>
        <mn> 8.44 </mn>
        <mo> &InvisibleTimes; </mo>
        <maligngroup/>
        <mi> x </mi>
      </mrow>
      <maligngroup/>
      <mo> + </mo>
      <mrow>
```

```
            <maligngroup/>
            <mn> 55 </mn>
            <mo> &InvisibleTimes; </mo>
            <maligngroup/>
            <mi> y </mi>
         </mrow>
      </mrow>
      <maligngroup/>
      <mo> = </mo>
      <maligngroup/>
      <mn> 0 </mn>
   </mtd>
   <mtd>
      <mrow>
         <mrow>
            <maligngroup/>
            <mn> 3.1 </mn>
            <mo> &InvisibleTimes; </mo>
            <maligngroup/>
            <mi> x </mi>
         </mrow>
         <maligngroup/>
         <mo> - </mo>
         <mrow>
            <maligngroup/>
            <mn> 0.7 </mn>
            <mo> &InvisibleTimes; </mo>
            <maligngroup/>
            <mi> y </mi>
         </mrow>
      </mrow>
      <maligngroup/>
      <mo> = </mo>
      <maligngroup/>
      <mrow>
         <mo> - </mo>
         <mn> 1.1 </mn>
      </mrow>
   </mtd>
</mtable>
```

3.5.5.9 Further details of alignment elements

The alignment elements `maligngroup` and `malignmark` can occur outside of alignment scopes, where they are ignored. The rationale behind this is that in situations in which MathML is generated, or copied from another document, without knowing whether it will be placed inside an alignment scope, it would be inconvenient for this to be an error.

An `mtable` element can be given the attribute `alignmentscope=false` to cause its columns not to act as alignment scopes. In general, this attribute has the syntax `(true | false)`

+; if its value is a list of boolean values, each boolean value applies to one column, with the last value repeated if necessary to cover additional columns, or with extra values ignored. Columns that are not alignment scopes are part of the alignment scope surrounding the `mtable` element, if there is one. Use of `alignmentscope=false` allows nested tables to contain `malignmark` elements for aligning the inner table in the surrounding alignment scope.

As discussed above, processing of alignment for content elements is not well-defined, since MathML does not specify how content elements should be rendered. However, many MathML applications are likely to find it convenient to internally convert content elements to presentation elements that render the same way. Thus, as a general rule, even if a renderer does not perform such conversions internally, it is recommended that the alignment elements should be processed as if it did perform them.

A particularly important case for renderers to handle gracefully is the interaction of alignment elements with the `matrix` content element, since this element may or may not be internally converted to an expression containing an `mtable` element for rendering. To partially resolve this ambiguity, it is suggested, but not required, that if the `matrix` element is converted to an expression involving an `mtable` element, that the `mtable` element be given the attribute `alignmentscope=false`, which will make the interaction of the `matrix` element with the alignment elements no different than that of a generic presentation element (in particular, it will allow it to contain `malignmark` elements that operate within the alignment scopes created by the columns of an `mtable` that contains the `matrix` element in one of its table cells).

The effect of alignment elements within table cells that have non-default values of the `columnspan` or `rowspan` attributes is not specified, except that such use of alignment elements is not an error. Future versions of MathML may specify the behavior of alignment elements in such table cells.

The effect of possible linebreaking of an `mtable` element on the alignment elements is not specified.

3.5.5.10 A simple alignment algorithm

A simple algorithm by which a MathML applications can perform the alignment specified in this section is given here. Since the alignment specification is deterministic (except for the definition of the left and right edges of a character), any correct MathML alignment algorithm will have the same behavior as this one. Each `mtable` column (alignment scope) can be treated independently; the algorithm given here applies to one `mtable` column, and takes into account the alignment elements, the `groupalign` attribute described in this section, and the `columnalign` attribute described under `mtable` (section 3.5.1).

First, a rendering is computed for the contents of each table cell in the column, using zero width for all `malignmark` and `malignmark` elements. The final rendering will be identical except for horizontal shifts applied to each alignment group and/or table cell. The positions of alignment points specified by any `malignmark` elements are noted, and the remaining alignment points are determined using `groupalign` values.

For each alignment group, the horizontal positions of the left edge, alignment point, and right edge are noted, allowing the width of the group on each side of the alignment point (left and right) to be determined. The sum of these two ‘side-widths’, i.e. the sum of the

widths to the left and right of the alignment point, will equal the width of the alignment group.

Second, each column of alignment groups, from left to right, is scanned. The *i*th scan covers the *i*th alignment group in each table cell containing any alignment groups. Table cells with no alignment groups, or with fewer than *i* alignment groups, are ignored. Each scan computes two maximums over the alignment groups scanned: the maximum width to the left of the alignment point, and the maximum width to the right of the alignment point, of any alignment group scanned.

The sum of all the maximum widths computed (two for each column of alignment groups) gives one total width, which will be the width of each table cell containing alignment groups. Call the maximum number of alignment groups in one cell *n*; each such cell's width is divided into 2*n* adjacent sections, called L(*i*) and R(*i*) for *i* from 1 to *n*, using the 2*n* maximum side-widths computed above; for each *i*, the width of all sections called L(*i*) is the maximum width of any cell's *i*th alignment group to the left of its alignment point, and the width of all sections called R(*i*) is the maximum width of any cell's *i*th alignment group to the right of its alignment point.

The alignment groups are then positioned in the unique way that places the part of each *i*th group to the left of its alignment point in a section called L(*i*), and places the part of each *i*th group to the right of its alignment point in a section called R(*i*). This results in the alignment point of each *i*th group being on the boundary between adjacent sections L(*i*) and R(*i*), so that all alignment points of *i*th groups have the same horizontal position.

The widths of the table cells that contain no alignment groups were computed as part of the initial rendering, and may be different for each cell, and different from the single width used for cells containing alignment groups. The maximum of all the cell widths (for both kinds of cells) gives the width of the table column as a whole.

The position of each cell in the column is determined by the applicable part of the value of the `columnalign` attribute of the innermost surrounding `mtable`, `mtr`, or `mtd` element that has an explicit value for it, as described in the sections on those elements. This may mean that the cells containing alignment groups will be shifted within their column, in addition to their alignment groups having been shifted within the cells as described above, but since each such cell has the same width, it will be shifted the same amount within the column, thus maintaining the vertical alignment of the alignment points of the corresponding alignment groups in each cell.

3.6 Enlivening Expressions

3.6.1 Bind Action to Sub-Expression (`maction`)

There are many ways in which it might be desirable to make mathematical content active. Adding a link to a MathML sub-expressions is one basic kind of interactivity section 7.1.5. However, many other kinds of interactivity cannot be easily accommodated by generic linking mechanisms. For example, in lengthy mathematical expressions, the ability to 'fold' expressions might be provided, i.e. a renderer might allow a reader to toggle between an ellipsis and a much longer expression that it represents.

To provide a mechanism for binding actions to expressions, MathML provides the `maction` element. This element accepts any number of sub-expressions as arguments, and the following attributes:

Name	values	default
actiontype	(described below)	(required attribute, no default value)
selection	positive-integer	1

By default, MathML applications that do not recognize the specified `actiontype` should render the selected sub-expression as defined below. If no selected sub-expression exists, it is a MathML error; the appropriate rendering in that case is as described in section 7.2.2 on the treatment of MathML errors.

Since a MathML-compliant application is not required to recognize any particular `actiontype`, an application can be fully MathML compliant just by implementing the above-described default behavior.

The `selection` attribute is provided for those `actiontype`s that permit someone viewing a document to select one of several sub-expressions for viewing. Its value should be a positive integer that indicates one of the sub-expressions of the `maction` element, numbered from 1 to the number of children of the element. When this is the case, the sub-expression so indicated is defined to be the 'selected sub-expression' of the `maction` element; otherwise the 'selected sub-expression' does not exist, which is an error. When the `selection` attribute is not specified (including for actiontypes for which it makes no sense), its default value is 1, so the selected sub-expression will be the first sub-expression.

Furthermore, as described in chapter 7, if a MathML application responds to a user command to copy a MathML sub-expression to the environment's 'clipboard', any `maction` elements present in what is copied should be given selection attributes that correspond to their selection state in the MathML rendering at the time of the copy command.

A suggested list of `actiontype`s and their associated actions is given below. Keep in mind, however, that this list is mainly for illustration, and recognized values and behaviors will vary from application to application.

<maction actiontype="toggle" selection="positive-integer" > (first expression) (second expression)... </maction> For this action type, a renderer would alternately display the given expressions, cycling through them when a reader clicked on the active expression, starting with the selected expression and updating the `selection` attribute value as described above. Typical uses would be for exercises in education, ellipses in long computer algebra output, or to illustrate alternate notations. Note that the expressions may be of significantly different size, so that size negotiation with the browser may be desirable. If size negotiation is not available, scrolling, elision, panning, or some other method may be necessary to allow full viewing.

<maction actiontype="statusline"> (expression) (message) </maction> In this case, the renderer would display the expression in context on the screen. When a reader clicked on the expression or moved the mouse over it, the renderer would send a rendering of the message to the browser statusline. Since most browsers in the forseeable future are likely to be limited to displaying text on their statusline, authors would presumably use plain text in an `mtext` element for the message in most circumstances. For non-`mtext` messages, renderers might provide a natural language translation of the markup, but this is not required.

<maction actiontype="tooltip"> (expression) (message) </maction> Here the renderer would also display the expression in context on the screen. When the mouse pauses over the expression for a long enough delay time, the renderer displays a rendering of the message in a pop-up 'tooltip' box near the expression. These

message boxes are also sometimes called ‘balloon help’ boxes. Presumably authors would use plain text in an `mtext` element for the message in most circumstances. For non-`mtext` messages, renderers may provide a natural language translation of the markup if full MathML rendering is not practical, but this is not required.

<maction actiontype="highlight" other="color=’#ff0000’"> expression </maction> <maction actiontype="highli

In this case, a renderer might highlight the enclosed expression on a ‘mouse-over’ event. In the example given above, use is being made of the ‘other’ attribute to pass non-standard attributes to renderers that support them, without violating the MathML DTD (see section 7.2.3). The ‘color’ attribute changes the color of the characters in the presentation, while the ‘background’ attribute changes the color of the background behind the characters.

<maction actiontype="menu" selection="1" > (menu item 1) (menu item 2) ... </maction>

This action type instructs a renderer to provide a pop up menu. This allows a one-to-many linking capability. Note that the menu items may be other <maction actiontype="menu">...</maction> expressions, thereby allowing nested menus.

Chapter 4

Content Markup

4.1 Introduction

4.1.1 The Intent of Content Markup

As has been noted in the introductory section of this recommendation, mathematics can be distinguished by its use of a (relatively) formal language, mathematical notation. However, mathematics and its presentation should not be viewed as one and the same thing. Mathematical sums or products exist and are meaningful to many applications completely without regard to how they are rendered aurally or visually. The intent of the content markup in the Mathematical Markup Language is to provide an explicit encoding of the underlying mathematical structure of an expression, rather than any particular rendering for the expression.

There are many reasons for providing a specific encoding for content. Even a disciplined and systematic use of presentation tags cannot properly capture this semantic information. This is because without additional information it is impossible to decide if a particular presentation was chosen deliberately to encode the mathematical structure or simply to achieve a particular visual or aural effect. Furthermore, an author using the same encoding to deal with both the presentation and mathematical structure might find a particular presentation encoding unavailable simply because convention had reserved it for a different semantic meaning.

The difficulties stem from the fact that there are many to one mappings from presentation to semantics and vice versa. For example the mathematical construct 'H multiplied by e' is often encoded using an explicit operator as in $H \times e$. In different presentational contexts, the multiplication operator might be invisible '$H\,e$', or rendered as the spoken word 'times'. Generally, many different presentations are possible depending on the context and style preferences of the author or reader. Thus, given '$H\,e$' out of context it may be impossible to decide if this is the name of a chemical or a mathematical product of two variables H and e.

Mathematical presentation also changes with culture and time: some expressions in combinatorial mathematics today have one meaning to an Russian mathematician, and quite another to a French mathematician; see section 5.4.1 for an example. Notations may lose currency, for example the use of musical sharp and flat symbols to denote maxima and minima [Chaundy1954]. A notation in use in 1644 for the multiplication mentioned above was ■He [Cajori1928].

When we encode the underlying mathematical structure explicitly, without regard to how it is presented aurally or visually, we are able to interchange information more precisely with

those systems that are able to manipulate the mathematics. In the trivial example above, such a system could substitute values for the variables *H* and *e* and evaluate the result. Further interesting application areas include interactive textbooks and other teaching aids.

4.1.2 The Scope of Content Markup

The semantics of general mathematical notation is not a matter of consensus. It would be an enormous job to systematically codify most of mathematics - a task that can never be complete. Instead, MathML makes explicit a relatively small number of commonplace mathematical constructs, chosen carefully to be sufficient in a large number of applications. In addition, it provides a mechanism for associating semantics with new notational constructs. In this way, mathematical concepts that are not in the base collection of elements can still be encoded (section 4.2.6).

The base set of content elements are chosen to be adequate for simple coding of most of the formulas used from kindergarten to the end of high school in the United States, and probably beyond through the first two years of college, that is up to A-Level or Baccalaureate level in Europe. Subject areas covered to some extent in MathML are:

- arithmetic, algebra, logic and relations
- calculus and vector calculus
- set theory
- sequences and series
- elementary classical functions
- statistics
- linear algebra

It is not claimed, or even suggested, that the proposed set of elements is complete for these areas, but the provision for author extensibility greatly alleviates any problem omissions from this finite list might cause.

4.1.3 Basic Concepts of Content Markup

The design of the MathML content elements are driven by the following principles:

- The expression tree structure of a mathematical expression should be directly encoded by the MathML content elements.
- The encoding of an expression tree should be explicit, and not dependent on the special parsing of PCDATAor on additional processing such as operator precedence parsing.
- The basic set of mathematical content constructs that are provided should have default mathematical semantics.
- There should be a mechanism for associating specific mathematical semantics with the constructs.

The primary goal of the content encoding is to establish explicit connections between mathematical structures and their mathematical meanings. The content elements correspond directly to parts of the underlying mathematical expression tree. Each structure has an associated default semantics and there is a mechanism for associating new mathematical definitions with new constructs.

Significant advantages to the introduction of content-specific tags include:

- Usage of presentation elements is less constrained. When mathematical semantics are inferred from presentation markup, processing agents must either be quite sophisticated, or they run the risk of inferring incomplete or incorrect semantics when irregular constructions are used to achieve a particular aural or visual effect.
- It is immediately clear which kind of information is being encoded simply by the kind of elements that are used.
- Combinations of semantic and presentation elements can be used to convey both the appearance and its mathematical meaning much more effectively than simply trying to infer one from the other.

Expressions described in terms of content elements must still be rendered. For common expressions, default visual presentations are usually clear. 'Take care of the sense and the sounds will take care of themselves' wrote Lewis Carroll [Carroll1871]. Default presentations are included in the detailed description of each element occurring in section 4.4.

To accomplish these goals, the MathML content encoding is based on the concept of an expression tree. A content expression tree is constructed from a collection of more primitive objects, referred to herein as containers and operators. MathML possesses a rich set of predefined container and operator objects, as well as constructs for combining containers and operators in mathematically meaningful ways. The syntax and usage of these content elements and constructions is described in the next section.

4.2 Content Element Usage Guide

Since the intent of MathML content markup is to encode mathematical expressions in such a way that the mathematical structure of the expression is clear, the syntax and usage of content markup must be consistent enough to facilitate automated semantic interpretation. There must be no doubt when, for example, an actual sum, product or function application is intended and if specific numbers are present, there must be enough information present to reconstruct the correct number for purposes of computation. Of course, it is still up to a MathML-compliant processor to decide what is to be done with such a content-based expression, and computation is only one of many options. A renderer or a structured editor might simply use the data and its own built-in knowledge of mathematical structure to render the object. Alternatively, it might manipulate the object to build a new mathematical object. A more computationally oriented system might attempt to carry out the indicated operation or function evaluation.

The purpose of this section is to describe the intended, consistent usage. The requirements involve more than just satisfying the syntactic structure specified by an XML DTD. Failure to conform to the usage as described below will result in a MathML error, even though the expression may be syntactically valid according to the DTD.

In addition to the usage information contained in this section, section 4.4 gives a complete listing of each content element, providing reference information about their attributes, syntax, examples and suggested default semantics and renderings. The rules for using presentation markup within content markup are explained in section 5.2.3. An informal EBNF grammar describing the syntax for the content markup is given in appendix B.

4.2.1 Overview of Syntax and Usage

MathML content encoding is based on the concept of an expression tree. As a general rule, the terminal nodes in the tree represent basic mathematical objects, such as numbers, vari-

ables, arithmetic operations and so on. The internal nodes in the tree generally represent some kind of function application or other mathematical construction that builds up a compound object. Function application provides the most important example; an internal node might represent the application of a function to several arguments, which are themselves represented by the terminal nodes underneath the internal node.

The MathML content elements can be grouped into the following categories based on their usage:

- containers
- operators and functions
- qualifiers
- relations
- conditions
- semantic mappings

These are the building blocks out of which MathML content expressions are constructed. Each category is discussed in a separate section below. In the remainder of this section, we will briefly introduce some of the most common elements of each type, and consider the general constructions for combining them in mathematically meaningful ways.

4.2.1.1 Constructing Mathematical Objects

Content expression trees are built up from basic mathematical objects. At the lowest level, leaf nodes are encapsulated in non-empty elements that define their type. Numbers and symbols are marked by the token elements `cn` and `ci`. More elaborate constructs such as sets, vectors and matrices are also marked using elements to denote their types, but rather than containing data directly, these container elements are constructed out of other elements. Elements are used in order to clearly identify the underlying objects. In this way, standard XML parsing can be used and attributes can be used to specify global properties of the objects.

The containers such as `<cn>12345<cn/>` `<ci>x</ci>` and `<csymbol definitionURL="mySymbol.htm" encoding="text">S</csymbol>` represent mathematical numbers , identifiers and externally defined symbols. Below, we will look at `operator` elements such as `plus` or `sin`, which provide access to the basic mathematical operations and functions applicable to those objects. Additional containers such as `set` for sets, and `matrix` for matrices are provided for representing a variety of common compound objects.

For example, the number 12345 is encoded as

```
<cn>12345</cn>
```

The attributes and `PCDATA` content together provide the data necessary for an application to parse the number. For example, a default base of 10 is assumed, but to communicate that the underlying data was actually written in base 8, simply set the `base` attribute to 8 as in

```
<cn base="8">12345</cn>
```

while the complex number 3 + 4i can be encoded as

```
<cn type="complex">3<sep/>4</cn>
```

Such information makes it possible for another application to easily parse this into the correct number.

As another example, the scalar symbol *v* is encoded as

```
<ci>v</ci>
```

By default, `ci` elements represent elements from a commutative field (see appendix C). If a vector is intended then this fact can be encoded as

```
<ci type="vector">v</ci>
```

This invokes default semantics associated with the `vector` element, namely an arbitrary element of a finite-dimensional vector space.

By using the `ci` and `csymbol` elements we have made clear that we are referring to a mathematical identifier or symbol but this does not say anything about how it should be rendered. By default a symbol is rendered as if the `ci` or `csymbol` element were actually the presentation element `mi` (see section 3.2.2). The actual rendering of a mathematical symbol can be made as elaborate as necessary simply by using the more elaborate presentational constructs (as described in chapter 3) in the body of the `ci` or `csymbol` element.

The default rendering of a simple `cn`-tagged object is the same as for the presentation element `mn` with some provision for overriding the presentation of the `PCDATA` by providing explicit `mn` tags. This is described in detail in section 4.4.

The issues for compound objects such as sets, vectors and matrices are all similar to those outlined above for numbers and symbols. Each such object has global properties as a mathematical object that impact how they are to be parsed. This may affect everything from the interpretation of operations that are applied to them through to how to render the symbols representing them. These mathematical properties are captured by setting attribute values.

4.2.1.2 Constructing General Expressions

The notion of constructing a general expression tree is essentially that of applying an operator to sub-objects. For example, the sum $a + b$ can be thought of as an application of the addition operator to two arguments *a* and *b*. In MathML, elements are used for operators for much the same reason that elements are used to contain objects. They are recognized at the level of XML parsing, and their attributes can be used to record or modify the intended semantics. For example, with the MathML `plus` element, setting the `definitionURL` and `encoding` attributes as in

```
<plus definitionURL="www.vnbooks.com/VectorCalculus.htm"
     encoding="text"/>
```

can communicate that the intended operation is vector-based.

There is also another reason for using elements to denote operators. There is a crucial semantic distinction between the function itself and the expression resulting from applying that function to zero or more arguments which must be captured. This is addressed by making the functions self-contained objects with their own properties and providing an explicit `apply` construct corresponding to function application. We will consider the `apply` construct in the next section.

MathML contains many pre-defined operator elements, covering a range of mathematical subjects. However, an important class of expressions involve unknown or user-defined functions and symbols. For these situations, MathML provides a general `csymbol` element, which is discussed below.

4.2.1.3 The *apply* construct

The most fundamental way of building up a mathematical expression in MathML content markup is the `apply` construct. An `apply` element typically applies an operator to its arguments. It corresponds to a complete mathematical expression. Roughly speaking, this means a piece of mathematics that could be surrounded by parentheses or 'logical brackets' without changing its meaning.

For example, $(x + y)$ might be encoded as

```
<apply>
  <plus/>
  <ci> x </ci>
  <ci> y </ci>
</apply>
```

The opening and closing tags of `apply` specify exactly the scope of any operator or function. The most typical way of using `apply` is simple and recursive. Symbolically, the content model can the described as:

```
<apply> op a b </apply>
```

where the operands a and b are containers or other content-based elements themselves, and op is an operator or function. Note that since `apply` is a container, this allows `apply` constructs to be nested to arbitrary depth.

An `apply` may in principle have any number of operands:

```
<apply> op a b [c...] <apply>
```

For example, $(x + y + z)$ can be encoded as

```
<apply>
  <plus/>
  <ci> a </ci>
  <ci> b </ci>
  <ci> c </ci>
</apply>
```

Mathematical expressions involving a mixture of operations result in nested occurrences of `apply` For example, $ax + b$ would be encoded as

```
<apply>
  <plus/>
  <apply>
    <times/>
    <ci> a </ci>
    <ci> x </ci>
  </apply>
```

```
  <ci> b </ci>
</apply>
```

There is no need to introduce parentheses or to resort to operator precedence in order to parse the expression correctly. The `apply` tags provide the proper grouping for the re-use of the expressions within other constructs. Any expression enclosed by an `apply` element is viewed as a single coherent object.

An expression such as $(F + G)(x)$ might be a product, as in

```
<apply>
  <times/>
  <apply>
    <plus/>
    <ci> F </ci>
    <ci> G </ci>
  </apply>
  <ci> x </ci>
</apply>
```

or it might indicate the application of the function $F + G$ to the argument x. This is indicated by constructing the sum

```
<apply>
  <plus/>
  <ci> F </ci>
  <ci> G </ci>
</apply>
```

and applying it to the argument x as in

```
<apply>
  <apply>
    <plus/>
    <ci> F </ci>
    <ci> G </ci>
    </apply>
  <ci> x </ci>
</apply>
```

Both the function and the arguments may be simple identifiers or more complicated expressions.

In MathML 1.0 , another construction closely related to the use of the `apply` element with operators and arguments was the `reln` element. The `reln` element was used to denote that a mathematical relation holds between its arguments, as opposed to applying an operator. Thus, the MathML markup for the expression $x < y$ was given in MathML 1.0 by:

```
<reln>
  <lt/>
  <ci> x </ci>
  <ci> y </ci>
</reln>
```

In MathML 2.0, the `apply` construct is used with all operators, including logical operators. The expression above becomes

```
<apply>
  <lt/>
  <ci> x </ci>
  <ci> y </ci>
</apply>
```

in MathML 2.0. The use of `reln` with relational operators is supported for reasons of backwards compatibility, but deprecated. Authors creating new content are encouraged to use `apply` in all cases.

4.2.1.4 Explicitly defined functions and operators

The most common operations and functions such as `plus` and `sin` have been predefined explicitly as empty elements (see section 4.4). They have `type` and `definitionURL` attributes, and by changing these attributes, the author can record that a different sort of algebraic operation is intended. This allows essentially the same notation to be re-used for a discussion taking place in a different algebraic domain.

Due to the nature of mathematics the notation must be extensible. The key to extensibility is the ability of the user to define new functions and other symbols to expand the terrain of mathematical discourse.

It is always possible to create arbitrary expressions, and then to use them as symbols in the language. Their properties can then be inferred directly from that usage as was done in the previous section. However, such an approach would preclude being able to encode the fact that the construct was a known symbol, or to record its mathematical properties except by actually using it. The `csymbol` element is used as a container to construct a new symbol in much the same way that `ci` is used to construct an identifier. (Note that 'symbol' is used here in the abstract sense and has no connection with any presentation of the construct on screen or paper). The difference in usage is that `csymbol` should refer to some mathematically defined concept with an external definition referenced via the `definitionURL` attribute, whereas `ci` is used for identifiers that are essentially 'local' to the MathML expression and do not use any external definition mechanism. The target of the `definitionURL` attribute on the `csymbol` element may encode the definition in any format: the particular encoding in use is given by the `encoding` attribute

To use `csymbol` to describe a completely new function, we write for example

```
<csymbol definitionURL="www.vnbooks.com/VectorCalculus.htm"
        encoding="text">
  <ci>Christoffel</ci>
</csymbol>
```

The `definitionURL` attribute specifies a URI that provides a written definition for the `Christoffel` symbol. Suggested default definitions for the content elements of MathML appear in appendix C in a format based on OpenMath, although there is no requirement that a particular format be used. The role of the `definitionURL` attribute is very similar to the role of definitions included at the beginning of many mathematical papers, and which often just refer to a definition used by a particular book.

MathML 1.0 supported the use of the `fn` to encode the fact that a construct is explicitly being used as a function or operator. To record the fact that $F + G$ is being used semantically as if it were a function, it was encoded as:

```
<fn>
  <apply>
    <plus/>
    <ci>F</ci>
    <ci>G</ci>
  </apply>
</fn>
```

This usage, although allowed in MathML 2.0 for reasons of backwards compatibility, is now deprecated. The fact that a construct is being used as an operator is clear from the position of the construct as the first child of the `apply` If it is required to add additional information to the construct, it should be wrapped in a `semantics` element, for example:

```
<semantics definitionURL="www.mathslib.com/vectorfuncs/plus.htm"
          encoding="Mathematica 4.0">
  <apply>
    <plus/>
    <ci>F</ci>
    <ci>G</ci>
  </apply>
</semantics>
```

MathML 1.0 supported the use of `definitionURL` with `fn` to refer to external definitions for user-defined functions. This usage, although allowed for reasons of backwards compatibility, is deprecated in MathML 2.0 in favour of using `csymbol` to define the function, and then `apply` to link the function to its arguments. For example:

```
  <apply>
    <csymbol definitionURL="http://www.defs.org/function_spaces.html#my_def"
           encoding="text">
      <ci>BigK</ci>
    </csymbol>
    <ci>x</ci>
    <ci>y</ci>
  </apply>
```

4.2.1.5 The inverse construct

Given functions, it is natural to have functional inverses. This is handled by the `inverse` element.

Functional inverses can be problematic from a mathematical point of view in that it implicitly involves the definition of an inverse for an arbitrary function F. Even at the K-through-12 level the concept of an inverse F^{-1} of many common functions F is not used in a uniform way. For example, the definitions used for the inverse trigonometric functions may differ slightly depending on the choice of domain and/or branch cuts.

MathML adopts the view: if F is a function from a domain D to D', then the inverse G of F is a function over D' such that $G(F(x)) = x$ for x in D. This definition does not assert that such an inverse exists for all or indeed any x in D, or that it is single-valued anywhere. Also, depending on the functions involved, additional properties such as $F(G(y)) = y$ for y in D' may hold.

The `inverse` element is applied to a function whenever an inverse is required. For example, application of the inverse sine function to x, i.e. $\sin^{-1}(x)$, is encoded as:

```
<apply>
   <apply> <inverse/> <sin/> </apply>
   <ci> x </ci>
</apply>
```

While `arcsin` is one of the predefined MathML functions, an explicit reference to $\sin^{-1}(x)$ might occur in a document discussing possible definitions of `arcsin`

4.2.1.6 The declare construct

Consider a document discussing the vectors $A = (a, b, c)$ and $B = (d, e, f)$, and later including the expression $V = A + B$. It is important to be able to communicate the fact that wherever A and B are used they represent a particular vector. The properties of that vector may determine aspects of operators such as `plus`

The simple fact that A is a vector can be communicated by using the markup

```
<ci type="vector">A</ci>
```

but this still does not communicate, for example, which vector is involved or its dimensions.

The `declare` construct is used to associate specific properties or meanings with an object. The actual declaration itself is not rendered visually (or in any other form). However, it indirectly impacts the semantics of all affected uses of the declared object.

The scope of a declaration is, by default, local to the MathML element in which the declaration is made. If the `scope` attribute of the `declare` element is set to `global`, the declaration applies to the entire MathML expression in which it appears.

The uses of the `declare` element range from resetting default attribute values to associating an expression with a particular instance of a more elaborate structure. Subsequent uses of the original expression (within the scope of the `declare`) play the same semantic role as would the paired object.

For example, the declaration

```
<declare>
  <ci> A </ci>
  <vector>
    <ci> a </ci>
    <ci> b </ci>
    <ci> c </ci>
  </vector>
</declare>
```

specifies that A stands for the particular vector (a, b, c) so that subsequent uses of A as in $V = A + B$ can take this into account. When `declare` is used in this way, the actual encoding

```
<apply>
  <eq/>
  <ci> V </ci>
  <apply>
    <plus/>
    <ci> A </ci>
    <ci> B </ci>
  </apply>
</apply>
```

remains unchanged but the expression can be interpreted properly as vector addition.

There is no requirement to declare an expression to stand for a specific object. For example, the declaration

```
<declare type="vector">
  <ci> A </ci>
</declare>
```

specifies that *A* is a vector without indicating the number of components or the values of specific components. The possible values for the `type` attribute include all the predefined container element names such as `vector`, `matrix` or `set` (see section 4.3.2.9).

4.2.1.7 The lambda construct

The lambda calculus allows a user to construct a function from a variable and an expression. For example, the lambda construct underlies the common mathematical idiom illustrated here:

Let f be the function taking x to $x^2 + 2$

There are various notations for this concept in mathematical literature, such as $\lambda(x, F(x)) = F$ or $\lambda(x, [F]) = F$, where x is a free variable in F.

This concept is implemented in MathML with the `lambda` element. A lambda construct with n internal variables is encoded by a `lambda` element with n+1 children. All but the last child must be `bvar` elements containing the identifiers of the internal variables. The last child is an expression defining the function. This is typically an `apply` but can also be any container element.

The following constructs $\lambda(x, \sin(x+1))$:

```
<lambda>
  <bvar><ci> x </ci></bvar>
  <apply>
    <sin/>
    <apply>
      <plus/>
      <ci> x </ci>
      <cn> 1 </cn>
    </apply>
  </apply>
</lambda>
```

To use `declare` and `lambda` to construct the function f for which $f(x) = x^2 + x + 3$ use:

```
<declare type="fn">
  <ci> f </ci>
  <lambda>
    <bvar><ci> x </ci></bvar>
    <apply>
      <plus/>
      <apply>
        <power/>
        <ci> x </ci>
        <cn> 2 </cn>
      </apply>
      <ci> x </ci>
      <cn> 3 </cn>
    </apply>
  </lambda>
</declare>
```

The following markup declares and constructs the function J such that $J(x, y)$ is the integral from x to y of t^4 with respect to t.

```
<declare type="fn">
  <ci> J </ci>
  <lambda>
    <bvar><ci> x </ci></bvar>
    <bvar><ci> y </ci></bvar>
    <apply> <int/>
      <bvar>
        <ci> t </ci>
      </bvar>
      <lowlimit>
        <ci> x </ci>
      </lowlimit>
      <uplimit>
        <ci> y </ci>
      </uplimit>
      <apply>   <power/>
        <ci>t</ci>
        <cn>4</cn>
      </apply>
    </apply>
  </lambda>
</declare>
```

The function J can then in turn be applied to an argument pair.

4.2.1.8 The use of qualifier elements and the condition construct

The last example of the preceding section illustrates the use of qualifier elements `lowlimit`, `uplimit`, and `bvar` used in conjunction with the `int` element. A number of common mathematical constructions involve additional data that is either implicit in conventional nota-

tion, such as a bound variable, or thought of as part of the operator rather than an argument, as is the case with the limits of a definite integral.

Content markup uses qualifier elements in conjunction with a number of operators, including integrals, sums, series, and certain differential operators. Qualifier elements appear in the same `apply` element with one of these operators. In general, they must appear in a certain order, and their precise meaning depends on the operators being used. For details, see section 4.2.3.2.

The qualifier element `bvar` is also used in another important MathML construction. The `condition` element is used to place conditions on bound variables in other expressions. This allows MathML to define sets by rule, rather than enumeration, for example. The following markup, for instance, encodes the set $x \mid x < 1$:

```
<set>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply>
      <lt/>
      <ci> x </ci>
      <cn> 1 </cn>
    </apply>
  </condition>
</set>
```

4.2.1.9 Rendering of Content elements

While the primary role of the MathML content element set is to directly encode the mathematical structure of expressions independent of the notation used to present the objects, rendering issues cannot be ignored. Each content element has a default rendering, given in section section 4.4, and several mechanisms (including section 4.3.3.2) are provided for associating a particular rendering with an object.

4.2.2 Containers

Containers provide a means for the construction of mathematical objects of a given type.

Tokens	`ci`, `cn`, `csymbol`
Constructors	`interval`, `list`, `matrix`, `matrixrow`, `set`, `vector`, `apply`, `reln`, `fn`, `lambda`
Specials	`declare`

4.2.2.1 Tokens

Token elements are typically the leaves of the MathML expression tree. Token elements are used to indicate mathematical identifiers, numbers and symbols.

It is also possible for the canonically empty operator elements such as `exp`, `sin` and `cos` to be leaves in an expression tree. The usage of operator elements is described in section 4.2.3.

cn The `cn` element is the MathML token element used to represent numbers. The supported types of numbers include: `real`, `integer`, `rational`, `complex-cartesian`, and `complex-polar`, with `real` being the default type. An attribute `base` (with default value `10`) is used to help specify how the content is to be parsed. The content itself is essentially `PCDATA`, separated by `<sep/>` when two parts are needed in order to fully describe a number. For example, the real number 3 is constructed by `<cn type="real"> 3 </cn>`, while the rational number 3/4 is constructed as `<cn type="rational"> 3<sep/>4 </cn>`. The detailed structure and specifications are provided in section 4.4.1.1.

ci The `ci` element, or 'content identifier' is used to construct a variable, or an identifier. A `type` attribute indicates the type of object the symbol represents. Typically, `ci` represents a real scalar, but no default is specified. The content is either `PCDATA` or a general presentation construct (see section 3.1.5). For example,

```
<ci>
<msub>
  <mi>c</mi>
  <mn>1</mn>
</msub>
</ci>
```

encodes an atomic symbol that displays visually as c_1 which, for purposes of content, is treated as a single symbol representing a real number. The detailed structure and specifications is provided in section 4.4.1.2.

csymbol The `csymbol` element, or 'content symbol' is used to construct a symbol whose semantics are not part of the core content elements provided by MathML, but defined externally. `csymbol` does not make any attempt to describe how to map the arguments occurring in any application of the function into a new MathML expression. Instead, it depends on its `definitionURL` attribute to point to a particular meaning, and the `encoding` attribute to give the syntax of this definition. The content of a `csymbol` is either `PCDATA` or a general presentation construct (see section 3.1.5). For example,

```
<csymbol definitionURL="www.vnbooks.com/ContDiffFuncs.htm"
        encoding="text">
<msup>
  <mi>C</mi>
  <mn>2</mn>
</msup>
</csymbol>
```

encodes an atomic symbol that displays visually as C^2 and that, for purposes of content, is treated as a single symbol representing the space of twice-differentiable continuous functions. The detailed structure and specifications is provided in section 4.4.1.3.

4.2.2.2 Constructors

MathML provides a number of elements for combining elements into familiar compound objects. The compound objects include things like lists, sets. Each constructor produces a new type of object.

interval The `interval` element is described in detail in section 4.4.2.4. It denotes an

interval on the real line with the values represented by its children as end points. The `closure` attribute is used to qualify the type of interval being represented. For example,

```
<interval closure="open-closed">
  <ci> a </ci>
  <ci> b </ci>
</interval>
```

represents the open-closed interval often written (*a*,*b*].

set and list The `set` and `list` elements are described in detail in section 4.4.6.1 and section 4.4.6.2. Typically, the child elements of a possibly empty `list` element are the actual components of an ordered list. For example, an ordered list of the three symbols *a*, *b*, and *c* is encoded as

```
<list> <ci> a </ci> <ci> b </ci> <ci> c </ci> </list>
```

Alternatively, `bvar` and `condition` elements can be used to define lists where membership depends on satisfying certain conditions. An `order` attribute, which is used to specify what ordering is to be used. When the nature of the child elements permits, the ordering defaults to a numeric or lexicographic ordering. Sets are structured much the same as lists except that there is no implied ordering and the `type` of set may be `normal` or `multiset` with `multiset` indicating that repetitions are allowed. For both sets and lists, the child elements must be valid MathML content elements. The type of the child elements is not restricted. For example, one might construct a list of equations, or inequalities.

matrix and matrixrow The `matrix` element is used to represent mathematical matrices. It is described in detail in section 4.4.10.2. It has zero or more child elements, all of which are `matrixrow` elements. These in turn expect zero or more child elements that evaluate to algebraic expressions or numbers. These sub-elements are often real numbers, or symbols as in

```
<matrix>
  <matrixrow> <cn> 1 </cn> <cn> 2 </cn> </matrixrow>
  <matrixrow> <cn> 3 </cn> <cn> 4 </cn> </matrixrow>
</matrix>
```

The `matrixrow` elements must always be contained inside of a matrix, and all rows in a given matrix must have the same number of elements. Note that the behavior of the `matrix` and `matrixrow` elements is substantially different from the `mtable` and `mtr` presentation elements.

vector The `vector` element is described in detail in section 4.4.10.1. It constructs vectors from an *n*-dimensional vector space so that its *n* child elements typically represent real or complex valued scalars as in the three-element vector

```
<vector>
  <apply>
    <plus/>
    <ci> x </ci>
    <ci> y </ci>
  </apply>
  <cn> 3 </cn>
  <cn> 7 </cn>
</vector>
```

apply The `apply` element is described in detail in section 4.4.2.1. Its purpose is apply a function or operator to its arguments to produce an an expression representing an element of the range of the function. It is involved in everything from forming sums such as $a + b$ as in

```
<apply>
  <plus/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

through to using the sine function to construct $\sin(a)$ as in

```
<apply>
  <sin/>
  <ci> a </ci>
</apply>
```

or constructing integrals. Its usage in any particular setting is determined largely by the properties of the function (the first child element) and as such its detailed usage is covered together with the functions and operators in section 4.2.3.

reln The `reln` element is described in detail in section 4.4.2.2. It was used in MathML 1.0 to construct an expression such as $a = b$, as in

```
<reln><eq/>
  <ci> a </ci>
  <ci> b </ci>
</reln>
```

indicating an intended comparison between two mathematical values. MathML 2.0 takes the view that this should be regarded as the application of a boolean function, and as such could be constructed using `apply` The use of `reln` with logical operators is supported for reasons of backwards compatibility, but deprecated in favour of `apply`

fn The `fn` element was used in MathML 1.0 to make explicit the fact that an expression is being used as a function or operator. This is allowed in MathML 2.0 for backwards compatibility, but is deprecated, as the use of an expression as a function or operator is clear from its position as the first child of an `apply` `fn` is discussed in detail in section 4.4.2.3.

lambda The `lambda` element is used to construct a user-defined function from an expression and one or more free variables. The lambda construct with n internal variables takes n+1 children. The first (second, up to n) is a `bvar` containing the identifiers of the internal variables. The last is an expression defining the function. This is typically an `apply` but can also be any container element. The following constructs $\lambda(x, \sin x)$

```
<lambda>
  <bvar><ci> x </ci></bvar>
  <apply>
    <sin/>
    <ci> x </ci>
  </apply>
</lambda>
```

The following constructs the constant function $\lambda(x, 3)$

```
<lambda>
```

```
    <bvar><ci> x </ci></bvar>
    <cn> 3 </cn>
  </lambda>
```

4.2.2.3 Special Constructs

The `declare` construct is described in detail in section 4.4.2.8. It is special in that its entire purpose is to modify the semantics of other objects. It is not rendered visually or aurally.

The need for declarations arises any time a symbol (including more general presentations) is being used to represent an instance of an object of a particular type. For example, you may wish to declare that the symbolic identifier *V* represents a vector.

The declaration

```
<declare type="vector"><ci>V</ci></declare>
```

resets the default type attribute of `<ci>V</ci>` to `vector` for all affected occurrences of `<ci>V</ci>`. This avoids having to write `<ci type="vector">V</ci>` every time you use the symbol.

More generally, `declare` can be used to associate expressions with specific content. For example, the declaration

```
<declare>
  <ci>F</ci>
  <lambda>
    <bvar><ci> U </ci></bvar>
    <apply>
      <int/>
      <bvar><ci> x </ci></bvar>
      <lowlimit><cn> 0 </cn></lowlimit>
      <uplimit><ci> a </ci></uplimit>
      <ci> U </ci>
    </apply>
  </lambda>
</declare>
```

associates the symbol *F* with a new function defined by the `lambda` construct. Within the scope where the declaration is in effect, the expression

```
<apply>
  <ci>F</ci>
  <ci> U </ci>
</apply>
```

stands for the integral of *U* from 0 to *a*.

The `declare` element can also be used to change the definition of a function or operator. For example, if the URL `http://.../MathML:noncommutplus` described a non-commutative plus operation encoded in Maple syntax, then the declaration

```
<declare definitionURL="http://.../MathML:noncommutplus"
```

```
       encoding="Maple V">
  <plus/>
</declare>
```

would indicate that all affected uses of `plus` are to be interpreted as having that definition of `plus`

4.2.3 Functions, Operators and Qualifiers

The operators and functions defined by MathML can be divided into categories as shown in the table below.

unary arithmetic	`exp`, `factorial`, `minus`, `abs`, `conjugate`, `arg`, `real`, `imaginary`
unary logical	`not`
unary functional	`inverse`, `ident`
unary elementary classical functions	`sin`, `cos`, `tan`, `sec`, `csc`, `cot`, `sinh`, `cosh`, `tanh`, `sech`, `csch`, `coth`, `arcsin`, `a`
unary linear algebra	`determinant`, `transpose`
unary calculus and vector calculus	`divergence`, `grad`, `curl`, `laplacian`
unary set-theoretic	`card`
binary arithmetic	`quotient`, `divide`, `minus`, `power`, `rem`
binary logical	`implies`, `equivalent`, `approx`
binary set operators	`setdiff`
binary linear algebra	`vectorproduct`, `scalarproduct`, `outerproduct`
n-ary arithmetic	`plus`, `times`, `max`, `min`, `gcd`
n-ary statistical	`mean`, `sdev`, `variance`, `median`, `mode`
n-ary logical	`and`, `or`, `xor`
n-ary linear algebra	`selector`
n-ary set operator	`union`, `intersect`
n-ary functional	`fn`, `compose`
integral, sum, product operators	`int`, `sum`, `product`
differential operator	`diff`, `partialdiff`
quantifier	`forall`, `exists`

From the point of view of usage, MathML regards functions (for example `sin` and `cos`) and operators (for example `plus` and `times`) in the same way. MathML predefined functions and operators are all canonically empty elements.

Note that the `csymbol` element can be used to construct a user-defined symbol that can be used as a function or operator.

4.2.3.1 Predefined functions and operators

MathML functions can be used in two ways. They can be used as the operator within an `apply` element, in which case they refer to a function evaluated at a specific value. For example,

```
<apply>
  <sin/>
  <cn>5</cn>
</apply>
```

denotes a real number, namely sin(5).

MathML functions can also be used as arguments to other operators, for example

```
<apply>
  <plus/><sin/><cos/>
</apply>
```

denotes a function, namely the result of adding the sine and cosine functions in some function space. (The default semantic definition of `plus` is such that it infers what kind of operation is intended from the type of its arguments.)

The number of child elements in the `apply` is defined by the element in the first (i.e. operator) position.

Unary operators are followed by exactly one other child element within the `apply`

Binary operators are followed by exactly two child elements.

N-ary operators are followed by zero or more child elements.

The one exception to these rules is that `declare` elements may be inserted in any position except the first. `declare` elements are not counted when satisfying the child element count for an `apply` containing a unary or binary operator element.

Integral, sum, product and differential operators are discussed below in section 4.2.3.2.

4.2.3.2 Operators taking Qualifiers

The table below contains the qualifiers and the operators taking qualifiers in MathML.

qualifiers	`lowlimit`, `uplimit`, `bvar`, `degree`, `logbase`, `interval`, `condition`
operators	`int`, `sum`, `product`, `root`, `diff`, `partialdiff`, `limit`, `log` `moment`, `min`, `max`, `forall`, `exists`

Operators taking qualifiers are canonically empty functions that differ from ordinary empty functions only in that they support the use of special qualifier elements to specify their meaning more fully. They are used in exactly the same way as ordinary operators, except that when they are used as operators, certain qualifier elements are also permitted to be in the enclosing `apply` They always precede the argument if it is present. If more than one qualifier is present, they appear in the order `bvar`, `lowlimit`, `uplimit`, `interval`, `condition`, `degree`, `logbase`. A typical example is:

```
<apply>
  <int/>
  <bvar><ci>x</ci></bvar>
  <lowlimit><cn>0</cn></lowlimit>
  <uplimit><cn>1</cn></uplimit>
  <apply>
      <power/>
    <ci>x</ci>
    <cn>2</cn>
  </apply>
</apply>
```

It is also valid to use qualifier schema with a function not applied to an argument. For example, a function acting on integrable functions on the interval [0,1] might be denoted:

```
<fn>
  <apply>
    <int/>
    <bvar><ci>x</ci></bvar>
    <lowlimit><cn>0</cn></lowlimit>
    <uplimit><cn>1</cn></uplimit>
  </apply>
</fn>
```

The meaning and usage of qualifier schema varies from function to function. The following list summarizes the usage of qualifier schema with the MathML functions taking qualifiers.

int The `int` function accepts the `lowlimit`, `uplimit`, `bvar`, `interval` and `condition` schemata. If both `lowlimit` and `uplimit` schema are present, they denote the limits of a definite integral. The domain of integration may alternatively be specified using interval or condition. The `bvar` schema signifies the variable of integration. When used with `int`, each qualifier schema is expected to contain a single child schema; otherwise an error is generated.

diff The `diff` function accepts the `bvar` schema. The `bvar` schema specifies with respect to which variable the derivative is being taken. The `bvar` may itself contain a `degree` schema that is used to specify the order of the derivative, i.e. a first derivative, a second derivative, etc. For example, the second derivative of f with respect to x is:

```
<apply>
  <diff/>
  <bvar>
    <ci> x </ci>
    <degree>
      <cn> 2 </cn>
    </degree>
  </bvar>
  <apply><fn><ci>f</ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

partialdiff The `partialdiff` function accepts zero or more `bvar` schemata. The `bvar` schema specify with respect to which variables the derivative is being taken. The `bvar` elements may themselves contain `degree` schemata that are used to specify the order of the derivative. Variables specified by multiple `bvar` elements will be used in order as the variable of differentiation in mixed partials. When used with `partialdiff`, the `degree` schema is expected to contain a single child schema. For example,

```
<apply>
  <partialdiff/>
  <bvar><ci>x</ci></bvar>
  <bvar><ci>y</ci></bvar>
  <fn><ci>f</ci></fn>
</apply>
```

denote the mixed partial $(\mathrm{d}^2 / \mathrm{d}x\mathrm{d}y)\, f$.

sum, product The `sum` and `product` functions accept the `bvar`, `lowlimit`, `uplimit`, `interval` and `condition` schemata. If both `lowlimit` and `uplimit` schemata are present, they denote the limits of the sum or product. The limits may alternatively be specified using the `interval` or `condition` schema. The `bvar` schema signifies the index variable in the sum or product. A typical example might be:

```
<apply>
  <sum/>
  <bvar><ci>i</ci></bvar>
  <lowlimit><cn>0</cn></lowlimit>
  <uplimit><cn>100</cn></uplimit>
  <apply>
    <power/>
    <ci>x</ci>
    <ci>i</ci>
  </apply>
</apply>
```

When used with `sum` or `product`, each qualifier schema is expected to contain a single child schema; otherwise an error is generated.

limit The `limit` function accepts zero or more `bvar` schemata, and optional `condition` and `lowlimit` schemata. A `condition` may be used to place constraints on the `bvar`. The `bvar` schema denotes the variable with respect to which the limit is being taken. The `lowlimit` schema denotes the limit point. When used with `limit`, the `bvar` and `lowlimit` schemata are expected to contain a single child schema; otherwise an error is generated.

log The `log` function accepts only the `logbase` schema. If present, the `logbase` schema denotes the base with respect to which the logarithm is being taken. Otherwise, the log is assumed to be base 10. When used with `log` the `logbase` schema is expected to contain a single child schema; otherwise an error is generated.

moment The `moment` function accepts only the `degree` schema. If present, the `degree` schema denotes the order of the moment. Otherwise, the moment is assumed to be the first order moment. When used with `moment`, the `degree` schema is expected to contain a single child schema; otherwise an error is generated.

min, max The `min` and `max` functions accept a `bvar` schema in cases where the maximum or minimum is being taken over a set of values specified by a `condition` schema together with an expression to be evaluated on that set. In MathML1.0, the `bvar` element was optional when using a `condition`; if a `condition` element containing a single variable was given by itself following a `min` or `max` operator, the variable was implicitly assumed to be bound, and the expression to be maximized or minimized (if absent) was assumed to be the single bound variable. This usage is deprecated in MathML 2.0 in favour of explicitly stating the bound variable(s) and the expression to be maximised in all cases. The `min` and `max` elements may also be applied to a list of values in which case no qualifier schemata are used. For examples of all three usages, see section 4.4.3.4.

forall, exists The universal and existential quantifier operators `forall` and `exists` are used in conjuction with one or more `bvar` schemata to represent simple logical assertions. There are two ways of using the logical quantifier operators. The first usage is for representing a simple, quantified assertion. For example, the statement ‘there exists $x < 9$’ would be represented as:

```
<apply>
  <exists/>
  <bvar><ci> x </ci></bvar>
  <apply><lt/>
    <ci> x </ci><cn> 9 </cn>
  </apply>
</apply>
```

The second usage is for representing implications. Hypotheses are given by a `condition` element following the bound variables. For example the statement 'for all $x < 9$, $x < 10$' would be represented as:

```
<apply>
  <forall/>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><lt/>
      <ci> x </ci><cn> 9 </cn>
    </apply>
  </condition>
  <apply><lt/>
    <ci> x </ci><cn> 10 </cn>
  </apply>
</apply>
```

Note that in both usages one or more `bvar` qualifiers are mandatory.

4.2.4 Relations

binary relation	`neq`, `equivalent`, `approx`
binary logical relation	`implies`
binary set relation	`in`, `notin`, `notsubset`, `notprsubset`
binary series relation	`tendsto`
n-ary relation	`eq`, `leq`, `lt`, `geq`, `gt`
n-ary set relation	`subset`, `prsubset`

The MathML content tags include a number of canonically empty elements which denote arithmetic and logical relations. Relations are characterized by the fact that, if an external application were to evaluate them (MathML does not specify how to evaluate expressions), they would typically return a truth value. By contrast, operators generally return a value of the same type as the operands. For example, the result of evaluating $a < b$ is either true or false (by contrast, 1 + 2 is again a number).

Relations are bracketed with their arguments using the `apply` element in the same way as other functions. In MathML 1.0, relational operators were bracketed using `reln`. This usage, although still supported, is now deprecated in favour of `apply`. The element for the relational operator is the first child element of the `apply`. Thus, the example from the preceding paragraph is properly marked up as:

```
<apply>
  <lt/>
  <ci>a</ci>
  <ci>b</ci>
```

</apply>

It is an error to enclose a relation in an element other than `apply` or `reln`.

The number of child elements in the `apply` is defined by the element in the first (i.e. relation) position.

Unary relations are followed by exactly one other child element within the `apply`

Binary relations are followed by exactly two child elements.

N-ary relations are followed by zero or more child elements.

The one exception to these rules is that `declare` elements may be inserted in any position except the first. `declare` elements are not counted when satisfying the child element count for an `apply` containing a unary or binary relation element.

4.2.5 Conditions

condition `condition`

The `condition` element is used to define the 'such that' construct in mathematical expressions. Condition elements are used in a number of contexts in MathML. They are used to construct objects like sets and lists by rule instead of by enumeration. They can be used with the `forall` and `exists` operators to form logical expressions. And finally, they can be used in various ways in conjunction with certain operators. For example, they can be used with and `int` element to specify domains of integration, or to specify argument lists for operators like `min` and `max`.

The `condition` element is always used together with one or more `bvar` elements.

The exact interpretation depends on the context, but generally speaking, the `condition` element is used to restrict the permissible values of a bound variable appearing in another expression to those that satisfy the relations contained in the `condition`. Similarly, when the `condition` element contains a `set`, the values of the bound variables are restricted to that set.

A condition element contains a single child that is either a `apply` or a `reln` element (deprecated). Compound conditions are indicated by applying relations such as `and` inside the child of the condition.

4.2.5.1 Examples

The following encodes 'there exists x such that $x^5 < 3$'.

```
<apply>
  <exists/>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><lt/>
      <apply>
        <power/>
        <ci>x</ci>
        <cn>5</cn>
```

```
      </apply>
      <cn>3</cn>
    </apply>
  </condition>
</apply>
```

The next example encodes ‘for all x in N there exists prime numbers p, q such that $p + q = 2x$’.

```
<apply>
  <forall/>
  <bvar><ci>x</ci></bvar>
  <condition>
    <apply><in/>
         <ci>x</ci>
         <csymbol encoding="text" definitionURL="www.naturalnums.htm>N</csymbol>
       </apply>
  </condition>

  <apply><exists/>
     <bvar><ci>p</ci></bvar>
     <bvar><ci>q</ci></bvar>
     <condition>
       <apply><and/>
          <apply><in/><ci>p</ci>
            <csymbol encoding="text" definitionURL="www.primes.htm>P</csymbol>
          </apply>
          <apply><in/><ci>q</ci>
            <csymbol encoding="text" definitionURL="www.primes.htm>P</csymbol>
          </apply>
          <apply><eq/>
             <apply><plus/><ci>p</ci><ci>q</ci></apply>
             <apply><times/><cn>2</cn><ci>x</ci></apply>
          </apply>
      </condition>
  </apply>
</apply>
```

A third example shows the use of quantifiers with `condition`. The following markup encodes ‘there exists $x < 3$ such that $x^2 = 4$’.

```
<apply>
  <exists/>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><lt/><ci>x</ci><cn>3</cn></apply>
  </condition>
  <apply>
    <eq/>
    <apply>
      <power/><ci>x</ci><cn>2</cn>
    </apply>
```

```
    <cn>4</cn>
  </apply>
</apply>
```

4.2.6 Syntax and Semantics

mappings `semantics`, `annotation`, `annotation-xml`

The use of content markup rather than presentation markup for mathematics is sometimes referred to as semantic tagging [Buswell1996]. The parse-tree of a valid element structure using MathML content elements corresponds directly to the expression tree of the underlying mathematical expression. We therefore regard the content tagging itself as encoding the syntax of the mathematical expression. This is, in general, sufficient to obtain some rendering and even some symbolic manipulation (e.g. polynomial factorization).

However, even in such apparently simple expressions as $X + Y$, some additional information may be required for applications such as computer algebra. Are X and Y integers, or functions, etc.? 'Plus' represents addition over which field? This additional information is referred to as semantic mapping. In MathML, this mapping is provided by the `semantics`, `annotation` and `annotation-xml` elements.

The `semantics` element is the container element for the MathML expression together with its semantic mappings. `semantics` expects a variable number of child elements. The first is the element (which may itself be a complex element structure) for which this additional semantic information is being defined. The second and subsequent children, if any, are instances of the elements `annotation` and/or `annotation-xml`.

The `semantics` tags also accepts the `definitionURL` and `encoding` attributes for use by external processing applications. One use might be a URI for a semantic content dictionary, for example. Since the semantic mapping information might in some cases be provided entirely by the `definitionURL` attribute, the `annotation` or `annotation-xml` elements are optional.

The `annotation` element is a container for arbitrary data. This data may be in the form of text, computer algebra encodings, C programs, or whatever a processing application expects. `annotation` has an attribute `encoding` defining the form in use. Note that the content model of `annotation` is `PCDATA`, so care must be taken that the particular encoding does not conflict with XML parsing rules.

The `annotation-xml` element is a container for semantic information in well-formed XML. For example, an XML form of the OpenMath semantics could be given. Another possible use here is to embed, for example, the presentation tag form of a construct given in content tag form in the first child element of `semantics` (or vice versa). `annotation-xml` has an attribute `encoding` defining the form in use.

For example:

```
<semantics>
  <apply>
  <divide/>
    <cn>123</cn>
    <cn>456</cn>
```

```
  </apply>
  <annotation encoding="Mathematica">
    N[123/456, 39]
  </annotation>
  <annotation encoding="TeX">
    $0.26973684210526315789473684210526315789 4\ldots$
  </annotation>
  <annotation encoding="Maple">
    evalf(123/456, 39);
  </annotation>
  <annotation-xml encoding="MathML-Presentation">
    <mrow>
      <mn> 0.269736842105263157894 </mn>
      <mover accent='true'>
        <mn> 736842105263157894 </mn>
        <mo> &OverBar; </mo>
      </mover>
    </mrow>
  </annotation-xml>
  <annotation-xml encoding="OpenMath">
    <OMA>...</OMA>
  </annotation-xml>
</semantics>
```

where `OMA`is the element defining the additional semantic information.

Of course, providing an explicit semantic mapping at all is optional, and in general would only be provided where there is some requirement to process or manipulate the underlying mathematics.

4.2.7 Semantic Mappings

Although semantic mappings can easily be provided by various proprietary, or highly specialized encodings, there are no widely available, non-proprietary standard schemes for semantic mapping. In part to address this need, the goal of the OpenMath effort is to provide a platform-independent, vendor-neutral standard for the exchange of mathematical objects between applications. Such mathematical objects include semantic mapping information. The OpenMath group has defined an SGML syntax for the encoding of this information [OpenMath1996]. This element set could provide the basis of one `annotation-xml` element set.

An attractive side of this mechanism is that the OpenMath syntax is specified in XML, so that a MathML expression together with its semantic annotations can be validated using XML parsers.

4.2.8 MathML element types

MathML functions, operators and relations can all be thought of as mathematical functions if viewed in a sufficiently abstract way. For example, the standard addition operator can be regarded as a function mapping pairs of real numbers to real numbers. Similarly, a relation can be thought of as a function from some space of ordered pairs into the set of values true, false. To be mathematically meaningful, the domain and range of a function

must be precisely specified. In practical terms, this means that functions only make sense when applied to certain kinds of operands. For example, thinking of the standard addition operator, it makes no sense to speak of 'adding' a set to a function. Since MathML content markup seeks to encode mathematical expressions in a way that can be unambiguously evaluated, it is no surprise that the types of operands is an issue.

MathML specifies the types of arguments in two ways. The first way is by providing precise instructions for processing applications about the kinds of arguments expected by the MathML content elements denoting functions, operators and relations. These operand types are defined in a dictionray of default semantic bindings for content elements, which is given in appendix C. For example, the MathML content dictionary specifies that for real scalar arguments the plus operator is the standard commutative addition operator over a field. The elements `cn` has a `type` attribute with a default value of `real` Thus some processors will be able to use this information to verify the validity of the indicated operations.

Issue (ci-default-type): Shouldn't `ci` have `none` or 'unspecified' as the default type?

Although MathML specifies the types of arguments for functions, operators and relations, and provides a mechanism for typing arguments, a MathML-compliant processor is not required to do any type checking. In other words, a MathML processor will not generate errors if argument types are incorrect. If the processor is a computer algebra system, it may be unable to evaluate an expression, but no MathML error is generated.

4.3 Content Element Attributes

4.3.1 Content Element Attribute Values

Content element attributes are all of the type `CDATA`, that is, any character string will be accepted as valid. In addition, each attribute has a list of predefined values, which a content processor is expected to recognize and process. The reason that the attribute values are not formally restricted to the list of predefined values is to allow for extension. A processor encountering a value (not in the predefined list) which it does not recognize may validly process it as the default value for that attribute.

4.3.2 Attributes Modifying Content Markup Semantics

Each attribute is followed by the elements to which it can be applied.

4.3.2.1 *base*

cn indicates numerical base of the number. Predefined values: any numeric string. The default value is `10`

4.3.2.2 *closure*

interval indicates closure of the interval. Predefined values: `open`, `closed`, `open-closed`, `closed-open`. The default value is `closed`

4.3.2.3 *definitionURL*

csymbol, declare, semantics, any operator element points to an external definition of the semantics of the symbol or construct being declared. The value is a URL or URI that should point to some kind of definition. This definition overrides the MathML default semantics. At present, MathML does not specify the format in which external semantic definitions should be given. In particular, there is no requirement that the target of the URI be loadable and parsable.An external definition could, for example, define the semantics in human-readable form. Ideally, in most situations the definition pointed to by the `definitionURL` attribute would be some standard, machine-readable format. However, there are several reasons why MathML does not require such a format. First, no such format currently exists. There are several projects underway to develop and implement standard semantic encoding formats, most notably the OpenMath effort. But by nature, the development of a comprehensive system of semantic encoding is a very large enterprise, and while much work has been done, much additional work remains. Therefore, even though the `definitionURL` is designed and intended for use with a formal semantic encoding language such as OpenMath, it is premature to require any one particular format. Another reason for leaving the format of the `definitionURL` attribute unspecified is that there will always be situations where some non-standard format is preferable. This is particularly true in situations where authors are describing new ideas. It is anticipated that in the near term, there will be a variety of renderer-dependent implementations of the `definitionURL` attribute. For example, a translation tool might simply prompt the user with the specified definition in situations where the proper semantics have been overridden, and in this case, human-readable definitions will be most useful. Other software may utilize OpenMath encodings. Still other software may use proprietary encodings, or look for definitions in any of several formats. As a consequence, authors need to be aware that there is no guarantee a generic renderer will be able to take advantage of information pointed to by the `definitionURL` attribute. Of course, when widely-accepted standardized semantic encodings are available, the definitions pointed to can be replaced without modifying the original document. However, this is likely to be labor intensive. There is no default value for the `definitionURL` attribute, i.e. the semantics are defined within the MathML fragment, and/or by the MathML default semantics.

4.3.2.4 *encoding*

annotation, annotation-xml, csymbol, semantics, all operator elements indicates the encoding of the annotation, or in the case of `csymbol`, `semantics` and operator elements, the syntax of the target referred to by `definitionURL`. Pre-defined values are `MathML-Presentation`, `MathML-Content`. Other typical values: `TeX`, `OpenMath`. The default value is "", i.e. unspecified.

4.3.2.5 *nargs*

declare indicates number of arguments for function declarations. Pre-defined values: `nary` or any numeric string. The default value is `1`

4.3.2.6 *occurrence*

declare indicates occurrence for operator declarations. Pre-defined values: `prefix`, `infix`, `function-model`. The default value is `function-model`

4.3.2.7 *order*

list indicates ordering on the list. Predefined values: `lexicographic`, `numeric`. The default value is `numeric`

4.3.2.8 *scope*

declare indicates scope of applicability of the declaration. Pre-defined values: `local`, `global`.

- `local` means the containing MathML element.
- `global` means the containing `math` element.

The default value is `local`. At present, declarations cannot affect anything outside of the containing `math` element. Ideally, one would like to make document-wide declarations by setting the value of the `scope` attribute to be `global-document`. However, the proper mechanism for document-wide declarations very much depends on details of the way in which XML will be embedded in HTML, future XML style sheet mechanisms, and the underlying Document Object Model. Since these supporting technologies are still in flux at present, the MathML specification does not include `global-document` as a pre-defined value of the `scope` attribute. It is anticipated, however, that this issue will be revisited in future revisions of MathML as supporting technologies stabilize. In the near term, MathML implementors that wish to simulate the effect of a document-wide declaration are encouraged to pre-process documents in order to distribute document-wide declarations to each individual `math` element in the document.

4.3.2.9 *type*

cn indicates type of the number. Predefined values: `integer`, `rational`, `real`, `float`, `complex`, `complex-polar`, `complex-cartesian`, `constant`. The default value is `real`. Notes. Each data type implies that the data adheres to certain formating conventions, detailed below. If the data fails to conform to the expected format, an error is generated. Details of the individual formats are:

real A real number is presented in decimal notation. Decimal notation consists of an optional sign ('+' or '-') followed by a string of digits possibly separated into an integer and a fractional part by a 'decimal point'. Some examples are 0.3, 1, and -31.56. If a different `base` is specified, then the digits are interpreted as being digits computed to that base. A real number may also be presented in scientific notation. Such numbers have two parts (a mantissa and an exponent) separated by 'e'. The first part is a real number, while the second part is an integer exponent indicating a power of the base. For example, 12.3e5 represents 12.3 times 10^5.

integer An integer is represented by an optional sign followed by a string of 1 or more 'digits'. What a 'digit' is depends on the `base` attribute. If `base` is present, it specifies the base for the digit encoding, and it specifies it base ten. Thus `base`='16' specifies a hex encoding. When `base`> 10, letters are

added in alphabetical order as digits. The legitimate values for `base` are therefore between 2 and 36.

rational A rational number is two integers separated by `<sep/>`. If `base` is present, it specifies the base used for the digit encoding of both integers.

complex-cartesian A complex number is of the form two real point numbers separated by `<sep/>`

complex-polar A complex number is specified in the form of a magnitude and an angle (in radians). The raw data is in the form of two real numbers separated by `<sep/>`

constant The `constant` type is used to denote named constants. For example, an instance of `<cn type="constant">&pi;</cn>` should be interpreted as having the semantics of the mathematical constant Pi. The data for a constant `cn` tag may be one of the following common constants:

Symbol	Value
`&pi;`	The usual `&pi;` of trigonometry: approximately 3.141592653...
`&ExponentialE;` (or `&ee;`)	The base for natural logarithms: approximately 2.718281828 ...
`&ImaginaryI;` (or `&ii;`)	Square root of -1
`&gamma;`	Euler's constant: approximately 0.5772156649...
`&infin;` (or `&infty;`)	Infinity. Proper interpretation varies with context
`&true;`	the logical constant `true`
`&false;`	the logical constant `false`
`&NotANumber;` (or `&NaN;`)	represents the result of an ill-defined floating point division

ci indicates type of the identifier. Predefined values: `integer`, `rational`, `real`, `float`, `complex`, `complex-polar`, `complex-cartesian`, `constant`, or the name of any content element. The meaning of the various attribute values is the same as that listed above for the `cn` element. The default value is "", i.e. unspecified.

declare indicates type of the identifier being declared. Predefined values: any content element name. The default value is `ci`, i.e. a generic identifier

set indicates type of the set. Predefined values: `normal`, `multiset`. `multiset` indicates that repetitions are allowed. The default value is `normal`.

tendsto indicates the direction from which the limiting value is approached. Predefined values: `above`, `below`, `two-sided`. The default value is `above`

4.3.3 Attributes Modifying Content Markup Rendering

4.3.3.1 *type*

The `type` attribute, in addition to conveying semantic information, can be interpreted to provide rendering information. For example in

```
<ci type="vector">V</ci>
```

a renderer could display a bold *V* for the vector.

4.3.3.2 General Attributes

All content elements support the following general attributes that can be used to modify the rendering of the markup.

- `class`

- `style`
- `id`
- `other`

The `class`, `style` and `id` attributes are intended for compatibility with Cascading Style Sheets (CSS), as described in section 2.3.4.

Content or semantic tagging goes along with the (frequently implicit) premise that, if you know the semantics, you can always work out a presentation form. When an author's main goal is to mark up re-usable, evaluatable mathematical expressions, the exact rendering of the expression is probably not critical, provided that it is easily understandable. However, when an author's goal is more along the lines of providing enough additional semantic information to make a document more accessible by facilitating better visual rendering, voice rendering, or specialized processing, controlling the exact notation used becomes more of an issue.

MathML elements accept an attribute `other` (see section 7.2.3), which can be used to specify things not specifically documented in MathML. On content tags, this attribute can be used by an author to express a preference between equivalent forms for a particular content element construct, where the selection of the presentation has nothing to do with the semantics. Examples might be

- inline or displayed equations
- script-style fractions
- use of x with a dot for a derivative over dx/dt

Thus, if a particular renderer recognized a display attribute to select between script-style and display-style fractions, an author might write

```
<apply other='display="scriptstyle"'>
  <divide/>
  <mn> 1 </mn>
  <mi> x </mi>
</apply>
```

to indicate that the rendering $1/x$ is preferred.

The information provided in the `other` attribute is intended for use by specific renderers or processors, and therefore, the permitted values are determined by the renderer being used. It is legal for a renderer to ignore this information. This might be intentional, in the case of a publisher imposing a house style, or simply because the renderer does not understand them, or is unable to carry them out.

4.4 The Content Markup Elements

This section provides detailed descriptions of the MathML content tags. They are grouped in categories that broadly reflect the area of mathematics from which they come, and also the grouping in the MathML DTD. There is no linguistic difference in MathML between operators and functions. Their separation here and in the DTD is for reasons of historical usage.

When working with the content elements, it can be useful to keep in mind the following.

- The role of the content elements is analogous to data entry in a mathematical system. The information that is provided is there to facilitate the successful parsing of an expression as the intended mathematical object by a receiving application.
- MathML content elements do not by themselves 'perform' any mathematical evaluations or operations. They do not 'evaluate' in a browser and any 'action' that is ultimately taken on those objects is determined entirely by the receiving mathematical application. For example, editing programs and applications geared to computation for the lower grades would typically leave 3 + 4 as is, whereas computational systems targeting a more advanced audience might evaluate this as 7. Similarly, some computational systems might evaluate sin(0) to 0, whereas others would leave it unevaluated. Yet other computational systems might be unable to deal with pure symbolic expressions sin(*x*) and may even regard it as a data entry error. None of this has any bearing on the correctness of the original MathML representation. Where evaluation is mentioned at all in the descriptions below, it is merely to help clarify the meaning of the underlying operation.
- Apart from the instances where there is an explicit interaction with presentation tagging, there is no required rendering (visual or aural) - only a suggested default. As such, the presentations that are included in this section are merely to help communicate to the reader the intended mathematical meaning by association with the same expression written in a more traditional notation.

The available content elements are:

- token elements
  - `cn`
  - `ci`
  - `csymbol` (MathML 2.0)
- basic content elements
  - `apply`
  - `reln` (deprecated)
  - `fn` (deprecated for externally defined functions)
  - `interval`
  - `inverse`
  - `sep`
  - `condition`
  - `declare`
  - `lambda`
  - `compose`
  - `ident`
- arithmetic, algebra and logic
  - `quotient`
  - `exp`
  - `factorial`
  - `divide`
  - `max` and `min`
  - `minus`
  - `plus`
  - `power`
  - `rem`
  - `times`
  - `root`

  - `gcd`
  - `and`
  - `or`
  - `xor`
  - `not`
  - `implies`
  - `forall`
  - `exists`
  - `abs`
  - `conjugate`
  - `arg` (MathML 2.0)
  - `real` (MathML 2.0)
  - `imaginary` (MathML 2.0)
- relations
  - `eq`
  - `neq`
  - `gt`
  - `lt`
  - `geq`
  - `leq`
  - `equivalent` (MathML 2.0)
  - `approx` (MathML 2.0)
- calculus and vector calculus
  - `int`
  - `diff`
  - `partialdiff`
  - `lowlimit`
  - `uplimit`
  - `bvar`
  - `degree`
  - `divergence` (MathML 2.0)
  - `grad` (MathML 2.0)
  - `curl` (MathML 2.0)
  - `laplacian` (MathML 2.0)
- theory of sets
  - `set`
  - `list`
  - `union`
  - `intersect`
  - `in`
  - `notin`
  - `subset`
  - `prsubset`
  - `notsubset`
  - `notprsubset`
  - `setdiff`
  - `card` (MathML 2.0)
- sequences and series
  - `sum`
  - `product`

  - limit
  - tendsto
- elementary classical functions
  - exp
  - ln
  - log
  - sin
  - cos
  - tan
  - sec
  - csc
  - cot
  - sinh
  - cosh
  - tanh
  - sech
  - csch
  - coth
  - arcsin
  - arccos
  - arctan
  - arccosh
  - arccot
  - arccoth
  - arccsc
  - arccsch
  - arcsec
  - arcsech
  - arcsinh
  - arctanh
- statistics
  - mean
  - sdev
  - variance
  - median
  - mode
  - moment
- linear algebra
  - vector
  - matrix
  - matrixrow
  - determinant
  - transpose
  - selector
  - vectorproduct (MathML 2.0)
  - scalarproduct (MathML 2.0)
  - outerproduct (MathML 2.0)
- semantic mapping elements
  - annotation
  - semantics

– annotation-xml

4.4.1 Token Elements

4.4.1.1 Number (*cn*)

Discussion

The cn element is used to specify actual numerical constants. The content model must provide sufficient information that a number may be entered as data into a computational system. By default, it represents a signed real number in base 10. Thus, the content normally consists of PCDATA restricted to a sign, a string of decimal digits and possibly a decimal point, or alternatively one of the predefined symbolic constants such as π

The cn element uses the attribute type to represent other types of numbers such as, for example, integer, rational, real or complex, and uses the attribute base to specify the numerical base.

In addition to simple PCDATA, cn accepts as content PCDATA separated by the (empty) element sep. This determines the different parts needed to construct a rational or complex-cartesian number.

The cn element may also contain arbitrary presentation markup in its content (see chapter 3) so that its presentation can be very elaborate.

Alternative input notations for numbers are possible, but must be explicitly defined by using the definitionURL and encoding attributes, to refer to a written specification of how a sequence of real numbers separated by <sep/> should be interpreted.

Attributes

All attributes are CDATA:

type Allowed values are real, integer, rational, complex-cartesian, complex-polar, constant

base Number (CDATA for XML DTD) between 2 and 36.

definitionURL URL or URI pointing to an alternative definition.

encoding Syntax of the alternative definition.

Examples

```
<cn type="real"> 12345.7 </cn>
<cn type="integer"> 12345 </cn>
<cn type="integer" base="16"> AB3 </cn>
<cn type="rational"> 12342 <sep/> 2342342 </cn>
<cn type="complex-cartesian"> 12.3 <sep/> 5 </cn>
<cn type="complex-polar"> 2 <sep/> 3.1415 </cn>
<cn type="constant">  &pi; </cn>
```

Default Rendering

By default, contiguous blocks of `PCDATA` contained in `cn` elements should render as if it were wrapped in an `mn` presentation element. Similarly, presentation markup contained in a `cn` element should render as it normally would. A mixture of `PCDATA` and presentation markup should render as if it were contained wrapped in an `mrow` element, with contiguous blocks of `PCDATA` wrapped in `mn` elements.

However, not all mathematical systems that encounter content based tagging do visual or aural rendering. The receiving applications are free to make use of a number in the manner it normally handles numerical data. Some systems might simplify the rational number 12342/2342342 to 6171/1171171 while pure floating point based systems might approximate this as 0.5269085385e-2. All numbers might be re-expressed in base 10. The role of MathML is simply to record enough information about the mathematical object and its structure so that it may be properly parsed.

The following renderings of the above MathML expressions are included both to help clarify the meaning of the corresponding MathML encoding and as suggestions for authors of rendering applications. In each case, no mathematical evaluation is intended or implied.

- 12345.7
- 12345
- $AB3_{16}$
- 12342 / 2342342
- 12.3 + 5 i
- Polar(2 , 3.1415)
- π

4.4.1.2 Identifier (*ci*)

Discussion

The `ci` element is used to name an identifier in a MathML expression (for example a variable). Such names are used to identify mathematical objects. By default they are assumed to represent complex scalars. The `ci` element may contain arbitrary presentation markup in its content (see chapter 3) so that its presentation as a symbol can be very elaborate.

The `ci` element uses the `type` attribute to specify the type of object that it represents. Valid types include `integer`, `rational`, `real`, `float`, `complex`, `constant`, and more generally, any of the names of the MathML container elements (e.g. `vector`) or their type values. The `definitionURL` and `encoding` attributes can be used to extend the definition of `ci` to include other types. For example, a more advanced use might require a `complex-vector`.

Examples

```
1. <ci> x </ci>
2. <ci type="vector"> V </ci>
3. <ci>
     <msub>
       <mi>x</mi>
       <mi>a</mi>
     </msub>
   </ci>
```

Default Rendering

If the content of a `ci` element is tagged using presentation tags, that presentation is used. If no such tagging is supplied then the `PCDATA` content would typically be rendered as if it were the content of an `mi` element. A renderer may wish to make use of the value of the type attribute to improve on this. For example, a symbol of type `vector` might be rendered using a bold face. Typical renderings of the above symbols are:

1. x
2. V
3. x_i

4.4.1.3 Externally defined symbol (*csymbol*)

Discussion

The `csymbol` element allows a writer to create an element in MathML whose semantics are externally defined (i.e. not in the core MathML content). The element can then be used in a MathML expression as for example an operator or constant. Attributes are used to give the syntax and location of the external definition of the symbol semantics.

Use of `csymbol` for referencing external semantics can be contrasted with use of the `semantics` to attach additional information in-line (ie. within the MathML fragment) to a MathML construct. See section 4.2.6

Attributes

All attributes are `CDATA`:

definitionURL Pointer to external definition of the semantics of the symbol. MathML does not specify a particular syntax in which this definition should be written.

encoding Gives the syntax of the definition pointed to by definitionURL. An application can then test the value of this attribute to determine whether it is able to process the target of the `definitionURL`. This syntax might be text, or a formal syntax such as OpenMath.

Examples

```
<!-- reference to OpenMath formal syntax definition of Bessel function -->
<apply>
  <csymbol encoding="OpenMath"
          definitionURL="www.openmath.org/cds/BesselFunctions.ocd">
    <msub><mi>J</mi><mn>0</mn></msub>
  </csymbol>
  <ci>y</ci>
</apply>

<!-- reference to human readable text description of Boltzmann's constant -->
<csymbol encoding="text"
         definitionURL="www.uni.edu/universalconstants/Boltzmann.htm">
  k
</csymbol>
```

Default Rendering

By default, a contiguous block of PCDATA contained in a csymbol element should render as if it were wrapped in an mo presentation element. Similarly, presentation markup contained in a csymbol element should render as it normally would. A mixture of PCDATA and presentation markup should render as if it were contained wrapped in an mrow element, with contiguous blocks of PCDATA wrapped in mo elements. The examples above would render by default as

- $J_0(y)$
- k

As csymbol is used to support reference to externally defined semantics, it is a MathML error to have embedded content MathML elements within the csymbol element.

4.4.2 Basic Content Elements

4.4.2.1 Apply (*apply*)

Discussion

The apply element allows a function or operator to be applied to its arguments. Nearly all expression construction in MathML content markup is carried out by applying operators or functions to arguments. The first child of apply is the operator, to be applied, with the other child elements as arguments.

The apply element is conceptually necessary in order to distinguish between a function or operator, and an instance of its use. The expression constructed by applying a function to 0 or more arguments is always an element from the range of the function.

Proper usage depends on the operator that is being applied. For example, the plus operator may have zero or more arguments. while the minus operator requires one or two arguments to be properly formed.

If the object being applied as a function is not already one of the elements known to be a function (such as fn, sin or plus) then it is treated as if it were the contents of an fn element.

Some operators such as diff and int make use of 'named' arguments. These special arguments are elements that appear as children of the apply element and identify 'parameters' such as the variable of differentiation or the domain of integration. These elements are discussed further in section 4.2.3.2.

Examples

```
1. <apply>
     <factorial/>
     <cn>3</cn>
   </apply>
2. <apply>
    <plus/>
    <cn>3</cn>
    <cn>4</cn>
   </apply>
```

```
3. <apply>
     <sin/>
     <ci>x</ci>
   </apply>
```

Default Rendering

A mathematical system that has been passed an `apply` element is free to do with it whatever it normally does with such mathematical data. It may be that no rendering is involved (e.g. a syntax validator), or that the 'function application' is evaluated and that only the result is rendered (e.g. sin(0) → 0).

When an unevaluated 'function application' is rendered there are a wide variety of appropriate renderings. The choice often depends on the function or operator being applied. Applications of basic operations such as `plus` are generally presented using an infix notation while applications of `sin` would use a more traditional functional notation such as sin (*x*). Consult the default rendering for the operator being applied.

Applications of user-defined functions (see `csymbo`,`lfn`) that are not evaluated by the receiving or rendering application would typically render using a traditional functional notation unless an alternative presentation is specified using the `semantics` tag.

4.4.2.2 Relation (*reln*)

Discussion

The `reln` element was used in MathML 1.0 to construct an equation or relation. Relations were constructed in a manner exactly analogous to the use of `apply` This usage is deprecated in MathML 2.0 in favour of the more generally usable `apply`

The first child of `reln` is the relational operator, to be applied, with the other child elements acting as arguments. See section 4.2.4 for further details.

Examples and Usage

```
<reln>
  <eq/>
  <ci> a </ci>
  <ci> b </ci>
</reln>
<reln>
  <lt/>
  <ci> a </ci>
  <ci> b </ci>
</reln>
```

Default Rendering

1. $a = b$
2. $a < b$

4.4.2.3 Function (*fn*)

Discussion

The `fn` element makes explicit the fact that a more general (possibly constructed) MathML object is being used in the same manner as if it were a pre-defined function such as `sin` or `plus`

`fn` has exactly one child element, used to give the name (or presentation form) of the function. When `fn` is used as the first child of an apply, the number of following arguments is determined by the contents of the `fn`.

In MathML 1.0, `fn` was also the primary mechanism used to extend the collection of 'known' mathematical functions. This usage is now deprecated in favour of the more generally applicable `csymbol` element. (New functions may also be introduced by using `declare` in conjunction with a `lambda` expression.)

Examples

```
1. <fn><ci> L </ci> </fn>
2. <apply>
     <fn>
     <apply>
       <plus/>
       <ci> f </ci>
       <ci> g </ci>
     </apply>
     </fn>
     <ci>z</ci>
   </apply>
```

Default Rendering

An `fn` object is rendered in the same way as its content. A rendering application may add additional adornments such as +parentheses to clarify the meaning.

1. L
2. $(f + g)z$

4.4.2.4 Interval (*interval*)

Discussion

The `interval` element is used to represent simple mathematical intervals of the real number line. It takes an attribute `closure`, which can take on any of the values `open`, `closed`, `open-closed`, or `closed-open`, with a default value of `closed`

More general domains are constructed by using the `condition` and `bvar` elements to bind free variables to constraints.

The `interval` element expects either two child elements that evaluate to real numbers or one child element that is a `condition` defining the `interval`

Examples

```
1. <interval>
     <ci> a </ci>
     <ci> b </ci>
   </interval>
2. <interval closure="open-closed">
     <ci> a </ci>
     <ci> b </ci>
   </interval>
```

Default Rendering

a, *b*

1. (*a*, *b*]

4.4.2.5 Inverse (*inverse*)

Discussion

The `inverse` element is applied to a function in order to construct a generic expression for the functional inverse of that function. (See also the discussion of `inverse` in section 4.2.1.5). As with other MathML functions, `inverse` may either be applied to arguments, or it may appear alone, in which case it represents an abstract inversion operator acting on other functions.

A typical use of the `inverse` element is in an HTML document discussing a number of alternative definitions for a particular function so that there is a need to write and define $f^{(-1)}$(x). To associate a particular definition with $f^{(-1)}$, use the `definitionURL` and `encoding` attributes.

Examples

```
1. <apply>
     <inverse/>
     <ci> f </ci>
   </apply>
2. <apply>
     <inverse definitionURL="../MyDefinition.htm" encoding="text"/>
     <ci> f </ci>
   </apply>
3. <apply>
      <apply><inverse/>
         <ci type="matrix"> a </ci>
      </apply>
      <ci> A </ci>
   </apply>
```

Default Rendering

The default rendering for a functional inverse makes use of a parenthesized exponent as in $f^{(-1)}(x)$.

4.4.2.6 Separator (*sep*)

Discussion

The `sep` element is to separate `PCDATA` into separate tokens for parsing the contents of the various specialized forms of the `cn` elements. For example, `sep` is used when specifying the real and imaginary parts of a complex number (see section 4.4.1). If it occurs between MathML elements, it is a MathML error.

Examples

```
<cn type="complex"> 3 <sep/> 4 </cn>
```

Default Rendering

The `sep` element is not directly rendered (see section 4.4.1).

4.4.2.7 Condition (*condition*)

Discussion

The `condition` element is used to place a condition on one or more free variables or identifiers. The conditions may be specified in terms of relations that are to be satisfied by the variables, including general relationships such as set membership.

It is used to define general sets and lists in situations where the elements cannot be explicitly enumerated. Condition contains either a single `apply` or `reln` element; the `apply` element is used to construct compound conditions. For example, it is used below to describe the set of all x such that $x < 5$. See the discussion on sets in section 4.4.6. See section 4.2.5 for further details.

Examples

```
1. <condition>
     <apply><in/><ci> x </ci><ci type="set"> R </ci></apply>
   </condition>
2. <condition>
     <apply> <and/>
       <apply><gt/><ci> x </ci><cn> 0 </cn></apply>
       <apply><lt/><ci> x </ci><cn> 1 </cn></apply>
     </apply>
   </condition>
3. <apply>
     <max/>
     <bvar><ci> x </ci></bvar>
     <condition>
       <apply> <and/>
         <apply><gt/><ci> x </ci><cn> 0 </cn></apply>
         <apply><lt/><ci> x </ci><cn> 1 </cn></apply>
       </apply>
```

```
    </condition>
    <apply>
      <minus/>
      <ci> x </ci>
      <apply>
        <sin/>
        <ci> x </ci>
      </apply>
    </apply>
  </apply>
```

Default Rendering

1. $x \in \mathbb{R}$

2. $x > 0 \wedge x < 1$

3. $\max_x \{ x - \sin x \mid 0 < x < 1 \}$

4.4.2.8 Declare (*declare*)

Discussion

The `declare` construct has two primary roles. The first is to change or set the default attribute values for a specific mathematical object. The second is to establish an association between a 'name' and an object. Once a declaration is in effect, the 'name' object acquires the new attribute settings, and (if the second object is present) all the properties of the associated object.

The various attributes of the `declare` element assign properties to the object being declared or determine where the declaration is in effect.

By default, the scope of a declaration is 'local' to the surrounding container element. Setting the value of the `scope` attribute to `global` extends the scope of the declaration to the enclosing `math` element. As discussed in section 4.3.2.8, MathML contains no provision for making document-wide declarations at present, though it is anticipated that this capability will be added in future revisions of MathML, when supporting technologies become available. `declare` takes one or two children. The first child, which is mandatory, is a `ci` containing the identifier being declared:

```
<declare type="vector"> <ci> V </ci> </declare>
```

The second child, which is optional, is a constructor initialising the variable:

```
<declare type="vector">
  <ci> V </ci>
  <vector>
    <cn> 1 </cn><cn> 2 </cn><cn> 3 </cn>
  </vector>
</declare>
```

The constructor type and the type of the element declared must agree. For example, if the type attribute of the declaration is `fn`, the second child (constructor) must be an element

equivalent to an `fn` element (This would include actual `fn` elements, `lambda` elements and any of the defined function in the basic set of content tags.) If no type is specified in the declaration then the type attribute of the declared name is set to the type of the constructor (second child) of the declaration. The type attribute of the declaration can be especially useful in the special case of the second element being a semantic tag.

Attributes

All attributes are `CDATA`

`type` defines the MathML element type of the identifier declared.
`scope` defines the scope of application of the declaration.
`nargs` number of arguments for function declarations.
`occurrence` describes operator usage as `prefix`, `infix` or `function-model` indications.
`definitionURL` URI pointing to detailed semantics of the function.
`encoding` syntax of the detailed semantics of the function.

Examples

The declaration

```
<declare type="fn" nargs="2" scope="local">
  <ci> f </ci>
  <apply>
    <plus/>
    <ci> F </ci><ci> G </ci>
  </apply>
</declare>
```

declares f to be a two-variable function with the property that $f(x,y) = (F + G)(x,y)$.

The declaration

```
<declare type="fn">
  <ci> J </ci>
  <lambda>
    <bvar><ci> x </ci></bvar>
    <apply><ln/>
      <ci> x </ci>
    </apply>
  </lambda>
</declare>
```

associates the name J with a one-variable function defined so that $J(x) = \ln y$. (Note that because of the type attribute of the `declare` element, the second argument must be something of type `fn`, namely a known function like `sin`, an `fn` construct, or a `lambda` construct.)

The `type` attribute on the declaration is only necessary if if the type cannot be inferred from the type of the second argument.

Even when a declaration is in effect it is still possible to override attributes values selectively as in `<ci type="integer"> V </ci>`. This capability is needed in order to write statements of the form ‘Let S be a member of S’.

Default Rendering

Since the `declare` construct is not directly rendered, most declarations are likely to be invisible to a reader. However, declarations can produce quite different effects in an application which evaluates or manipulates MathML content. While the declaration

```
<declare>
  <ci> v </ci>
  <vector>
    <cn> 1 </cn>
    <cn> 2 </cn>
    <cn> 3 </cn>
  </vector>
</declare>
```

is active the symbol *v* acquires all the properties of the vector, and even its dimension and components have meaningful values. This may affect how *v* is rendered by some applications, as well as how it is treated mathematically.

4.4.2.9 Lambda (*lambda*)

Discussion

The `lambda` element is used to construct a user-defined function from an expression and one or more free variables. The lambda construct with *n* internal variables takes *n*+1 children. The first *n* children identify the variables that are used as placeholders in the last child for actual parameter values. See section 4.2.2.2 for further details.

Examples

The following markup represents $\lambda(x, \sin x+1)$.

```
<lambda>
  <bvar><ci> x </ci></bvar>
  <apply><sin/>
    <apply>
      <plus/>
      <ci> x </ci>
      <cn> 1 </cn>
    </apply>
  </apply>
</lambda>
```

The following examples constructs a one argument function in which the argument *b* specifies the upper bound of a specific definite integral.

```
<lambda>
  <bvar><ci> b </ci></bvar>
  <apply>
    <int/>
    <bvar>
       <ci> x </ci>
```

```
    </bvar>
    <lowlimit>
       <ci> a </ci>
    </lowlimit>
    <uplimit>
       <ci> b </ci>
    </uplimit>
    <apply><fn><ci> f </ci></fn>
       <ci> x </ci>
    </apply>
  </apply>
</lambda>
```

Such constructs are often used in conjunction with `declare` to construct new functions.

Default Rendering

$$\lambda(x, f(x))$$

4.4.2.10 Function composition (*compose*)

Discussion

The `compose` element represents the function composition operator. Note that MathML makes no assumption about the domain and range of the constituent functions in a composition; the domain of the resulting composition may be empty.

To override the default semantics for the `compose` element, or to associate a more specific definition for function composition, use the `definitionURL` and `encoding` attributes. See section 4.2.3 for further details.

Examples

The following markup represents $f \circ g$.

```
<apply>
  <compose/>
  <fn><ci> f </ci></fn>
  <fn><ci> g </ci></fn>
</apply>
```

The following markup represents $f \circ g \circ h$.

```
<apply>
  <compose/>
  <ci type="fn"> f </ci>
  <ci type="fn"> g </ci>
  <ci type="fn"> h </ci>
</apply>
```

The following examples both represent $(f \circ g)(x)$.

```
<apply>
  <apply><compose/>
    <fn><ci> f </ci></fn>
    <fn><ci> g </ci></fn>
  </apply>
  <ci> x </ci>
</apply>

<apply><fn><ci> f </ci></fn>
  <apply>
    <fn><ci> g </ci></fn>
  <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$f \circ g$$

4.4.2.11 Identity function (*ident*)

Discussion

The `ident` element represents the identity function. MathML makes no assumption about the function space in which the identity function resides. That is, proper interpretation of the domain (and hence range) of the identity function depends on the context in which it is used.

To override the default semantics for the `ident` element, or to associate a more specific definition, use the `definitionURL` and `encoding` attributes (see section 4.2.3).

Examples

The following markup encoded the expression $f \circ f^{-1} = \mathrm{id}$.

```
<apply>
  <eq/>
  <apply><compose/>
    <fn><ci> f </ci></fn>
    <apply><inverse/>
      <fn><ci> f </ci></fn>
    </apply>
  </apply>
  <ident/>
</apply>
```

Default Rendering

id

4.4.3 Arithmetic, Algebra and Logic

4.4.3.1 Quotient (*quotient*)

Discussion

The `quotient` element is the operator used for division modulo a particular base. When the `quotient` operator is applied to integer arguments *a* and *b*, the result is the 'quotient of *a* divided by *b*'. That is, `quotient` returns the unique integer, *q* such that $a = q\ b + r$. (In common usage, *q* is called the quotient and *r* is the remainder.)

The `quotient` element takes the attribute `definitionURL` and `encoding` attributes, which can be used override the default semantics.

The `quotient` element is a binary arithmetic operator (see section 4.2.3).

Example

```
<apply>
  <quotient/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

Various mathematical applications will use this data in different ways. Editing applications might choose an image such as shown below, while a computationally based application would evaluate it to 2 when *a*=13 and *b*=5.

Default Rendering

There is no commonly used notation for this concept. Some possible renderings are

1. quotient of *a* divided by *b*
2. integer part of *a*/*b*
3. $\lfloor a/b \rfloor$

4.4.3.2 Factorial (*factorial*)

Discussion

The `factorial` element is used to construct factorials.

The `factorial` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `factorial` element is a unary arithmetic operator (see section 4.2.3).

Example

```
<apply>
  <factorial/>
  <ci> n </ci>
</apply>
```

If this were evaluated at $n = 5$ it would evaluate to 120.

Default Rendering

n!

4.4.3.3 Division (*divide*)

Discussion

The `divide` element is the division operator.

The `divide` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `divide` element is a binary arithmetic operator (see section 4.2.3).

Example

```
<apply>

<divide/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

As a MathML expression, this does not evaluate. However, on receiving such an expression, some applications may attempt to evaluate and simplify the value. For example, when *a*=5 and *b*=2 some mathematical applications may evaluate this to 2.5 while others will treat is as a rational number.

Default Rendering

a/*b*

4.4.3.4 Maximum and minimum (*max*, *min*)

Discussion

The elements `max` and `min` are used to compare the values of their arguments. They return the maximum and minimum of these values respectively.

The `max` and `min` elements take the `definitionURL` and `encoding` attributes that can be used to override the default semantics.

The `max` and `min` elements are n-ary arithmetic operators (see section 4.2.3).

Examples

When the objects are to be compared explicitly they are listed as arguments to the function as in:

```
<apply>
  <max/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

The elements to be compared may also be described using bound variables with a `condition` element and an expression to be maximised, as in:

```
<apply>
  <min/>
  <bvar><ci>x</ci></bvar>
  <condition>
    <apply><notin/><ci> x </ci><ci type="set"> B </ci></apply>
  </condition>
  <apply>
     <power/>
    <ci> x </ci>
    <cn> 2 </cn>
  </apply>
</apply>
```

Note that the bound variable must be stated even if it might be implicit in conventional notation. In MathML1.0, the bound variable and expression to be evaluated (x) could be omitted in the example below: this usage is deprecated in MathML2.0 in favour of explicitly stating the bound variable and expression in all cases:

```
<apply>
  <bvar><ci>x</ci></bvar>
  <max/>
  <condition>
    <apply><and/>
      <apply><in/><ci>x</ci><ci type="set">B</ci></apply>
      <apply><notin/><ci>x</ci><ci type="set">C</ci></apply>
    </apply>
  </condition>
  <ci>x</ci>
</apply>
```

Default Rendering

1. $\max\{a, b\}$
2. $\min_x\{ x^2 \mid x \notin B \}$
3. $\max\{ x \in B \wedge x \notin C \}$

4.4.3.5 Subtraction (*minus*)

Discussion

The `minus` element is the subtraction operator.

The `minus` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `minus` element can be used as a unary arithmetic operator (e.g. to represent -x), or as a binary arithmetic operator (e.g. to represent x-y).

Example

```
<apply> <minus/>
   <ci> x </ci>
   <ci> y </ci>
</apply>
```

If this were evaluated at x=5 and y=2 it would yield 3.

Default Rendering

$x - y$

4.4.3.6 Addition (*plus*)

Discussion

The `plus` element is the addition operator.

The `plus` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `plus` element is an n-ary arithemtic operator (see section 4.2.3).

Example

```
<apply>
  <plus/>
  <ci> x </ci>
  <ci> y </ci>
  <ci> z </ci>
</apply>
```

If this were evaluated at x = 5, y = 2 and z = 1 it would yield 8.

Default Rendering

$x + y + z$

4.4.3.7 Exponentiation (*power*)

Discussion

The `power` element is a generic exponentiation operator. That is, when applied to arguments a and b, it returns the value the 'a to the power of b'.

The `power` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `power` element is an binary arithmetic operator (see section 4.2.3).

Example

```
<apply>
  <power/>
  <ci> x </ci>
  <cn> 3 </cn>
</apply>
```

If this were evaluated at $x = 5$ it would yield 125.

Default Rendering

x^3

4.4.3.8 Remainder (*rem*)

Discussion

The `rem` element is the operator that returns the 'remainder' of a division modulo a particular base. When the `rem` operator is applied to integer arguments *a* and *b*, the result is the 'remainder of *a* divided by *b*'. That is, `rem` returns the unique integer, *r* such that $a = q\,b + r$, where $r < q$. (In common usage, *q* is called the quotient and *r* is the remainder.)

The `rem` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `rem` element is a binary arithmetic operator (see section 4.2.3).

Example

```
<apply>
  <rem/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were evaluated at $a = 15$ and $b = 8$ it would yield 7.

Default Rendering

a mod *b*

4.4.3.9 Multiplication (*times*)

Discussion

The `times` element is the multiplication operator.

`times` takes the `definitionURL` and `encoding` attributes which can be used to override the default semantics.

Example

```
<apply> <times/>
   <ci> a </ci>
   <ci> b </ci>
</apply>
```

If this were evaluated at $a = 5.5$ and $b = 3$ it would yield 16.5.

Default Rendering

$a\ b$

4.4.3.10 Root (*root*)

Discussion

The `root` element is used to construct roots. The kind of root to be taken is specified by a `degree` element, which should be given as the first child of the `apply` element enclosing the `root` element. Thus, square roots correspond to the case where `degree` contains the value 2, cube roots correspond to 3, and so on. If no `degree` is present, a default value of 2 is used.

The `root` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `root` element is an operator taking qualifiers (see section 4.2.3.2).

Example

The *n*th root of *a* is is given by

```
<apply> <root/>
   <degree><ci type='integer'> n </ci></degree>
   <ci> a </ci>
</apply>
```

Default Rendering

$\sqrt[n]{a}$

4.4.3.11 Greatest common divisor (*gcd*)

Discussion

The `gcd` element is used to denote the greatest common divisor of its arguments.

The `gcd` takes the `definitionURL` and `encoding` attributes which can be used to override the default semantics.

The `gcd` element is an n-ary operator (see section 4.2.3).

Example

```
<apply> <gcd/>
   <ci> a </ci>
   <ci> b </ci>
   <ci> c </ci>
</apply>
```

If this were evaluated at $a = 15$, $b = 21$, $c = 48$ it would yield 3.

Default Rendering

$\gcd(a, b, c)$

4.4.3.12 And (*and*)

Discussion

The `and` element is the boolean 'and' operator.

The `and` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `and` element is an n-ary logical operator (see section 4.2.3).

Example

```
<apply>
  <and/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were evaluated and both a and b had truth values of `true`, then the value would be `true`

Default Rendering

$a \wedge b$

4.4.3.13 Or (*or*)

Discussion

The `or` element is the boolean 'or' operator.

The `or` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `or` element is an n-ary logical operator (see section 4.2.3).

Example

```
<apply>
  <or/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

Default Rendering

$$a \vee b$$

4.4.3.14 Exclusive Or (*xor*)

Discussion

The `xor` element is the boolean 'exclusive or' operator.

`xor` takes the `definitionURL` and `encoding` attributes which can be used to override the default semantics.

The `xor` element is an n-ary logical operator (see section 4.2.3).

Example

```
<apply>
  <xor/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

Default Rendering

$$a \operatorname{xor} b$$

4.4.3.15 Not (*not*)

The `not` operator is the boolean 'not' operator.

The `not` element takes the attribute `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `not` element is a unary logical operator (see section 4.2.3).

Example

```
<apply>
  <not/>
  <ci> a </ci>
</apply>
```

Default Rendering

$\neg a$

4.4.3.16 Implies (`implies`)

Discussion

The `implies` element is the boolean relational operator 'implies'.

The `implies` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `implies` element is a binary logical operator (see section 4.2.4).

Example

```
<apply>
  <implies/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Mathematical applications designed for the evaluation of such expressions would evaluate this to `true` when a = `false` and b = `true`

Default Rendering

$A \Rightarrow B$

4.4.3.17 Universal quantifier (`forall`)

The `forall` element represents the universal quantifier of logic. It must used in conjunction with one or more bound variables, an optional `condition` element, and an assertion, which may either take the form of an `apply` or `reln` element.

The `forall` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `forall` element is an quantifier (see section 4.2.3.2).

Examples

The first example encodes the sense of the expression 'for all x, x - x = 0'.

```
<apply>
  <forall/>
  <bvar><ci> x </ci></bvar>
  <apply><eq/>
    <apply>
      <minus/><ci> x </ci><ci> x </ci>
    </apply>
```

```
    <cn>0</cn>
  </apply>
</apply>
```

A more involved example, making use of an optional `condition` element encodes the sense of the expression: for all p, q in $\mathbb{Q}$ such that $p < q$, $p < q^2$.

```
<apply>
  <forall/>
  <bvar><ci> p </ci></bvar>
  <bvar><ci> q </ci></bvar>
  <condition>
    <apply><and/>
      <apply><in/><ci> p </ci><ci type="set"> Q </ci></apply>
      <apply><in/><ci> q </ci><ci type="set"> Q </ci></apply>
      <apply><lt/><ci> p </ci><ci> q </ci></apply>
    </apply>
  </condition>
  <apply><lt/>
      <ci> p </ci>
      <apply>
          <power/>
        <ci> q </ci>
        <cn> 2 </cn>
      </apply>
  </apply>
</apply>
```

A final example, utilizing both the `forall` and `exists` quantifiers, encodes the sense of the expression: for all $n > 0$, n in $\mathbb{Z}$, there exist x, y, z in $\mathbb{Z}$ such that $x^n + y^n = z^n$.

```
<apply>
  <forall/>
  <bvar><ci> n </ci></bvar>
  <condition>
    <apply><and/>
      <apply><gt/><ci> n </ci><cn> 0 </cn></apply>
      <apply><in/><ci> n </ci><ci type="set"> Z </ci></apply>
    </apply>
  </condition>
  <apply>
    <exists/>
    <bvar><ci> x </ci></bvar>
    <bvar><ci> y </ci></bvar>
    <bvar><ci> z </ci></bvar>
    <condition>
      <apply><and/>
        <apply><in/><ci> x </ci><ci type="set"> Z </ci></apply>
        <apply><in/><ci> y </ci><ci type="set"> Z </ci></apply>
        <apply><in/><ci> z </ci><ci type="set"> Z </ci></apply>
      </apply>
```

```
    </condition>
    <apply>
      <eq/>
      <apply>
        <plus/>
        <apply><power/><ci> x </ci><ci> n </ci></apply>
        <apply><power/><ci> y </ci><ci> n </ci></apply>
      </apply>
      <apply><power/><ci> z </ci><ci> n </ci></apply>
    </apply>
  </apply>
</apply>
```

Default Rendering

1. for all
2. ∀

4.4.3.18 Existential quantifier (*exists*)

The `exists` element represents the existential quantifier of logic. It must used in conjuction with one or more bound variables, an optional `condition` element, and an assertion, which may either take the form of an `apply` or `reln` element.

The `exists` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `exists` element is an quantifier (see section 4.2.3.2).

Example

The following example encodes the sense of the expression ‘there exists an *x* such that $f(x) = 0$’.

```
<apply>
  <exists/>
  <bvar><ci> x </ci></bvar>
  <apply><eq/>
    <apply>
      <fn><ci> f </ci></fn>
      <ci> x </ci>
    </apply>
    <cn>0</cn>
  </apply>
</apply>
```

Default Rendering

1. there exists
2. ∃

4.4.3.19 Absolute Value (*abs*)

The abselement represents the absolute value of a real quantity or the modulus of a complex quantity.

The abselement takes the definitionURLand encodingattributes, which can be used to override the default semantics.

The abselement is a unary arithmetic operator (see section 4.2.3).

Example

The following example encodes the absolute value of x.

```
<apply>
  <abs/>
  <ci> x </ci>
</apply>
```

Default Rendering

$|x|$

4.4.3.20 Complex conjugate (*conjugate*)

The conjugateelement represents the complex conjugate of a complex quantity.

The conjugateelement takes the definitionURLand encodingattributes, which can be used to override the default semantics.

The conjugateelement is a unary arithmetic operator (see section 4.2.3).

Example

The following example encodes the conjugate of $x + \mathrm{i}y$.

```
<apply>
  <conjugate/>
  <apply>
    <plus/>
    <ci> x </ci>
    <apply><times/>
      <cn> &ImaginaryI; </cn>
      <ci> y </ci>
    </apply>
  </apply>
</apply>
```

Default Rendering

$$\overline{x+\mathrm{i}y}$$

4.4.3.21 Argument (*arg*)

The `arg` operator (introduced in MathML 2.0)) gives the 'argument' of a complex number, which is the angle (in radians) it makes with the positive real axis. Real negative numbers have argument equal to $+\pi$.

The `arg` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `arg` element is a unary arithmetic operator (see section 4.2.3).

Example

The following example encodes the argument operation on $x + \mathrm{i}\, y$.

```
<apply>
  <arg/>
  <apply><plus/>
    <ci> x </ci>
    <apply><times/>
      <cn> &ImaginaryI; </cn>
      <ci> y </ci>
    </apply>
  </apply>
</apply>
```

Default Rendering

$$\arg(x + \mathrm{i}y)$$

4.4.3.22 Real part (*real*)

The `real` operator (introduced in MathML 2.0) gives the real part of a complex number, that is the x component in $x + \mathrm{i}\, y$

The `real` element takes the attributes `encoding, definitionURL` can be used to override the default semantics.

The `real` element is a unary arithmetic operator (see section 4.2.3).

Example

The following example encodes the real operation on $x + \mathrm{i}\, y$.

```
<apply>
  <real/>
  <apply><plus/>
    <ci> x </ci>
    <apply><times/>
      <cn> &ImaginaryI; </cn>
      <ci> y </ci>
    </apply>
```

```
  </apply>
</apply>
```

A MathML-aware evaluation system would return the *x* component, suitably encoded.

Default Rendering

$$\mathrm{real}(x + \mathrm{i}y)$$

4.4.3.23 Imaginary part (*imaginary*)

The `imaginary` operator (introduced in MathML 2.0) gives the imaginary part of a complex number, that is the y component in *x* + i *y*

The `imaginary` element takes the attributes `encoding, definitionURL` that can be used to override the default semantics.

The `imaginary` element is a unary arithmetic operator (see section 4.2.3).

Example

The following example encodes the imaginary operation on *x* + i*y*.

```
<apply>
  <imaginary/>
  <apply><plus/>
    <ci> x </ci>
    <apply><times/>
      <cn> &ImaginaryI; </cn>
      <ci> y </ci>
    </apply>
  </apply>
</apply>
```

A MathML-aware evaluation system would return the *y* component, suitably encoded.

Default Rendering

$$\mathrm{imaginary}(x + \mathrm{i}y)$$

4.4.4 Relations

4.4.4.1 Equals (*eq*)

Discussion

The `eq` element is the relational operator ‘equals’.

The `eq` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `equals` element is an n-ary relation (see section 4.2.3.2).

Example

```
<apply>
  <eq/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were tested at $a = 5.5$ and $b = 6$ it would yield the truth value `false`

Default Rendering

$a = b$

4.4.4.2 Not Equals (*neq*)

Discussion

The `neq` element is the 'not equal to' relational operator.

`neq` takes the `definitionURL` and `encoding` attributes which can be used to override the default semantics.

The `neq` element is an binary relation (see section 4.2.4).

Example

```
<apply>
  <neq/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were tested at $a = 5.5$ and $b = 6$ it would yield the truth value `true`

Default Rendering

$a \neq b$

4.4.4.3 Greater than (*gt*)

Discussion

The `gt` element is the 'greater than' relational operator.

The `gt` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `gt` element is an n-ary relation (see section 4.2.4).

Example

```
<apply>
  <gt/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were tested at $a = 5.5$ and $b = 6$ it would yield the truth value `false`

Default Rendering

$a > b$

4.4.4.4 Less Than (*lt*)

Discussion

The `lt` element is the 'less than' relational operator.

The `lt` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `lt` element is an n-ary relation (see section 4.2.4).

Example

```
<apply>
  <lt/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were tested at $a = 5.5$ and $b = 6$ it would yield the truth value 'true'.

Default Rendering

$a < b$

4.4.4.5 Greater Than or Equal (*geq*)

Discussion

The `geq` element is the relational operator 'greater than or equal'.

The `geq` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `geq` element is an n-ary relation (see section 4.2.4).

Example

```
<apply>
  <geq/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If this were tested for $a = 5.5$ and $b = 5.5$ it would yield the truth value `true`

Default Rendering

$$a \geq b$$

4.4.4.6 Less Than or Equal (*leq*)

Discussion

The `leq` element is the relational operator 'less than or equal'.

The `leq` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `leq` element is an n-ary relation (see section 4.2.4).

Example

```
<apply>
  <leq/>
  <ci> a </ci>
  <ci> b </ci>
</apply>
```

If $a = 5.4$ and $b = 5.5$ this will yield the truth value `true`

Default Rendering

$$a \leq b$$

4.4.4.7 Equivalent (*equivalent*)

Discussion

The `equivalent` element is the 'equivalence' relational operator.

The `equivalent` element takes the attributes `encoding, definitionURL` can be used to override the default semantics.

The `equivalent` element is an n-ary relation (see section 4.2.3.2).

Example

```
<apply>
  <equivalent/>
  <ci> a </ci>
  <apply>
    <not/>
    <apply> <not/> <ci> a </ci> </apply>
  </apply>
</apply>
```

This yields the truth value `true` for all values of *a*.

Default Rendering

$$a \equiv \neg(\neg a)$$

4.4.4.8 Approximately (*approx*)

Discussion

The `approx` element is the relational operator 'approximately equal'.

The `approx` element takes the attributes `encoding, definitionURL` that can be used to override the default semantics.

The `approx` element is a binary relation (see section 4.2.3.2).

Example

```
<apply>
  <approx/>
  <cn type="rational"> 22 <sep/> 7 </cn>
  <cn type="constant"> &pi; </cn>
</apply>
```

Default Rendering

$$a \approx b$$

4.4.5 Calculus and Vector Calculus

4.4.5.1 Integral (*int*)

Discussion

The `int` element is the operator element for an integral. The lower limit, upper limit and bound variable are given by (optional) child elements, `lowlimit`, `uplimit` and `bvar` in the enclosing `apply` element. The integrand is also specified as a child element of the enclosing `apply` element.

The domain of integration may alternatively be specified by using an `interval` element, or by a `condition` element. In such cases, if a bound variable of integration is intended, it must be specified explicitly. (The condition may involve more than one symbol.)

The `int` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `int` element is an operator taking qualifiers (see section 4.2.3.2).

Examples

This example specifies a `lowlimit`, `uplimit`, and `bvar`.

```
<apply>
  <int/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <lowlimit>
    <cn> 0 </cn>
  </lowlimit>
  <uplimit>
    <ci> a </ci>
  </uplimit>
  <apply>
    <fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

This example specifies the domain of integration with an `interval` element.

```
<apply>
  <int/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <interval>
     <ci> a </ci>
     <ci> b </ci>
   </interval>
  <apply><cos/>
    <ci> x </ci>
  </apply>
</apply>
```

The final example specifies the domain of integration with an `condition` element.

```
<apply>
  <int/>
  <bvar>
    <ci> x </ci>
  </bvar>
```

```
  <condition>
    <apply><in/>
      <ci> x </ci>
      <ci type="set"> D </ci>
    </apply>
  </condition>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$\int_0^a f(x)\,\mathrm{d}x$$

$$\int_a^b \cos x\,\mathrm{d}x$$

$$\int_{x\in D} f(x)\,\mathrm{d}x$$

4.4.5.2 Differentiation (*diff*)

Discussion

The `diff` element is the differentiation operator element for functions of a single real variable. The bound variable is given by a `bvar` element that is a child of the containing `apply` element. The `bvar` elements may also contain a `degree` element, which specifies the order of the derivative to be taken.

The `diff` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `diff` element is an operator taking qualifiers (see section 4.2.3.2).

Example

```
<apply>
  <diff/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$\frac{\mathrm{d}\, f(x)}{\mathrm{d}x}$$

4.4.5.3 Partial Differentiation (*partialdiff*)

Discussion

The `partialdiff` element is the partial differentiation operator element for functions of several real variables. The bound variables are given by `bvar` elements, which are children of the containing `apply` element. The `bvar` elements may also contain a `degree` element, which specifies the order of the partial derivative to be taken in that variable.

The `partialdiff` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `partialdiff` element is an operator taking qualifiers (see section 4.2.3.2).

Example

```
<apply>
  <partialdiff/>
  <bvar>
     <ci> x </ci>
     <degree>
        <cn> 2 </cn>
     </degree>
  </bvar>
  <bvar>
     <ci> y </ci>
  </bvar>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
    <ci> y </ci>
  </apply>
</apply>
```

Default Rendering

$$\frac{\partial^3}{\partial x^2 \partial y} f(x,y)$$

4.4.5.4 Lower limit (*lowlimit*)

Discussion

The `lowlimit` element is the container element used to indicate the ‘lower limit’ of an operator using qualifiers. For example, in an integral, it can be used to specify the lower limit of integration. Similarly, it is also used to specify the lower limit of an index for sums and products.

The meaning of the `lowlimit` element depends on the context it is being used in. For further details about how qualifiers are used in conjunction with operators taking qualifiers, consult section 4.2.3.2.

Example

```
<apply>
  <int/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <lowlimit>
    <ci> a </ci>
  </lowlimit>
  <uplimit>
    <ci> b </ci>
  </uplimit>
  <apply><fn><ci> f </ci></fn>
     <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

The default rendering of the `lowlimit` element and its contents depends on the context. In the preceding example, it should be rendered as a subscript to the integral sign:

$$\int_{a}^{b} f(x)\,\mathrm{d}x$$

Consult the descriptions of individual operators that make use of the `lowlimit` construct for default renderings.

4.4.5.5 Upper limit (*uplimit*)

Discussion

The `uplimit` element is the container element used to indicate the 'upper limit' of an operator using qualifiers. For example, in an integral, it can be used to specify the upper limit of integration. Similarly, it is also used to specify the upper limit of an index for sums and products.

The meaning of the `uplimit` element depends on the context it is being used in. For further details about how qualifiers are used in conjunction with operators taking qualifiers, consult section 4.2.3.2.

Example

```
<apply>
  <int/>
```

```
  <bvar>
     <ci> x </ci>
  </bvar>
  <lowlimit>
     <ci> a </ci>
  </lowlimit>
  <uplimit>
     <ci> b </ci>
  </uplimit>
  <apply><fn><ci> f </ci></fn>
     <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

The default rendering of the `uplimit` element and its contents depends on the context. In the preceding example, it should be rendered as a superscript to the integral sign:

$$\int_{a}^{b} f(x)\,\mathrm{d}x$$

Consult the descriptions of individual operators that make use of the `uplimit` construct for default renderings.

4.4.5.6 Bound variable (*bvar*)

Discussion

The `bvar` element is the container element for the 'bound variable' of an operation. For example, in an integral it specifies the variable of integration. In a derivative, it indicates which variable with respect to which a function is being differentiated. When the `bvar` element is used to quantify a derivative, the `bvar` element may contain a child `degree` element that specifies the order of the derivative with respect to that variable. The `bvar` element is also used for the internal variable in sums and products and for the bound variable used with the universal and existential quantifiers `forall` and `exists`

The meaning of the `bvar` element depends on the context it is being used in. For further details about how qualifiers are used in conjunction with operators taking qualifiers, consult section 4.2.3.2.

Examples

```
<apply>
  <diff/>
  <bvar>
    <ci> x </ci>
    <degree>
      <cn> 2 </cn>
    </degree>
  </bvar>
```

```
  <apply>
    <power/>
    <ci> x </ci>
    <cn> 4 </cn>
  </apply>
</apply>

<apply>
  <int/>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><in/><ci> x </ci><ci> D </ci></apply>
  </condition>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

The default rendering of the `bvar` element and its contents depends on the context. In the preceding examples, it should be rendered as the x in the dx of the integral, and as the x in the denominator of the derivative symbol:

$$\frac{\mathrm{d}x^4}{\mathrm{d}x^2}$$

$$\int_{x \in D} f(x)\,\mathrm{d}x$$

Note that in the case of the derivative, the default rendering of the `degree` child of the `bvar` element is as an exponent.

Consult the descriptions of individual operators that make use of the `bvar` construct for default renderings.

4.4.5.7 Degree (*degree*)

Discussion

The `degree` element is the container element for the ‘degree’ or ‘order’ of an operation. There are a number basic mathematical constructs that come in families, such as derivatives and moments. Rather than introduce special elements for each of these families, MathML uses a single general construct, the `degree` element for this concept of ‘order’.

The meaning of the `degree` element depends on the context it is being used in. For further details about how qualifiers are used in conjunction with operators taking qualifiers, consult section 4.2.3.2.

Example

```
<apply>
  <partialdiff/>
```

```
  <bvar>
    <ci> x </ci>
    <degree>
      <ci> n </ci>
    </degree>
  </bvar>
  <bvar>
    <ci> y </ci>
    <degree>
      <ci> m </ci>
    </degree>
  </bvar>
  <apply><sin/>
    <apply> <times/>
      <ci> x </ci>
      <ci> y </ci>
    </apply>
  </apply>
</apply>
```

Default Rendering

The default rendering of the `degree` element and its contents depends on the context. In the preceding example, the `degree` elements would be rendered as the exponents in the differentiation symbols:

$$\frac{\partial^{n+m}}{\partial x^n \partial y^m} \sin(xy)$$

Consult the descriptions of individual operators that make use of the `degree` construct for default renderings.

4.4.5.8 Divergence (*`divergence`*)

Discussion

The `divergence` element is the vector calculus divergence operator, often called div.

The `divergence` element takes the attributes `encoding, definitionURL` that can be used to override the default semantics.

The `divergence` element is an unary calculus operator (see section 4.2.3).

Example

```
<apply>
<divergence/>
  <ci> a </ci>
</apply>
```

If a is a vector field defined inside a closed surface S enclosing a volume V, then the divergence of a is given by

```
<apply>
  <limit/>
  <bvar>
    <ci> V </ci>
  </bvar>
  <condition>
    <apply>
      <tendsto/>
      <ci> V </ci>
      <cn> 0 </cn>
    </apply>
  </condition>
  <apply>
    <divide/>
    <apply><int encoding="text" definitionURL="SurfaceIntegrals.htm"/>
      <bvar>
        <ci> S</ci>
      </bvar>
    <ci> a </ci>
    </apply>
    <ci> V </ci>
  </apply>
</apply>
```

Default Rendering

$$\operatorname{div} a$$

4.4.5.9 Gradient (*grad*)

Discussion

The `grad` element is the vector calculus gradient operator, often called grad.

The `grad` element takes the attributes `encoding, definitionURL` that can be used to override the default semantics.

The `grad` element is an unary calculus operator (see section 4.2.3).

Example

```
<apply>
  <grad/>
  <ci> f</ci>
</apply>
```

Where for example f is a scalar function and $f(x,y,z) = k$ defines a surface S

Default Rendering

$$\operatorname{grad} f$$

4.4.5.10 Curl (*curl*)

Discussion

The curl element is the vector calculus curl operator.

The curl element takes the attributes encoding, definitionURL can be used to override the default semantics.

The curl element is an unary calculus operator (see section 4.2.3).

Example

```
<apply>
  <curl/>
  <ci> a </ci>
</apply>
```

Where for example *a* is a vector.

Default Rendering

curl *a*

4.4.5.11 Laplacian (*laplacian*)

Discussion

The laplacian element is the vector calculus laplacian operator.

The laplacian element takes the attributes encoding, definitionURL can be used to override the default semantics.

The laplacian element is an unary calculus operator (see section 4.2.3).

Example

```
<apply>
  <eq/>
  <apply><laplacian/>
    <ci> f </ci>
  </apply>
  <apply>
    <divergence/>
    <apply><grad/>
      <ci> f </ci>
    </apply>
  </apply>
</apply>
```

Where for example f is a vector

Default Rendering

$$\nabla^2 f$$

4.4.6 Theory of Sets

4.4.6.1 Set (*set*)

Discussion

The `set` element is the container element that constructs a set of elements. The elements of a set can be defined either by explicitly listing the elements, or by using the `bvar` and `condition` elements.

The `set` element is a constructor element (see section 4.2.2.2).

Examples

```
<set>
  <ci> b </ci>
  <ci> a </ci>
  <ci> c </ci>
</set>
```

```
<set>
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><lt/>
      <ci> x </ci>
      <cn> 5 </cn>
    </apply>
  </condition>
</set>
```

Default Rendering

1. a, b, c

2. $x \mid x < 5$

4.4.6.2 List (*list*)

Discussion

The `list` element is the container element that constructs a list of elements. Elements can be defined either by explicitly listing the elements, or by using the `bvar` and `condition` elements.

Lists differ from sets in that there is an explicit order to the elements. Two orders are supported: lexicographic and numeric. The kind of ordering that should be used is specified by the `order` attribute.

The `list` element is a constructor element (see section 4.2.2.2).

Examples

```
<list>
  <ci> a </ci>
  <ci> b </ci>
  <ci> c </ci>
</list>
```

```
<list order="numeric">
  <bvar><ci> x </ci></bvar>
  <condition>
    <apply><lt/>
      <ci> x </ci>
      <cn> 5 </cn>
    </apply>
  </condition>
</list>
```

Default Rendering

1. [*a*, *b*, *c*]

2. [$x \mid x < 5$]

4.4.6.3 Union (*union*)

Discussion

The `union` element is the operator for a set-theoretic union or join of two (or more) sets.

The `union` attribute takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `union` element is an n-ary set operator (see section 4.2.3).

Example

```
<apply>
  <union/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$A \cup B$

4.4.6.4 Intersect (*intersect*)

Discussion

The `intersect` element is the operator for the set-theoretic intersection or meet of two (or more) sets.

The `intersect` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `intersect` element is an n-ary set operator (see section 4.2.3).

Example

```
<apply>
  <intersect/>
  <ci type="set"> A </ci>
  <ci type="set"> B </ci>
</apply>
```

Default Rendering

$$A \cap B$$

4.4.6.5 Set inclusion (*in*)

Discussion

The `in` element is the relational operator used for a set-theoretic inclusion (‘is in’ or ‘is a member of’).

The `in` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `in` element is a binary set relation (see section 4.2.4).

Example

```
<apply>
  <in/>
  <ci> a </ci>
  <ci type="set"> A </ci>
</apply>
```

Default Rendering

$$a \in A$$

4.4.6.6 Set exclusion (*notin*)

Discussion

The `notin` element is the relational operator element used for set-theoretic exclusion ('is not in' or 'is not a member of').

The `notin` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `notin` element is a binary set relation (see section 4.2.4).

Example

```
<apply>
  <notin/>
  <ci> a </ci>
  <ci> A </ci>
</apply>
```

Default Rendering

$$a \notin A$$

4.4.6.7 Subset (*subset*)

Discussion

The `subset` element is the relational operator element for a set-theoretic containment ('is a subset of').

The `subset` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `subset` element is an n-ary set relation (see section 4.2.4).

Example

```
<apply>
  <subset/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$$A \subseteq B$$

4.4.6.8 Proper Subset (*prsubset*)

Discussion

The prsubset element is the relational operator element for set-theoretic proper containment (‘is a proper subset of’).

The prsubset element takes the definitionURL and encoding attributes, which can be used to override the default semantics.

The subset element is an n-ary set relation (see section 4.2.4).

Example

```
<apply>
  <prsubset/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$$A \subset B$$

4.4.6.9 Not Subset (*notsubset*)

Discussion

The notsubset element is the relational operator element for the set-theoretic relation ‘is not a subset of’.

The notsubset element takes the definitionURL and encoding attributes, which can be used to override the default semantics.

The notsubset element is a binary set relation (see section 4.2.4).

Example

```
<apply>
  <notsubset/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$$A \not\subset B$$

4.4.6.10 Not Proper Subset (*notprsubset*)

Discussion

The `notprsubset` element is the operator element for the set-theoretic relation 'is not a proper subset of'.

The `notprsubset` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `notprsubset` element is a binary set relation (see section 4.2.4).

Example

```
<apply>
  <notprsubset/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$$A \not\subset B$$

4.4.6.11 Set Difference (*setdiff*)

Discussion

The `setdiff` element is the operator element for a set-theoretic difference of two sets.

The `setdiff` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics. `setdiff` is a binary operator.

The `setdiff` element is a binary set operator (see section 4.2.3).

Example

```
<apply>
  <setdiff/>
  <ci> A </ci>
  <ci> B </ci>
</apply>
```

Default Rendering

$$A \setminus B$$

4.4.6.12 Cardinality (*card*)

Discussion

The `card` element is the operator element for deriving the size or cardinality of a set

The `card` element takes the attributes `definitionURL, encoding` and can be used to override the default semantics.

The `card` element is a unary set operator (see section 4.2.3).

Example

```
<apply>
  <eq/>
  <apply><card/>
    <ci> A </ci>
  </apply>
  <ci> 5 </ci>
</apply>
```

where A is a set with 5 elements.

Default Rendering

$|A|$

4.4.7 Sequences and Series

4.4.7.1 Sum (*sum*)

Discussion

The `sum` element denotes the summation operator. Upper and lower limits for the sum, and more generally a domains for the bound variables are specified using `uplimit`, `lowlimit` or a `condition` on the bound variables. The index for the summation is specified by a `bvar` element.

The `sum` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `sum` element is an operator taking qualifiers (see section 4.2.3.2).

Examples

```
<apply>
  <sum/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <lowlimit>
    <ci> a </ci>
  </lowlimit>
```

```
  <uplimit>
    <ci> b </ci>
  </uplimit>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>

<apply>
  <sum/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <condition>
    <apply> <in/>
      <ci> x </ci>
      <ci type="set"> B </ci>
    </apply>
  </condition>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$\sum_{x=a}^{b} f(x)$$

$$\sum_{x \in B} f(x)$$

4.4.7.2 Product (*product*)

Discussion

The `product` element denotes the product operator. Upper and lower limits for the product, and more generally a domains for the bound variables are specified using `uplimit`, `lowlimit` or a `condition` on the bound variables. The index for the product is specified by a `bvar` element.

The `product` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `product` element is an operator taking qualifiers (see section 4.2.3.2).

Examples

```
<apply>
  <product/>
  <bvar>
```

```
    <ci> x </ci>
  </bvar>
  <lowlimit>
    <ci> a </ci>
  </lowlimit>
  <uplimit>
    <ci> b </ci>
  </uplimit>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>

<apply>
  <product/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <condition>
    <apply> <in/>
      <ci> x </ci>
      <ci type="set"> B </ci>
    </apply>
  </condition>
  <apply><fn><ci> f </ci></fn>
    <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$\prod_{x=a}^{b} f(x)$$

$$\prod_{x \in B} f(x)$$

4.4.7.3 Limit (*limit*)

Discussion

The `limit` element represents the operation of taking a limit of a sequence. The limit point is expressed by specifying a `lowlimit` and a `bvar`, or by specifying a `condition` on one or more bound variables.

The `limit` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `limit` element is an operator taking qualifiers (see section 4.2.3.2).

Examples

```
<apply>
  <limit/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <lowlimit>
    <cn> 0 </cn>
  </lowlimit>
  <apply><sin/>
    <ci> x </ci>
  </apply>
</apply>

<apply>
  <limit/>
  <bvar>
    <ci> x </ci>
  </bvar>
  <condition>
    <apply>
      <tendsto type="above"/>
      <ci> x </ci>
      <ci> a </ci>
    </apply>
  </condition>
  <apply><sin/>
     <ci> x </ci>
  </apply>
</apply>
```

Default Rendering

$$\lim_{x \to 0} \sin x$$

$$\lim_{x \to a^{+}} \sin x$$

4.4.7.4 Tends To (*tendsto*)

Discussion

The `tendsto` element is used to express the relation that a quantity is tending to a specified value.

The `tendsto` element takes the attributes `type` to set the direction from which the the limiting value is approached and the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `tendsto` element is a binary relational operator (see section 4.2.4).

Examples

```
<apply>
  <tendsto type="above"/>
  <apply>
    <power/>
    <ci> x </ci>
    <cn> 2 </cn>
  </apply>
  <apply>
    <power/>
    <ci> a </ci>
    <cn> 2 </cn>
  </apply>
</apply>
```

To express $(x, y) \to (f(x, y), g(x,y))$, one might use vectors, as in:

```
<apply>
  <tendsto/>
  <vector>
     <ci> x </ci>
     <ci> y </ci>
  </vector>
  <vector>
    <apply><fn><ci> f </ci></fn>
      <ci> x </ci>
      <ci> y </ci>
    </apply>
    <apply><fn><ci> g </ci></fn>
      <ci> x </ci>
      <ci> y </ci>
    </apply>
  </vector>
</apply>
```

Default Rendering

$x^2 \to a^2$

$(x,y) \to (f(x,y),g(x,y))$

4.4.8 Elementary classical functions

The names of the common trigonometric functions supported by MathML are listed below. Since their standard interpretations are widely known, they are discussed as a group.

4.4.8.1 Discussion

These operator elements denote the standard trigonometrical functions.

These elements all take the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

They are all unary trigonometric operators. (see section 4.2.3).

```
sin      cos      tan
sec      csc      cot
sinh     cosh     tanh
sech     csch     coth
arcsin   arccos   arctan
arccosh  arccot   arccoth
arccsc   arccsch  arcsec
arcsech  arcsinh  arctanh
```

4.4.8.2 Examples

```
<apply>
  <sin/>
  <ci> x </ci>
</apply>

<apply>
  <sin/>
  <apply>
    <plus/>
    <apply><cos/>
      <ci> x </ci>
    </apply>
    <apply>
      <power/>
      <ci> x </ci>
      <cn> 3 </cn>
    </apply>
  </apply>
</apply>
```

4.4.8.3 Default Rendering

sin x

$\sin(\cos x + x^3)$

4.4.8.4 Exponential (*exp*)

Discussion

The `exp` element represents the exponential function associated with the inverse of the `ln` function. In particular, exp(1) is approximately 2.718281828.

`exp` takes the `definitionURL` and `encoding` attributes which may be used to override the default semantics.

The `exp` element is a unary arithmetic operator (see section 4.2.3).

Example

```
<apply>
  <exp/>
  <ci> x </ci>
</apply>
```

Default Rendering

e^{x}

4.4.8.5 Natural Logarithm (*ln*)

Discussion

The `ln` element is the natural logarithm operator.

The `ln` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `ln` element is an unary calculus operator (see section 4.2.3).

Example

```
<apply>
  <ln/>
  <ci> a </ci>
</apply>
```

If $a = e$ this will yield the value 1.

Default Rendering

ln *a*

4.4.8.6 Logarithm (*log*)

Discussion

The `log` element is the operator that returns a logarithm to a given base. The base may be specified using a `logbase` element, which should be the first element following `log` i.e. the second child of the containing `apply` element. If the `logbase` element is not present, a default base of 10 is assumed.

The `log` element takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

The `log` element can be used as either an operator taking qualifiers or a unary calculus operator (see section 4.2.3.2).

Example

```
<apply>
  <log/>
  <logbase>
    <cn> 3 </cn>
  </logbase>
  <ci> x </ci>
</apply>
```

This markup represents ‘the base 3 logarithm of x’. For natural logarithms base e, the `ln` element should be used instead.

Default Rendering

$\log_3 x$

4.4.9 Statistics

4.4.9.1 Mean (*mean*)

Discussion

`mean` is the operator element for a mean or average.

`mean` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`mean` is an n-ary operator (see section 4.2.3).

```
<apply>
  <mean/>
  <ci> X </ci>
</apply>
```

Default Rendering

$\bar{X}$ or $\langle X \rangle$

4.4.9.2 Standard Deviation (*sdev*)

Discussion

`sdev` is the operator element for the standard deviation.

`sdev` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`sdev` is an n-ary operator (see section 4.2.3).

```
<apply>
  <sdev/>
  <ci> X </ci>
</apply>
```

Default Rendering

$$\sigma(X)$$

4.4.9.3 Variance (*variance*)

Discussion

`variance` is the operator element for the statistical variance.

`variance` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`variance` is an n-ary operator (see section 4.2.3).

```
<apply>
  <variance/>
  <ci> X </ci>
</apply>
```

Default Rendering

$$\sigma(X)^2$$

4.4.9.4 Median (*median*)

Discussion

`median` is the operator element for the median .

`median` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`median` is an n-ary operator (see section 4.2.3).

```
<apply>
  <median/>
  <ci> X </ci>
</apply>
```

Default Rendering

median(X)

4.4.9.5 Mode (*mode*)

Discussion

`mode` is the operator for the statistical mode.

`mode` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`mode` is an n-ary operator (see section 4.2.3).

```
<apply>
  <mode/>
  <ci> X </ci>
</apply>
```

Default Rendering

mode(X)

4.4.9.6 Moment (*moment*)

Discussion

The `moment` element represents statistical moments. Use `degree` for the *n* in '*n*-th moment'.

`moment` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`moment` is an operator taking qualifiers (see section 4.2.3.2).

```
<apply>
  <moment/>
  <degree>
    <cn> 3 </cn>
  </degree>
  <ci> X </ci>
</apply>
```

Default Rendering

$\langle X^3 \rangle$

4.4.10 Linear Algebra

4.4.10.1 Vector (*vector*)

Discussion

`vector` is the container element for a vector. The child elements form the components of the vector.

For purposes of interaction with matrices and matrix multiplication, vectors are regarded as equivalent to a matrix consisting of a single column, and the transpose of a vector behaves the same as a matrix consisting of a single row.

Example

`vector` is a constructor element (see section 4.2.2.2).

```
<vector>
  <cn> 1 </cn>
  <cn> 2 </cn>
  <cn> 3 </cn>
  <ci> x </ci>
</vector>
```

Default Rendering

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ x \end{pmatrix}$$

(1, 2, 3, *x*)

4.4.10.2 Matrix (*matrix*)

Discussion

The `matrix` element is the container element for matrix rows, which are represented by `matrixrow`. The `matrixrow` s contain the elements of a matrix.

Example

`matrix` is a constructor element (see section 4.2.2.2).

```
<matrix>
  <matrixrow>
    <cn> 0 </cn> <cn> 1 </cn> <cn> 0 </cn>
  </matrixrow>
  <matrixrow>
    <cn> 0 </cn> <cn> 0 </cn> <cn> 1 </cn>
  </matrixrow>
  <matrixrow>
    <cn> 1 </cn> <cn> 0 </cn> <cn> 0 </cn>
  </matrixrow>
</matrix>
```

Default Rendering

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

4.4.10.3 Matrix row (*matrixrow*)

Discussion

The `matrixrow` element is the container element for the rows of a matrix.

Example

`matrixrow` is a constructor element (see section 4.2.2.2).

```
<matrixrow>
  <cn> 1 </cn>
  <cn> 2 </cn>
</matrixrow>
<matrixrow>
  <cn> 3 </cn>
  <ci> x </ci>
</matrixrow>
```

Default Rendering

Matrix rows are not directly rendered by themselves outside of the context of a matrix.

4.4.10.4 Determinant (*determinant*)

Discussion

The `determinant` element is the operator for constructing the determinant of a matrix.

`determinant` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`determinant` is a unary operator (see section 4.2.3).

```
<apply>
  <determinant/>
  <ci type="matrix"> A </ci>
</apply>
```

Default Rendering

det A

4.4.10.5 Transpose (*transpose*)

Discussion

The `transpose` element is the operator for constructing the transpose of a matrix.

`transpose` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

Example

`transpose` is a unary operator (see section 4.2.3).

```
<apply>
  <transpose/>
  <ci type="matrix"> A </ci>
</apply>
```

Default Rendering

A^t

4.4.10.6 Selector (*selector*)

Discussion

The `selector` element is the operator for indexing into vectors matrices and lists. It accepts one or more arguments. The first argument identifies the vector, matrix or list from which the selection is taking place, and the second and subsequent arguments, if any, indicate the kind of selection taking place.

When `selector` is used with a single argument, it should be interpreted as giving the sequence of all elements in the list, vector or matrix given. The ordering of elements in the sequence for a matrix is understood to be first by column, then by row. That is, for a matrix $(a_{i,j})$, where the indices denote row and column, the ordering would be $a_{1,1}$, $a_{1,2}$, ... $a_{2,1}$, $a_{2,2}$ ... etcetera.

When three arguments are given, the last one is ignored for a list or vector, and in the case of a matrix, the second and third arguments specify the row and column of the selected element.

When two arguments are given, and the first is a vector or list, the second argument specifies an element in the list or vector. When a matrix and only one index *i* is specified as in

```
<apply>
  <selector/>
  <matrix>
    <matrixrow>
      <cn> 1 </cn> <cn> 2 </cn>
    </matrixrow>
    <matrixrow>
      <cn> 3 </cn> <cn> 4 </cn>
    </matrixrow>
```

```
  </matrix>
  <cn> 1 </cn>
</apply>
```

it refers to the *i*-th matrixrow. Thus, the preceding example selects the following row:

```
<matrixrow> <cn> 1 </cn> <cn> 2 </cn> </matrixrow>
```

`selector` takes the `definitionURL` and `encoding` attributes, which can be used to override the default semantics.

`selector` is classified as an n-ary linear algebra operator even though it can take only one, two, or three arguments.

Example

```
<apply>
  <selector/>
  <ci type="matrix"> A </ci>
  <cn> 3 </cn>
  <cn> 2 </cn>
</apply>
```

Default Rendering

The `selector` construct renders the same as the expression which it selects.

4.4.10.7 Vector product (*`vectorproduct`*)

Discussion

The `vectorproduct` is the operator element for deriving the vector product of two vectors

The `vectorproduct` element takes the attributes `definitionURL, encoding` that can be used to override the default semantics.

The `vectorproduct` element is a binary vector operator (see section 4.2.3).

Example

```
<apply>
  <eq/>
  <apply><vectorproduct/>
    <ci type="vector"> A </ci>
    <ci type="vector"> B </ci>
  </apply>
  <apply><times/>
    <ci> a </ci>
    <ci> b </ci>
    <apply><sin/>
      <ci> &theta; </ci>
```

```
    </apply>
  </apply>
</apply>
```

where A and B are vectors, *a*, *b* are the magnitudes of A, B and θ is the angle between A and B.

Default Rendering

$A \times B$

4.4.10.8 Scalar product (*scalarproduct*)

Discussion

The `scalarproduct` is the operator element for deriving the scalar product of two vectors

The `scalarproduct` element takes the attributes `definitionURL, encoding` that can be used to override the default semantics.

The `scalarproduct` element is a binary vector operator (see section 4.2.3).

Example

```
<apply>
  <eq/>
  <apply><scalarproduct/>
    <ci type="vector"> A </ci>
    <ci type="vector">B </ci>
  </apply>
  <apply><times/>
    <ci> a </ci>
    <ci> b </ci>
    <apply><cos/>
      <ci> &theta; </ci>
    </apply>
  </apply>
</apply>
```

where A and B are vectors, *a*, *b* are the magnitudes of A, B and θ is the angle between A and B.

Default Rendering

$A.B$

4.4.10.9 Outer product (*outerproduct*)

Discussion

The `outerproduct` is the operator element for deriving the outer product of two vectors

The `outerproduct` element takes the attributes `definitionURL, encoding` that can be used to override the default semantics.

The `outerproduct` element is a binary vector operator (see section 4.2.3).

Example

```
<apply>
  <outerproduct/>
  <ci type="vector">A</ci>
  <ci type="vector">B</ci>
</apply>
```

where A and B are vectors.

Default Rendering

A.*B*

4.4.11 Semantic Mapping Elements

This section explains the use of the semantic mapping elements `semantics`, `annotation` and `annotation-xml`

4.4.11.1 Annotation (*`annotation`*)

Discussion

The `annotation` element is the container element for a semantic annotation in a non-XML format.

The `annotation` element takes the attribute `encoding` to define the encoding being used.

Example

The `annotation` element is a semantic mapping element. It is always used with `semantics`

```
<semantics>
  <apply>
    <plus/>
    <apply><sin/>
      <ci> x </ci>
    </apply>
    <cn> 5 </cn>
  </apply>
  <annotation encoding="TeX">
    \sin x + 5
  </annotation>
</semantics>
```

Default Rendering

None. The information contained in annotations may optionally be used by a renderer able to process the kind of annotation given.

4.4.11.2 Semantics (*semantics*)

Discussion

The `semantics` element is the container element that associates additional representations with a given MathML construct. The `semantics` element has as its first child the expression being annotated, and the subsequent children are the annotations. There is no restriction on the kind of annotation that can be attached using the semantics element. For example, one might give a TEX encoding, or computer algebra input in an annotation.

The representations that are XML based are enclosed in an `annotation-xml` element while those representations that are to be parsed as `PCDATA` are enclosed in an `annotation` element.

The `semantics` element takes the `definitionURL` and `encoding` attributes, which can be used to reference an external source for some or all of the semantic information.

An important purpose of the `semantics` construct is to associate specific semantics with a particular presentation, or additional presentation information with a content construct. The default rendering of a `semantics` element is the default rendering of its first child. When a MathML-presentation annotation is provided, a MathML renderer may optionally use this information to render the MathML construct. This would typically be the case when the first child is a MathML content construct and the annotation is provided to give a preferred rendering differing from the default for the content elements.

Use of `semantics` to attach additional information in-line to a MathML construct can be contrasted with use of the `csymbol` for referencing external semantics. See section 4.4.1.3

Examples and Usage

The `semantics` element is a semantic mapping element.

```
<semantics>
  <apply>
    <plus/>
    <apply>
      <sin/>
      <ci> x </ci>
    </apply>
    <cn> 5 </cn>
  </apply>
  <annotation encoding="Maple">
    sin(x) + 5
  </annotation>
  <annotation-xml encoding="MathML-Presentation">
    ...
    ...
  </annotation-xml>
  <annotation encoding="Mathematica">
    Sin[x] + 5
  </annotation>
  <annotation encoding="TeX">
    \sin x + 5
```

```
  </annotation>
  <annotation-xml encoding="OpenMath">
    <OMA>...</OMA>
  </annotation-xml>
</semantics>
```

Default Rendering

The default rendering of a `semantics` element is the default rendering of its first child.

4.4.11.3 XML-based annotation (*`annotation-xml`*)

Discussion

The `annotation-xml` container element is used to contain representations that are XML based. It is always used together with the `semantics` element, and takes the attribute `encoding` to define the encoding being used.

Issue (encoding): The various chapters use an interesting array of values for the `encoding` attribute, namely `Maple`, `MapleMathML-Content`, `MathML-Presentation`, `Mathematica 4.0`, `Mathematica`, `OpenMath`, `TeX`, `mathml`. We need some normative text here about what values this attribute can have, and we should have internal consistency among the chapters at the very least.

`annotation-xml` is a semantic mapping element.

Example

```
<semantics>
  <apply>
    <plus/>
    <apply><sin/>
      <ci> x </ci>
    </apply>
    <cn> 5 </cn>
  </apply>
  <annotation-xml encoding="OpenMath">
    <OMA><OMS name="plus" cd="arith1"/>
      <OMA><OMS name="sin" cd="transc1"/>
        <OMV name="x"/>
      </OMA>
      <OMI>5</OMI>
    </OMA>
  </annotation-xml>
</semantics>
```

See also the discussion of `semantics` above.

Default Rendering

None. The information may optionally be used by a renderer able to process the kind of annotation given.

Chapter 5

Combining Presentation and Content Markup

Presentation markup and content markup can be combined in one of two ways. The first manner is to intersperse content and presentation elements in what is essentially a single tree. This is called mixed markup. The second manner is to provide both an explicit presentation and an explicit content in a pair of trees. This is called parallel markup. This chapter describes both mixed and parallel markup, and how they may used in conjunction with style sheets and other tools.

5.1 Why Two Different Kinds of Markup?

Chapters 3 and 4 describe two kinds of markup for encoding mathematical material in documents.

Presentation markup captures notational structure. It encodes the notational structure of an expression in a sufficiently abstract way to facilitate rendering to various media. Thus, the same presentation markup can be rendered with relative ease on screen in either wide and narrow windows, in ASCII or graphics, in print, or it can be enunciated in a sensible way when spoken. It does this by providing information such as structured grouping of expression parts, classification of symbols, etc.

Presentation markup does not directly concern itself with the mathematical structure or meaning of an expression. In many situations, notational structure and mathematical structure are closely related, so a sophisticated processing application may be able to heuristically infer mathematical meaning from notational structure, provided sufficient context is known. However, in practice, the inference of mathematical meaning from mathematical notation must often be left to the reader.

Employing presentation tags alone may limit the ability to re-use a MathML object in another context, especially evaluation by external applications.

Content markup captures mathematical structure. It encodes mathematical structure in a sufficiently regular way in order to facilitate the assignment of mathematical meaning to an expression by application programs. Though the details of mapping from mathematical expression structure to mathematical meaning can be extremely complex, in practice, there is wide agreement about the conventional meaning of many basic mathematical constructions. Consequently, much of the meaning of a content expression is easily accessible to a processing application, independently of where or how it is displayed to the reader. In

many cases, content markup could be cut from a Web browser and pasted into a mathematical software tool (such as future versions of Axiom, Maple or Mathematica) with confidence that sensible values will be computed.

Since content markup is not directly concerned with how an expression is displayed, a renderer must infer how an expression should be presented to a reader. While a sufficiently sophisticated renderer and style-sheet mechanism could in principle allow a user to read mathematical documents using personalized notational preferences, in practice, rendering content expressions with notational nuances may still require human intervention of some sort.

Employing content tags alone may limit the ability of the author to precisely control how an expression is rendered.

It is important to emphasize that both content and presentation tags are necessary in order to provide the full expressive capability one would expect in a mathematical markup language. Often the same mathematical notation is used to represent several completely different concepts. For example, the notation x^i may be intended (in polynomial algebra) as the i-th power of the variable x, or (in vector analysis) as the i-th component of a vector x, for example in tensor calculus in general relativity. In other cases, the same mathematical concept may be displayed in one of various notations. For instance, the factorial of a number might be expressed with an exclamation mark, a Gamma function, or a Pochhammer symbol.

Thus the same notation may represent several mathematical ideas, and, conversely, the same mathematical idea often has several notations. In order to provide authors with the ability to precisely control notational nuances while at the same time encoding meanings in a machine-readable way, both content and presentation markup are needed.

In general, if it is important to control exactly how an expression is rendered, presentation markup will generally be more satisfactory. If it is important that the meaning of an expression can be interpreted dependably and automatically, then content markup will generally be more satisfactory.

5.2 Mixed Markup

MathML offers authors elements for both content and presentation markup. Whether to use one or the other, or a combination of both, depends on what aspects of rendering and interpretation an author wishes to control, and what kinds of re-use he or she wishes to facilitate.

5.2.1 Reasons to Mix Markup

In many common situations, an author or authoring tool may choose to generate either presentation or content markup exclusively. For example, a program for translating legacy documents would most likely generate pure presentation markup. Similarly, an educational software package might very well generate only content markup for evaluation in a computer algebra system. However, in many other situations, there are advantages to mixing both presentation and content markup within a single expression.

If an author is primarily presentation-oriented, interspersing some content markup will often produce more accessible, more re-usable results. For example, an author writing about linear algebra might write:

```
<mrow>
  <apply>
    <power/>
    <ci>x</ci><cn>2</cn>
  </apply>
  <mo>+</mo>
  <msup>
    <mi>v</mi><mn>2</mn>
  </msup>
</mrow>
```

where *v* is a vector and the superscript denotes a vector component, and *x* is a real variable. On account of the linear algebra context, a visually impaired reader may have directed his or her voice synthesis software to render superscripts as vector components. By explicitly encoding the power, the content markup yields a much better voice rendering than would likely happen by default.

If an author is primarily content-oriented, there are two reasons to intersperse presentation markup. First, using presentation markup provides a way of modifying or refining how a content expression is rendered. For example, one might write:

```
<apply>
  <in/>
  <ci><mi fontweight="bold">v</mi></ci>
  <ci>S</ci>
</apply>
```

In this case, the use of embedded presentation markup allows the author to specify that *v* should be rendered in boldface.

A second reason to intersperse presentation in content markup is that there is a continually growing list of areas of discourse that do not have pre-defined content elements for encoding their objects and operators. As a consequence, any system of content markup inevitably requires an extension mechanism that combines notation with semantics in some way. MathML content markup specifies several ways of attaching an external semantic definitions to content objects. However, it is necessary to use MathML presentation markup to specify how such user-defined semantic extensions should be rendered.

For example, the ‘rank’ operator from linear algebra is not included as a pre-defined MathML content element. Thus, to express the statement rank($u^T v$)=1 we use the `mo` presentation element inside a `ci` element to achieve the proper presentation, along with a `semantics` element to bind a semantic definition to the symbol. The `mo` element indicates to a renderer that it should use standard operator spacing around the content identifier ‘rank’, just as it would for ‘sin’ or ‘log’:

```
<apply>
  <eq/>
  <apply>
    <fn>
      <semantics>
        <ci><mo>rank</mo></ci>
```

```
      <annotation-xml encoding="OpenMath">
        <OMS cd="linalg1" name="rank"/>
      </annotation-xml>
    </semantics>
  </fn>
  <apply>
    <times/>
    <apply>
      <transpose/>
      <ci>u</ci>
    </apply>
    <ci>v</ci>
  </apply>
 </apply>
 <cn>1</cn>
</apply>
```

Here, the semantics of the presentation sub-expressions have been given using symbols from OpenMath content dictionaries (CD).

5.2.2 Combinations that are prohibited

The main consideration when presentation markup and content markup are mixed together in a single expression is that the result should still make sense. When both kinds of markup are contained in a presentation expression, this means it should be possible to render the resulting mixed expressions simply and sensibly. Conversely, when mixed markup appears in a content expression, it should be possible to simply and sensibly assign a semantic interpretation to the expression as whole. These requirements place a few natural constraints on how presentation and content markup can be mixed in a single expression, in order to avoid ambiguous or otherwise problematic expressions.

Two motivating examples illustrate the kinds of problems that must be avoided in mixed markup. Consider:

```
<mrow>
  <plus/> <mi> x </mi> <mi> y </mi>
</mrow>
```

In this example, the content element `plus`has been indiscriminately embedded in a presentation expression. Should the plus sign appear in its usual infix position, as it would in content markup, or should it render as the first thing in the row? Neither choice is very satisfactory, and consequently, this kind of mixing is not allowed. Similarly, consider:

```
<apply>
  <ci> x </ci> <mo> + </mo> <ci> y </ci>
</apply>
```

As before, the `mo`element is problematic. Should a renderer infer that the usual arithmetic operator is intended, and act as if the prefix content element `plus`had been used? Again, there is no compelling answer, and thus this kind of mixing of content and presentation markup is also prohibited.

5.2.3 Presentation Markup Contained in Content Markup

The use of presentation markup within content markup is limited to situations that do not effect the ability of content markup to unambiguously encode mathematical meaning. More specifically, presentation markup may only appear in content markup in three ways:

1. within `ci` and `cn` token elements
2. within the `csymbol` element
3. within the `semantics` element

Any other presentation markup occurring within a content markup is a MathML error. More detailed discussion of these three cases follows:

Presentation markup within token elements. The token elements `ci` and `cn` are permitted to contain any sequence of `PCDATA`, presentation elements, and `sep` empty elements. Contiguous blocks of `PCDATA` in `ci` and `cn` elements are rendered as if they were wrapped in `mi` elements. A contiguous block of `PCDATA` within `cn` should be rendered as if wrapped in `mn`. If a token element contains both `PCDATA` and presentation elements, contiguous blocks of `PCDATA` (if any) are treated as if wrapped in `mi` or `mn` elements as appropriate, and the resulting collection of presentation elements are rendered as if wrapped in an `mrow` element. The `sep` element is only meaningful in identifiers and numbers defined to be of complex type, where it separates `PCDATA` into real and imaginary parts. When a token elements contains both `sep` elements and presentation elements, the `sep` elements are ignored.

Presentation markup within the `csymbol` element. The `csymbol` element may contain either `PCDATA` interspersed with presentation markup, or content elements of the container type. It is a MathML error for a `csymbol` element to contain both presentation and content elements. When the `csymbol` element contains both raw data and presentation markup, the same rendering rules that apply to content elements of the token type should be used.

Presentation markup within the `semantics` element. One of the main purposes of the `semantics` element is to provide a mechanism for incorporating arbitrary MathML expressions into content markup in a semantically meaningful way. In particular, any valid presentation expression can be embedded in a content expression by placing it as the first child of a `semantics` element. The meaning of this wrapped expression should be indicated by one or more annotation elements also contained in the `semantics` element. Suggested rendering for a `semantics` element is discussed in section 4.2.6.

5.2.4 Content Markup Contained in Presentation Markup

Issue (text-unclear): Stéphane Dalmas has commented that the paragraph below is not clear enough.

The guiding principle for embedding content markup within presentation expressions is that the resulting expression should still have an unambiguous rendering. In general, this means that embedded content expressions must be semantically meaningful, since rendering of content markup depends on its meaning. This translates into the condition that elements of the content, operator and relation type are permitted, while elements of the qualifier and condition type are not.

As a rule, content elements other than containers derive part of their semantic meaning from the surrounding context, such as whether a `bvar` element is qualifying an integral, logical

quantifier or lambda expression. Another example would be whether a `degree` element occurs in a `root` or `partialdiff` element. Thus, in a presentation context, elements such as these do not have a clearly defined meaning, and hence there is no obvious choice for a rendering. Consequently, they are not allowed.

Issue (shorten-list): Stéphane Dalmas has suggested to list only those elements that cannot appear as a child.

The complete list of content elements that may appear as a child in a presentation element is: `ci`, `cn`, `csymbol`, `apply`, `lambda`, `reln`, `interval`, `list`, `matrix`, `matrixrow`, `set`, `vector`, `declare` (containers), `factorial`, `abs`, `conjugate`, `not`, `inverse`, `ident`, `exp`, `ln`, `log`, `sin`, `cos`, `tan`, `sec`, `csc`, `cot`, `sinh`, `cosh`, `tanh`, `sech`, `csch`, `coth`, `arcsin`, `arccos`, `arctan`, `determinant`, `transpose`, `quotient`, `divide`, `minus`, `power`, `rem`, `implies`, `setdiff`, `plus`, `times`, `max`, `min`, `gcd`, `mean`, `sdev`, `variance`, `median`, `mode`, `and`, `or`, `xor`, `selector`, `union`, `intersect`, `fn`, `compose`, `int`, `sum`, `product`, `diff`, `partialdiff`, `forall`, `exists` (operators), `neq`, `implies`, `in`, `notin`, `notsubset`, `notprsubset`, `tendsto`, `eq`, `leq`, `lt`, `geq`, `gt`, `subset`, `prsubset` (relations).

Note that within presentation markup, content expressions may only appear in locations where it is valid for any MathML expression to appear. In particular, content expressions may not appear within presentation token elements. In this regard mixing presentation and content are asymmetrical.

For rendering purposes, when a permitted content element appears within a presentation context, a processing application should treat it as if it were replaced with an `mrow` containing a presentation encoding of the rendering the application would ordinarily generate for that content markup. For example, consider:

```
<mfrac>
  <mi>x</mi>
  <interval closure="open-closed">
    <cn>1</cn>
    <cn>3</cn>
  </interval>
</mfrac>
```

In this case, a visual renderer would typically render the `interval` construct as (1,3]. Using presentation markup, this might be encoded as:

```
<mfenced close="]">
  <mn>1</mn>
  <mn>3</mn>
</mfenced>
```

Consequently, the original mixed markup should be visually rendered as

```
<mfrac>
  <mi>x</mi>
  <mfenced close="]">
    <mn>1</mn>
    <mn>3</mn>
  </mfenced>
</mfrac>
```

5.3 Parallel Markup

Some applications are able to make use of both presentation and content information. For these applications it is desirable to provide both forms of markup for the same mathematical expression. This is called parallel markup.

Parallel markup is achieved with the `semantics` element. Parallel markup for an expression can be used on its own, or can be incorporated as part of a larger content or presentation tree.

5.3.1 Top-level Parallel Markup

In many cases what is desired is to provide presentation markup and content markup for a mathematical expression as a whole. To achieve this, a single `semantics` element is used pairing two markup trees, with the first branch being the MathML presentation markup, and the second branch being the MathML content markup.

The following example encodes the boolean arithmetic expression $(a+b)(c+d)$ in this way.

```
<semantics>
  <mrow>
    <mrow><mo>(</mo><mi>a</mi> <mo>+</mo> <mi>b</mi><mo>)</mo></mrow>
    <mo>&InvisibleTimes;</mo>
    <mrow><mo>(</mo><mi>c</mi> <mo>+</mo> <mi>d</mi><mo>)</mo></mrow>
  </mrow>
  <annotation-xml encoding="MathML-Content">
    <apply><and/>
      <apply><xor/><ci>a</ci> <ci>b</ci></apply>
      <apply><xor/><ci>c</ci> <ci>d</ci></apply>
    </apply>
  </annotation-xml>
</semantics>
```

This example is non-trivial in the sense that the content markup could not be easily derived from the presentation markup alone.

5.3.2 Fine-grained Parallel Markup

Top-level pairing of independent presentation and content markup is sufficient for many, but not all, situations. Applications that allow treatment of sub-expressions of mathematical objects require the ability to associate presentation, content or information with the parts of an object with mathematical markup. Top-level pairing with a `semantics` element is insufficient in this type of situation; identification of a sub-expression in one branch of `semantics` element gives no indication of the corresponding parts in other branches.

The ability to identify corresponding sub-expressions is required in applications such as mathematical expression editors. In this situation, selecting a sub-expression on a visual display can identify a particular portion of a presentation markup tree. The application then needs to determine the corresponding annotations of the sub-expressions; in particular, the application requires the sub-expressions of the `annotation-xml` tree in MathML content notation.

It is, in principle, possible to provide annotations for each presentation node by incorporating `semantics` elements recursively.

```
<semantics>
  <mrow>
    <semantics>
      <mrow><mo>(</mo><mi>a</mi> <mo>+</mo> <mi>b</mi><mo>)</mo></mrow>
      <annotation-xml encoding="MathML-Content">
        <apply><plus/><ci>a</ci> <ci>b</ci></apply>
      </annotation-xml>
    </semantics>
    <mo>&InvisibleTimes;</mo>
    <semantics>
      <mrow><mo>(</mo><mi>c</mi> <mo>+</mo> <mi>d</mi><mo>)</mo></mrow>
      <annotation-xml encoding="MathML-Content">
        <apply><plus/><ci>c</ci> <ci>d</ci></apply>
      </annotation-xml>
    </semantics>
  </mrow>

  <annotation-xml encoding="MathML-Content">
    <apply><times/>
      <apply><plus/><ci>a</ci> <ci>b</ci></apply>
      <apply><plus/><ci>c</ci> <ci>d</ci></apply>
    </apply>
  </annotation-xml>
</semantics>
```

To be complete this example would be much more verbose, wrapping each of the individual leaves `mi`, `mo` and `mn` in a further seven `semantics` elements.

This approach is very general and works for all kinds of annotations (including non-MathML annotations and multiple annotations). It leads, however, to $O(n^2)$ increase in size of the document. This is therefore not a suitable approach for fine-grained parallel markup of large objects.

5.3.3 Parallel Markup via Cross-References: `id` and `xref`

To better accomodate applications that must deal with sub-expressions of large objects, MathML uses cross-references between the branches of a `semantics` element to identify corresponding sub-structures.

Cross-referencing is achieved using `id` and `xref` attributes within the branches of a containing `semantics` element. These attributes may optionally be placed on MathML elements of any type.

The following example shows this cross-referencing for the boolean arithmetic expression $(a+b)(c+d)$.

```
<semantics>
  <mrow id="E">
    <mrow id="E.1">
      <mo id="E.1.1">(</mo>
      <mi id="E.1.2">a</mi>
```

```
      <mo id="E.1.3">+</mo>
      <mi id="E.1.4">b</mi>
      <mo id="E.1.5">)</mo>
    </mrow>
    <mo id="E.2">&InvisibleTimes;</mo>
    <mrow id="E.3">
      <mo id="E.3.1">(</mo>
      <mi id="E.3.2">c</mi>
      <mo id="E.3.3">+</mo>
      <mi id="E.3.4">d</mi>
      <mo id="E.3.5">)</mo>
    </mrow>
  </mrow>

  <annotation-xml encoding="MathML-Content">
    <apply xref="E">
      <and xref="E.2"/>
      <apply xref="E.1">
        <xor xref="E.1.3"/><ci xref="E.1.2">a</ci><ci xref="E.1.4">b</ci>
      </apply>
      <apply xref="E.3">
        <xor xref="E.3.3"/><ci xref="E.3.2">c</ci><ci xref="E.3.4">d</ci>
      </apply>
    </apply>
  </annotation-xml>

  <annotation-xml encoding="OpenMath">
    <OMA xref="E">
      <OMS cd="logic" name="and" xref="E"/>
      <OMA xref="E.1">
        <OMS cd="logic" name="xor" xref="E.1.3"/>
        <OMV name="a" xref="E.1.2"/>
        <OMV name="b" xref="E.1.4"/>
      </OMA>
      <OMA xref="E.3">
        <OMS cd="logic" name="xor" xref="E.3.3"/>
        <OMV name="c" xref="E.3.2"/>
        <OMV name="d" xref="E.3.4"/>
      </OMA>
    </OMA>
  </annotation-xml>
</semantics>
```

where `OMA`, `OMS` and `OMV` are elements defined in the OpenMath standard for representing application, symbol and variable, respectively.

An `id` attribute and a corresponding `xref` appearing within the same `semantics` element create a correspondence between sub-expressions.

In creating these correspondences by cross-reference, all of the `id` attributes referenced by any `xref` must be in the same branch of an enclosing `semantics` element. This constraint guarantees that these correspondences do not create unintentional cycles. (Note that this

restriction does not exclude the use of `id` attributes within the other branches of the enclosing `semantics` element. It does, however, exclude references to these other `id` attributes originating in the same `semantics` element.)

There is no restriction on which branch of the `semantics` element may contain the destination `id` attributes. It is up to the application to determine which branch to use.

In general, there will not be a one-to-one correspondence between nodes in parallel branches. For example, a presentation tree may contain elements, such as parentheses, that have no correspondents in the content tree. It is therefore often useful to put the `id` attributes on the branch with the finest-grained node structure. Then all of the other branches will have `xref` attributes to some subset of the `id` attributes.

In absence of other criteria, the first branch of the `semantics` element is a sensible choice to contain the `id` attributes. Applications that add or remove annotations will then not have to re-assign attributes to the `semantics` trees.

In general, the use of `id` and `xref` attributes allows a full correspondence between subexpressions to be given in text that is at most a constant factor larger than the original. The direction of the references should not be taken to imply that sub-expression selection is intended to be permitted only on one child of the `semantics` element. It is equally feasible to select a subtree in any branch and to recover the corresponding subtrees of the other branches.

5.4 Tools, Style Sheets and Macros for Combined Markup

The interaction of presentation and content markup can be greatly enhanced through the use of various tools. While the set of tools and standards for working with XML applications is rapidly evolving at the present, we can already outline some specific techniques.

In general, the interaction of content and presentation is handled via transformation rules on MathML trees. These transformation rules are sometimes called ‘macros’. In principle, these rules can be expressed using any one of a number of mechanisms, including DSSSL, Java programs operating on a DOM, etc. We anticipate, however, that the principal mechanism for these transformations in most applications shall be XSLT.

In this section we discuss transformation rules for two specific purposes: for notational style sheets, and to simplify parallel markup.

5.4.1 Notational Style Sheets

Authors who make use of content markup may be required to deploy their documents in locales with notational conventions different than the default content rendering. It is therefore expected that transformation tools will be used to determine notations for content elements in different settings. Certain elements, e.g. `lambda`, `mean` and `transpose`, have widely varying common notations and will often require notational selection. Some examples of notational variations are given below.

- $\mathbf{V}$ versus $\vec{V}$
- $\tan x$ versus $\mathrm{tg}\, x$
- $\binom{n}{m}$ versus ${}_nC^m$ versus C^n_m versus C^m_n
- $a_0 + \frac{1|}{|a_1} + \ldots + \frac{1|}{|a_k}$ versus $[a_0, a_1, \ldots, a_k]$

Other elements, for example `plus`and `sin`, are less likely to require these features.

We observe that selection of notational style is sometimes necessary for correct understanding of documents by locale. For instance, the binomial coefficient C_m^n in French notation is equivalent to C_n^m in Russian notation.

A natural way for a MathML application to bind a particular notation to the set of content tags is with an XSLT style sheet [XSLT]. The examples of this section shall assume this is the mechanism to express style choices. (Other choices are equally possible, for example an application program may provide menus offering a number of rendering choices for all content tags.)

When writing XSLT style sheets for mathematical notation, some transformation rules can be purely local, while others will require multi-node context to determine the correct output notation. The following example gives a local transformation rule that could be included in a notational style sheet displaying open intervals as]*a*,*b*[rather than as (*a*,*b*).

```
<xsl:template match="interval">
  <mrow>
    <xsl:choose>
      <xsl:when test="@closure='closed'">
        <mfenced open="[" close="]" separators=",">
          <xsl:apply-templates/>
        </mfenced>
      </xsl:when>
      <xsl:when test="@closure='open'">
        <mfenced open="]" close="[" separators=",">
          <xsl:apply-templates/>
        </mfenced>
      </xsl:when>
      <xsl:when test="@closure='open-closed'">
        <mfenced open="]" close="]" separators=",">
          <xsl:apply-templates/>
        </mfenced>
      </xsl:when>
      <xsl:when test="@closure='closed-open'">
        <mfenced open="[" close="[" separators=",">
          <xsl:apply-templates/>
        </mfenced>
      </xsl:when>
      <xsl:otherwise>
        <mfenced open="[" close="]" separators=",">
          <xsl:apply-templates/>
        </mfenced>
      </xsl:otherwise>
    </xsl:choose>
  </mrow>
</xsl:template>
```

An example of a rule requiring context information would be:

```
<xsl:template match="apply[factorial]">
  <mrow>
    <xsl:choose>
```

```
      <xsl:when test="not(*[2]=ci) and not(*[2]=cn)">
        <mrow>
          <mo>(</mo>
          <xsl:apply-templates select="*[2]" />
          <mo>)</mo>
        </mrow>
      </xsl:when>
      <xsl:otherwise>
        <xsl:apply-templates select="*[2]" />
      </xsl:otherwise>
    </xsl:choose>
    <mo>!</mo>
  </mrow>
</xsl:template>
```

Other examples of context-dependent transformations would be, e.g. for the `apply`of a `plus`to render *a*-*b*+*c*, rather than *a*+ -*b*+*c*, or for the `apply`of a `power`to render $\sin^2 x$, rather than sin x^2.

Notational variation will occur both for built-in content elements as well as extensions. Notational style for extensions can be handled as described above, with rules matching the names of any extension tags, and with the content handling (in a content-faithful style sheet) proceeding as described in section 5.4.3.

5.4.2 Content-Faithful Transformations

There may be a temptation to view notational style sheets as a transformation from content markup to equivalent presentation markup. This viewpoint is explicitly discouraged, since information will be lost and content-oriented applications will not function properly.

We define a ‘content-faithful’ transformation to be a transformation that retains the original content in parallel markup (section 5.3).

Tools that support MathML should be ‘content-faithful’, and not gratuitously convert content elements to presentation elements in their processing. Notational style sheets should be content-faithful whenever they may be used in interactive applications.

It is possible to write content-faithful style sheets in a number of ways. Top-level parallel markup can be achieved by incorporating the following rules in an XSLT style sheet:

```
<xsl:template match="/">
   <semantics>
      <xsl:apply-templates/>

      <annotation-xml encoding="MathML-Content">
         <xsl:copy-of select="."/>
      </annotation-xml>
   </semantics>
</xsl:template>

<xsl:template match="*">
  <xsl:copy>
```

```
    <xsl:apply-templates/>
  </xsl:copy>
</xsl:template>
```

The notation would be generated by additional rules for producing presentation from content, such as those in section 5.4.1.

5.4.3 Style Sheets for Extensions

The presentation tags of MathML form a closed vocabulary of notational structures, but are quite rich and can be used to express a rendering of most mathematical notations. Although the basic layout schemata are rich enough, they may be composed to layout notations that are quite complex, or that might not have yet been considered. In this sense, the presentation ability of MathML is open-ended. It is often useful, however, to give a name to a new notational schema if it is going to be used often.

The content tags form a fixed vocabulary of concepts covering the types of mathematics seen in most common applications. It is not reasonable to expect users to compose existing MathML content tags to construct new content concepts. (This approach is frought with technical difficulties even for professional mathematicians.) Instead, it is anticipated that applications whose mathematical content concepts extend beyond what is offered by MathML, will use annotations within `semantics` elements, and that these annotations will use content description languages outside of MathML.

Often the naming of a notation and the identification of a new semantic concept are related. This allows a single transformation rule to capture both a presentation and a content markup for an expression. This is one of the areas of MathML that benefits most strongly from the use of macro processing. With any of the current document transformation standards, for instance XSLT or DSSSL, it is trivial to define rules that take, for example

```
<rank/>
```

and

```
<tr>X</tr>
```

and respectively transform them to

```
<semantics>
   <ci><mo>rank</mo></ci>
   <annotation-xml encoding="OpenMath">
      <OMS cd="linalg1" name="rank"/>
   </annotation-xml>
</semantics>
```

and

```
<apply>
 <transpose/>
 <ci>X</ci>
</apply>
```

The lengthy sample encoding of rank($u^T v$)=1, from section 5.2.1 could then be condensed to

```
<apply>
  <eq/>
  <apply>
    <rank/>
    <apply>
      <times/>
      <tr>u</tr>
      <ci>v</ci>
    </apply>
   </apply>
  <cn>1</cn>
</apply>
```

From this example we see how the combination of presentation and content markup could become much simpler and effective to generate as standard style-sheet libraries become available.

Chapter 6

Entities, Characters and Fonts

6.1 Introduction

6.1.1 The Intent of Entity Names

Notation has proved very important for mathematics. Mathematics has grown in part because of the succinctness and suggestiveness of its evolving notation. There have been many new signs evolved for use in mathematical notation, and mathematicians have not held back from making use of many symbols originally developed elsewhere. The result is that mathematics makes use of a very large collection of symbols. It is difficult to write mathematics fluently if these characters are not available for use in coding. It is difficult to read mathematics if glyphs are not available for presentation on specific display devices.

This situation poses a problem for the W3C Math Working Group. It does not fall naturally within the purview of a mathematics for HTML specification and DTD production to worry about more than the entities allowed in the DTD. Moreover, as experience has shown, a long list of entities with no means to display them is of little use, and a cause of frequent frustrations in trying use a standard. On the other hand, a large collection of glyphs or characters without a standard way to refer to them is not of much use either.

The W3C Math Working Group has therefore taken on directly specification of part of the full mechanism of proceeding from notation to final presentation, and is collaborating with organizations undertaking specification of the rest.

For instance, we try to use entity names that are contained in ISO TR 9573, which supersedes the ISO TR 8879 annex as far as mathematics is concerned. There are considerations of mathematical usage that do on occasion militate against this, and the TR 9573 lists need supplementing. We hope to be able to agree with the TR 9573 WG on suitable extensions, in the course of the revision of their document that they are presently undertaking.

The STIX project of the STIPUB group of scientific and technical publishers has also been working toward a common collection of mathematical symbols and names. The W3C Math Working Group expects to issue further updates on the matter of character entities as a consequence of this project's useful work. For the latest character tables and fonts information, see the W3C Math Working Group home page.

6.1.2 The STIX Project

The STIX project team leader, Nico Poppelier, is a member of the W3C Math Working Group. The STIX project, set up by the STIPUB group of publishers, aims to formulate

a collection of characters needed in the course of scientific and technical publishing. A database of characters in common use is being produced by collaborating publishing organizations. The team will propose to the Unicode consortium the additions to the next revision of the Unicode character set that this process shows are needed, together with the appropriate character codes. Finally the STIX project will commission the production of a complete set of fonts covering those Unicode characters for science and technology, to be made available to the public under license, but free of charge. The STIPUB group recognizes that easy availability of the characters and fonts greatly facilitates communication and publication.

6.1.3 Entity Listings

This chapter of the MathML Specification contains a listing of entities for use in MathML.

To provide more background on the characters used by mathematics we have used a larger comparative database showing codes and meanings in other common math environments. The W3C Math Working Group is very grateful to Elsevier Science and to Wolfram Research (makers of Mathematica ®) for making available to us so much useful data.

6.1.4 Non-Marking Entities

Some character entities, although important for the quality of print rendering do not directly have glyph marks that correspond. They are called here non-marking entities. Below we have a table of those adopted for the purposes of MathML. Their roles are discussed in chapter 3 and chapter 4, respectively. The values of the spaces given are recommendations. Some of these characters do not already have Unicode values. Arbitrary values up in the Private Zone E8 range have been assigned. The correspondence between the spacing values mentioned below and those in the Unicode descriptions are not exact, but are good matches.

6.1.5 Printing Entity Listings

Since the situation concerning availability of character codes from Unicode and under ISO 9573-13 is not yet fully clear at the time of writing, we have decided to proceed conservatively.

We have taken the ISO 9573-13 proposal, as conveyed to us from Anders Berglund, and have added a number of additional aliases based in the practice of the mathematical typesetting community. Thus the main influence outside ISO has been the names to be found in the TEX community.

To facilitate comprehension of a fairly large list of names, which totals over 2000 in this case, we offer the same information in more than one form.

We have entities listed by name and sample glyphs for all of them. Each entity name is accompanied by a code for a character grouping chosen from a list given below, a short verbal description, and a Unicode hex code if there is a corresponding sample glyph to be found in ISO 10646. Those codes beginning with the hex digit E, e.g. E321, indicate assignments to the private zone of Unicode. This indicates that the character in question is not at present an official Unicode character. It is highly recommended that authors use entity names instead of Unicode values, especially for those characters in the Unicode private zone, as those values may change. It is hoped that most of these characters will

Entity name	Unicode	Description
` `	0009	tabulator stop; horizontal tabulation
`
`	000A	force a line break; line feed
`&IndentingNewLine;`	E891	force a line break and indent appropriately on next line
``	E892	never break line here
`&GoodBreak;`	E893	if a linebreak is needed, here is a good spot
`&BadBreak;`	E894	if a linebreak is needed, try to avoid breaking here
`&Space;`	0020	one em of space in the current font
` `	00A0	space that is not a legal breakpoint
``	200B	space of no width at all
` `	200A	space of width 1/18 em
` `	2009	space of width 3/18 em
` `	2005	space of width 4/18 em
` `	E897	space of width 5/18 em
``	E898	space of width -1/18 em
``	E899	space of width -3/18 em
``	E89A	space of width -4/18 em
``	E89B	space of width -5/18 em
`⁣`	E89C	used as a separator, e.g. in indices (section 3.2.4
`⁣`	E89C	short form of `⁣`
`⁢`	E89E	marks multiplication when it is understood without a mark (section 3.2.4
`⁢`	E89E	short form of `⁢`
`⁡`	E8A0	character showing function application in presentation tagging (section 3.2
`⁡`	E8A0	short form of `⁡`

become officially endorsed by Unicode and ISO under its 10646 standard in due course. In any case we expect fonts for these characters to become publicly available as the use of MathML develops. If the entity name is an alias then a reference back to the ISO form is given if there is one, and to a preferred form if not. The ISO or preferred forms have references to their alternates where they exist.

Newly Revised. The entity listings by alphabetical and Unicode order in section 6.1.7 have now been brought more into line with the corresponding ISO character sets, in that if some part of a set is included then the entire set is included. Also, ISOCHEM has been dropped. These changes have also been reflected in the entity declarations in the DTD in appendix A.

The tables of character sets with glyphs given in section 6.1.8 have not been revised from the original tables. In cases where information from section 6.1.7 and section 6.1.8 conflict, the tables in section 6.1.6 and the DTD should be considered normative.

6.1.6 Special Constants

To commence we list separately a few of the special characters which MathML has seen fit to be a little radical in introducing. There are two for special constants and one for calculus. They too must have private Unicode values.

6.1.7 Alphabetical Lists

The first table offered is a very large ASCII listing of printing entity names, ordered alphabetically, with upper-case preceding lower-case as in ASCII order. The Unicode numbers

Entity name	Unicode	Description
ⅅ	F74B	D for use in differentials, e.g. within integrals
ⅅ	F74B	short form of ⅅ
ⅆ	F74C	d for use in differentials, e.g. within integrals
ⅆ	F74C	short form of ⅆ
ⅇ	F74D	e for use for the exponential base of the natural logarithms
ⅇ	F74D	short form of ⅇ
&false;	E8A7	logical constant false
ⅈ	F74E	i for use as a square root of -1
ⅈ	F74E	short form of ⅈ
&NotANumber;	E8AA	used in section 4.3.2.9
&true;	E8AB	logical constant true

beginning with E are arbitrary assignments in the Private Area where there is presently no Unicode character available. When there is no Unicode offered at all it is because the characters listed can be thought of as font variations of common Roman alphabetic characters.

There is also an ASCII listing of printing entities ordered by Unicode number. Next we have collections of the entities in entity sets which are similar to the groupings in the corresponding ISO documents.

6.1.8 ISO Entity Set Groupings

In addition, we list the above material in the groupings used by ISO 9573-13 with an additional grouping of aliases introduced. This table makes explicit the entity groupings and provides links to ASCII listings of the groups and HTML tabular listings which display the glyphs, insofar as they are to be had, as well.

6.1.8.1 ISO Symbol Entity Sets

The symbols for mathematics that ISO have considered are organized, for both historical and mnemonic reasons into groupings with somewhat descriptive names. In the tables below we reproduce the newly proposed versions of these groups and give the corresponding Unicode sample glyphs. For each ISO 9573-13 group we give first an Extended version in ASCII listing which includes aliases, then a similar listing with sample glyphs, then the Basic ISO 9573-13 entity set and its version with included glyphs. The entries are organized alphabetically by entity name.

It should be noted that the sample glyphs given here are in GIF files intended for viewing on a monitor's screen at 72dpi. They are not suitable for printing, and in particular do not constitute a set of fonts covering the symbols of mathematics. In addition, it is important to note that the Unicode numbers assigned in the private zone, beginning with hex digits E2 and above, are arbitrary and only used here to ensure that sample glyphs are available for display. They do not constitute suggested assignments of codes. Such a set of fonts is under development in more than one context. The MathML Working Group is engaged in ensuring that fonts will be readily publicly available.

This first block of entity sets includes mostly non-letter symbols, along with a few letters loaded with mathematical semantics. At the end of the block we have included the table MMALIAS of the aliases introduced by MathML, which mostly come from the TEX community, and MMEXTRA with the additional character entities added by MathML. Note that some of the blocks are place-holders for a possible future expansion of the tables.

Group	Descriptive Name
ISOAMSA	
ISOAMSB	
ISOAMSC	
ISOAMSN	
ISOAMSO	
ISOAMSR	
ISOTECH	
ISOPUB	
ISODIA	
ISONUM	
ISOBOX	
MMLALIAS	
MMLEXTRA	

6.1.8.2 ISO Entity Sets for Mathematics Alphabets

Mathematical literature displays the common use of particular font styles. Characters representing given letters which differ only in the glyph presentation are in principle not different for the purposes of a character registry such as Unicode, which is not supposed to take into account mere font differences. However usage has meant that both ISO and Unicode, like mathematics, recognize them as different entities. Therefore we include lists for Greek, script, open face (also known as double struck or blackboard bold), and fraktur (also known as gothic or German) fonts.

Group	Descriptive Name
ISOGRK3	
ISOMSCR	
ISOMOPF	
ISOMFRK	

6.1.8.3 Other ISO Font Entity Sets

For reference we provide a list of two additional ISO font entity sets which are really normally used for text.

Group	Descriptive Name
ISOCYR1	
ISOCYR2	

Chapter 7

The MathML Interface

To be effective, MathML must work well with a wide variety of renderers, processors, translators and editors. This chapter addresses some of the interface issues involved in generating and rendering MathML. Since MathML exists primarily to encode mathematics in Web documents, perhaps the most important interface issues are related to embedding MathML in HTML.

There are three kinds of interface issues that arise in embedding MathML in HTML. First, MathML must be semantically integrated into HTML. Browsers must recognize MathML markup as embedded XML content, and not as an HTML syntax error. This is primarily a question of managing namespaces in XML.

Second, MathML rendering must be integrated into browser software. Some browsers already implement MathML rendering natively, and one can expect more browsers will do so in the future. At the same time, other browsers have developed infrastructure to facilitate the rendering of MathML and other embedded XML content by embedded elements. While substantial progress has been made, further improvement in coordination between browsers and embedded elements will be necessary. For example, better support for coordinating initialization and size negotiation is needed, as is better support for high-resolution printing.

Third, other tools for generating and processing MathML must be able to intercommunicate. A number of MathML tools have been or are being developed, include editors, translators, computer algebra systems, and other scientific software. However, since MathML expressions tend to be lengthy, and prone to error when entered by hand, special emphasis must be given to insuring that MathML can be easily generated by user-friendly conversion and authoring tools, and that these tools work together in a dependable, platform and vendor independent way.

The W3C Math working group is committed to providing support to software vendors developing all kinds of MathML tools. The working group monitors the public mailing list www-math@w3.org, and will attempt to answer questions about the MathML specification. The working group also intends to stimulate the formation of MathML developer and user groups. For current information about MathML tools, applications and user support activities, consult the home page of the W3C Math Working Group.

7.1 Embedding MathML in HTML

MathML specifies a single top-level `math` element, which encapsulates each instance of MathML markup within an HTML page. As such, the `math` element provides an attachment point for information that affects a MathML expression as a whole.

In practice, the `math` element also serves as the interface for embedding MathML in HTML. In this capacity, the `math` element simultaneously signals the semantic inclusion of MathML (XML) content in HTML, and provides the necessary machinery for rendering its content in a browser either by invoking an embedded element, or by specifying parameters for a native renderer in the browser. Both semantic inclusion and rendering present a number of issues that extend beyond the scope of this specification.

In order to produce a complete and self-contained description of MathML, this document only specifies the attributes and usage of the `math` element as a top-level element for MathML, and not as an interface element. The W3C Math working group will continue working closely with other W3C activities to insure that emerging standards for embedding XML in HTML accommodate seamless integration of MathML in HTML. In section 7.1.2 we list requirements an interface element for MathML would have to meet in order to fully integrate MathML into HTML. However, it is important to note that the MathML specification is independent of embedding mechanisms.

7.1.1 The Top-Level `math` Element

As stated above, MathML specifies a single top-level `math` element. All other MathML content must be contained in a `math` element; equivalently, every valid, complete MathML expression must be contained in `<math>` tags. The `math` element must always be the outermost element in a MathML expression; it is an error for one `math` element to contain another.

Applications that return sub-expressions of other MathML expressions, for example as the result of a cut-and-paste operation, should always wrap them in `<math>` tags. The presence of enclosing `<math>` tags should be a reasonable heuristic test for MathML content. Similarly, applications which insert MathML expressions in other MathML expressions must take care to remove the `<math>` tags from the inner expressions.

The `math` element can contain an arbitrary number of children schemata. The children schemata render by default as if they were contained in a `mrow` element.

The attributes of the `math` element are:

class, id, style Provided for style sheet compatibility.

macros This attribute provides a way of pointing to external macro definition files. Macros are not part of the MathML specification, and it is anticipated that in the future, many uses of macros in MathML can be accommodated by XSL transformations [XSLT]. However, the `macros` attribute is provided to make possible future development of more streamlined, MathML-specific macro mechanisms. The value of this attribute is a sequence of URLs or URIs, separated by whitespace

mode (deprecated) The `mode` attribute specifies whether the enclosed MathML expression should be rendered in a display style or an in-line style. Allowed values are `display` and `inline` (default). This attribute is deprecated in favor of the standard CSS2 'display' property with the analogous `block` and `inline` values.

7.1.2 Requirements for a MathML Browser Interface

The top-level `math` element described in the preceding section is concerned with encapsulating MathML content and defining attributes that affect the entire enclosed expression. It is, in a sense, 'inward looking'. However, to render MathML properly in a browser, and to integrate it properly into an HTML document, an 'outward looking' interface element is also required. This interface element must be aware of its surrounding environment, and provide a mechanism for passing information between the browser and the MathML renderer.

As noted above, the MathML interface element and the MathML top-level element are in practice one and the same. The `math` element must serve both to encapsulate MathML content, and admit additional attributes for controlling how a MathML renderer should interact with the surrounding context, typically a browser.

While general mechanisms for embedding XML in HTML are beginning to be deployed, wide variations in strategy and level of implementation remain between vendors. Consequently, the remainder of this section describes attributes and functionality that would be highly desirable in a MathML interface element. In the near term, implementors attempting to provide interim solutions for rendering MathML in browsers should try to give authors some way of passing the following interface attributes to the renderer:

type The type attribute assigns a MIME type to the tag content. This attribute should ideally be used to select an embedded element to invoke, such as a Java applet, plug-in or ActiveX control, to render the tag content as described in the next section.

name Provided for scripting.

height, width, baseline Ideally, embedded elements should be able to dynamically negotiate height, width and baseline alignment with browsers. However, these optional attributes, which have a length as value, are suggested as an interim solution for software vendors that want to support MathML, but are unable to provide dynamic resizing and alignment.

overflow In cases where size negotiation is not possible or fails (for example in the case of an extremely long equation), this attribute is provided to suggest an alternative processing method to the renderer. Allowed values are

- **scroll** The window provides a viewport into the larger complete display of the mathematical expression. Horizontal or vertical scrollbars are added to the window as necessary to allow the viewport to be moved to a different position.
- **elide** The display is abbreviated by removing enough of it so that the remainder fits into the window. For example, a large polynomial might have the first and last terms displayed with '+ ... +' between them. Advanced renderers may provide a facility to zoom in on elided areas.
- **truncate** The display is abbreviated by simply truncating it at the right and bottom borders. It is recommended that some indication of truncation is made to the viewer.
- **scale** The fonts used to display the mathematical expression are chosen so that the full expression fits in the window. Note that this only happens if the expression is too large. In the case of a window larger than necessary, the expression is shown at its normal size within the larger window.

altimg, alttext These attributes provide graceful fall-backs for browsers that do not support embedded elements, or images respectively. The value of the first attribute is

an URL, and the value of the second one is a text string.

Attributes that apply to the MathML interface element necessarily take effect when the document is first loaded, and therefore suffer the limitation that they cannot change in response to reader interaction unless they are exposed in the Document Object Model (http://www.w3.org/TR/WD-DOM-Level-2) and subject to programmatic control. The `height` and `width`attributes are good examples; if the reader changes the current font size, the height and width of the embedded mathematical fragments also need to change.

At present, browser support for the DOM, and embedded element access to the DOM, is too limited to provide acceptable rendering for MathML. The W3C Math working group is working closely with the W3C DOM working group in an effort to provide better communication between embedded MathML renderers and browsers (see appendix E).

The basic requirements for communication between an embedded MathML renderer and a browser include:

- Embedded elements must be able to determine the ambient style parameters, including font characteristics, foreground and background colors, and link color schemes. Embedded elements must also be able to align themselves to an arbitrary baseline.
- Embedded elements must be able to detect and react to reader input. In particular, embedded elements must be able to dynamically resize themselves when the ambient font size changes.
- Embedded elements must be able to print in context, and at high resolution.

7.1.3 Invoking Embedded Objects as Renderers

In browsers where MathML is not natively supported, we anticipate that MathML rendering will be carried out via embedded objects such as plug-ins, applets, or helper applications. In the near term, the W3C Math working group advocates the use of MIME types to bind embedded MathML to renderers. Mechanisms for assigning MIME types already exist in HTML, and mechanisms for registering and automatically invoking embedded elements such as plug-ins based on MIME type already exist in Web browsers.

The `type`attribute, described in the previous section as a requirement for the MathML interface element, is intended to associate a MIME type with its content. The HTML element `META`is proposed as a means of specifying document-wide default MIME types for an element.

We propose a simple naming convention for MIME types that is flexible enough to accommodate several common situations:

- An author wishing to reach an audience as wide as possible might want MathML to be rendered by any available renderer.
- An author targeting a specific audience might want to indicate that a particular MathML renderer be used.
- A reader might wish to specify which of several available renderers should be used.

We propose that generic MathML be assigned the MIME type `text/mathml`and for browser registry, we suggest the standard file extension `.mml`be used. To invoke specific renderers, we suggest assigning a MIME type of the following format:

```
text/mathml-renderer
```

7.1.3.1 Example

A user downloads and installs renderer A, and registers it with the browser for the `text/mathml` MIME type to process generic MathML. However renderer A also accepts TEX as an input syntax, and therefore during the installation process, it requests to be registered for `application/x-tex` as well. Later, the user discovers renderer B provides additional features, such as the capability to cut and paste. Therefore, the user downloads, installs and registers renderer B for the MIME type `text/mathml-rendererB`.

An author then creates a document that contains the the following line in the document header:

```
<META Content-math-Type="text/mathml">
```

Later, the document contains the following expressions:

```
<math>
  <msup><mi>x</mi><mn>2</mn></msup>
</math>
<math type="text/mathml-rendererB">
  <mi>&alpha;</mi><mo>=</mo><mn>0.4</mn>
</math>
```

When our hypothetical reader views this document, renderer A is invoked to process the first expression, while renderer B is invoked for the second. Later, when our hypothetical reader later views a document with MIME type `application/x-tex`, renderer A is again invoke, this time in TEX processing mode.

7.1.4 Invoking Other Applications

Although rendering MathML expressions typically occurs in place in a Web browser, other MathML processing functions take place more naturally in other applications. Particularly common tasks include opening a MathML expression in an equation editor or computer algebra system.

At present, there is no standard way of specifying that embedded content should be rendered with one application, edited in another, and evaluated by a third. As work progresses on coordination between browsers and embedded elements and the Document Object Model (DOM), providing this kind of functionality should be a priority. Both authors and readers should be able to indicate a preference about what MathML application to use in a given context. For example, one might imagine that some mouse gesture over a MathML expression causes a browser to present the reader with a pop-up menu, showing the various kinds of MathML processing available on the system, and the MathML processors recommended by the author.

Since MathML will probably be widely generated by authoring tools, it is particularly important that opening a MathML expression in an editor should be easy to do and to implement. In many cases, it will be desirable for an authoring tool to record some information about its internal state along with a MathML expression, so that an author can pick up editing where he or she left off. The MathML specification does not explicitly contain provisions for recording information about the authoring tool. In some circumstances, it may be possible to include authoring tool information that applies to an entire document in the form of meta-data; interested readers are encouraged to consult the W3C Metadata Activity for current information about metadata and resource definition. For encoding authoring tool state information that applies to a particular MathML instance, readers are referred to the possible use of the `semantics` element for this purpose.

7.1.5 Mixing and Linking MathML and HTML

In order to be fully integrated into HTML, it should be possible not only to embed MathML in HTML, but also to embed HTML in MathML. However, the problem of supporting HTML in MathML presents many difficulties. Moreover, the problems are not specific to MathML; they are problems for XML applications in HTML generally. Therefore, at present, the MathML specification does not permit any HTML elements within a MathML expression, although this may be subject to change in a future revision of MathML, when mechanisms for embedding XML in HTML have been further developed.

In most cases, HTML elements either do not apply in mathematical contexts (headings, paragraphs, lists, etcetera), or MathML already provides equivalent or better functionality specifically tailored to mathematical content (tables, style changes, etcetera). However, there are two notable exceptions.

7.1.5.1 Linking

MathML has no element that corresponds to the HTML anchor element *a*. In HTML, anchors are used both to make links, and to provide locations to link to. MathML, as an XML application, defines links by the use of the XLink mechanism [XLink]. However, MathML at present does not provide a way for other documents to make links into a MathML expression. One reason for this omission is that linking into embedded XML content is better addressed as part of a general mechanism for embedding XML in HTML. Moreover, until browsers either natively implement MathML rendering, or substantially better coordination between embedded elements and browsers becomes possible, there is no reasonable way of implementing links into MathML expressions.

MathML linking elements are generic XML linking elements as described in the [XLink]. The reader is cautioned that this is as present still a working draft, and is therefore subject to future revision. Since the MathML linking mechanism is defined in terms of the XML linking specification, the same proviso holds for it as well.

A MathML element is designated as a link by the presence of the attribute `xlink:href` To use the attribute `xlink:href` it is also necessary to declare the appropriate namespace. Thus, a typical MathML link might look like:

```
<mrow xmlns:xlink="http://www.w3.org/XML/XLink/0.9"
     xlink:href="sample.xml">
  ...
</mrow>
```

MathML designates that almost all elements can be used as an XML linking element. The only elements that cannot serve as linking elements are those such as the `sep` element, which exist primarily to disambiguate other MathML constructs and in general do not correspond to any part of a typical visual rendering. The full list of exceptional elements that cannot be used as linking elements is given in the table below.

Table 7.1: MathML elements that cannot be linking elements.

`mprescripts`	`none`	`sep`
`malignmark`	`maligngroup`	

7.1.5.2 Images

The `IMG`element has no MathML equivalent. The decision to omit a general mechanism for image inclusion from MathML was based on several factors. First, a simple mechanism for including images in MathML along the lines of the `IMG`element would not be more closely tied to mathematical content or notation than the HTML `IMG`element itself. Therefore, such an element would likely be superseded by the `IMG`element if it becomes possible to mix XML and HTML generally.

Another reason for not providing an image facility is that MathML takes great pains to make the notational structure and mathematical content it encodes easily available to processors, whereas information contained in images is only available to a human reader looking at a visual representation. Thus, for example, in the MathML paradigm, it would be preferable to introduce new glyphs by the creation of special symbol fonts, rather than simply including them as images.

Finally, apart from the introduction of new glyphs, many of the situations where one might be inclined to use an image amount to some sort of labeled diagram. For example, knot diagrams, Venn diagrams, Dynkin diagrams, Feynman diagrams and complicated commutative diagrams all fall into this category. As such, their content would be better encoded via some combination of structured graphics and MathML markup. Because of the generality of the 'labeled diagram' construction, the definition of a markup language to encode such constructions extends beyond the scope of the current W3C Math activity. (See http://www.w3.org/Graphics for further W3C activity in this area.)

7.2 Generating, Processing and Rendering MathML

Information is increasingly generated, processed and rendered by software tools. The exponential growth of the Web is fueling the development of advanced systems for automatically searching, categorizing, and interconnecting information. Thus, although MathML can be written by hand and read by humans, the future of MathML is also tied to the ability to process it with software tools.

There are many different kinds of MathML editors, translators, processors and renderers. What it means to support MathML varies widely between applications. For example, the issues that arise with a MathML-compliant validating parser are very different from those for a MathML-compliant equation editor.

In this section, guidelines are given for describing different types of MathML support, and for quantifying the extent of MathML support in a given application. Developers, users and reviewers are encouraged to use these guidelines in characterizing products. The intention behind these guidelines is to facilitate reuse and interoperability between MathML applications by accurately characterizing their capabilities in quantifiable terms.

7.2.1 MathML Compliance

A valid MathML expression is an XML construct determined by the MathML DTD together with the additional requirements given in the specifications of the MathML document.

We define a 'MathML processor' to mean any application that can accept, produce, or 'roundtrip' a valid MathML expression. An example of an application that might round-trip

a MathML expression might be an editor that writes a new file even though no modifications are made.

We specify three forms of MathML compliance:

1. A MathML-input-compliant processor must accept all valid MathML expressions, and faithfully translate all MathML expressions into application-specific form allowing native application operations to be performed.
2. A MathML-output-compliant processor must generate valid MathML, faithfully representing all application-specific data.
3. A MathML-roundtrip-compliant processor must preserve MathML equivalence. Two MathML expressions are 'equivalent' if and only if both expressions have the same interpretation (as stated by the MathML DTD and specification) under any circumstances, by any MathML processor. Equivalence on an element-by-element basis is discussed elsewhere in this document.

Beyond the above definitions, the MathML specification makes no demands of individual processors. In order to guide developers, the MathML specification includes advisory material; for example, there are suggested rendering rules included in chapter 3. However, in general, developers are given wide latitude in interpreting what kind of MathML implementation is meaningful for their own particular application.

To clarify the difference between compliance and interpretation of what is meaningful, consider some examples:

1. In order to be MathML-input-compliant, a validating parser needs only to accept expressions, and return 'true' for expressions that are valid MathML. In particular, it need not render or interpret the MathML expressions at all.
2. A MathML computer-algebra interface based on content markup might choose to ignore all presentation markup. Provided the interface accepts all valid MathML expressions included those containing presentation markup, it would be technically correct to characterize the application as MathML-input-compliant.
3. A equation editor might have an internal data representation that makes it easy to export some equations as MathML but not others. If the editor exports the simple equations, and merely displays an error message to the effect that conversion failed for the others, it is still technically MathML-output-compliant.

As the previous examples show, to be useful, the concept of MathML compliance frequently involves a judgment about what parts of the language are meaningfully implemented, as opposed to parts that are merely processed in a technically correct way with respect to the definitions of compliance. This requires some mechanism for giving a quantitative statement about which parts of MathML are meaningfully implemented by a given application. To this end, the W3C Math working group has provided a test suite of MathML expressions at http://www.w3.org/Math/testsuite.

The test suite consists of a large number of MathML expressions categorized by markup category and dominant MathML element being tested. The existence of this test suite makes is possible, for example, to characterize quantitatively the hypothetical computer algebra interface mentioned above by saying that it is a MathML-input compliant processor which meaningfully implements MathML content markup, including all of the expressions given under http://www.w3.org/Math/testsuite/tests/4.

Developers who choose not to implement parts of the MathML specification in a meaningful way are encouraged to itemize the parts they leave out by referring to specific categories in the test suite.

For MathML-output-compliant processors, there is also a MathML validator online at http://www.w3.org/Math/validator. Developers of MathML-output-compliant processors are encouraged to verify their output using this validator.

Customers of MathML applications who wish to verify claims as to which parts of the MathML specification are implemented by an application are encouraged to use the test suites as a part of their decision processes.

7.2.2 Handling of Errors

If a MathML-input-compliant application receives input containing one or more elements with an illegal number or type of attributes or child schemata, it should nonetheless attempt to render all the input in an intelligible way, i.e. to render normally those parts of the input that were valid, and to render error messages (rendered as if enclosed in an `merror` element) in place of invalid expressions.

MathML-output-compliant applications such as editors and translators may choose to generate `merror` expressions to signal errors in their input. This is usually preferable to generating valid, but possibly erroneous, MathML.

7.2.3 Attribute for unspecified data

The MathML attributes described in the MathML specification are necessary for presentation and content markup. Ideally, the MathML attributes should be an open-ended list so that users can add specific attributes for specific renderers. However, this cannot be done within the confines of a single XML DTD. Although it can be done using extensions of the standard DTD, some authors will wish to use non-standard attributes while remaining strictly in compliance with the standard DTD.

To allow this, this specification also allows the attribute `other` for all elements, for use as a hook to pass on renderer-specific information. In particular, it can be used as a hook for passing information to audio renderers, computer algebra systems, and for pattern matching in any future macro/extension mechanism. This idea is used in other languages. For example, PostScript comments are widely used to pass information that is not part of PostScript.

At the same time, the intent of the `other` attribute is not to encourage software developers to use this as a loop-hole for circumventing the core conventions for MathML markup. We trust both authors and applications will use the `other` attribute judiciously.

The value of the `other` attribute should be a string containing an attribute list in valid XML format, e.g.

```
attr1="val1" attr2="val2"
```

or

```
attr1='val1' attr2='val2'
```

with appropriate escaping of quotes appearing inside the attribute values). Renderers that accept non-standard attributes directly should also accept them when they occur within the string value of the `other` attribute. This is not required for attributes specifically documented by the MathML standard.

7.3 Future Extensions

MathML is in its infancy; it is to be expected that MathML will need to be extended and revised in various ways. Some of these extensions can be easily foreseen; as noted repeatedly in this chapter, the mechanisms for fully integrating MathML into HTML are not yet developed, and these mechanisms may have a significant impact on some aspects of MathML.

Similarly, there are several kinds of functionality that are fairly obvious candidates for future MathML extensions. These include macros, style sheets, and perhaps a general facility for 'labeled diagrams'. However, there will no doubt be other desirable extensions to MathML that will only emerge as MathML is widely used. For these extensions, the W3C Math working group relies on the extensible architecture of XML, and the common sense of the larger Web community.

7.3.1 Macros and Style Sheets

The development of style-sheet mechanisms for XML is part of the ongoing XML activity of the World Wide Web Consortium. Both XSL and CSS are working to incorporate greater support for mathematics. Further, XSL can be used to provide a basic macro capability as well.

Macros, however, play a very important and useful role in encoding mathematical content and meaning. Moreover, it is difficult to devise a coherent, general macro system for MathML, because there are so many distinct applications for MathML macros. Therefore, a good direction for further work is the definition of a macro mechanism specifically tailored to MathML, in addition to participating in general ongoing activities in the areas of XML style sheets and macro facilities.

Some of the possible uses of MathML macros include:

Abbreviation One common use of macros is for abbreviation. Authors needing to repeat some complicated but constant notation can define a macro. This greatly facilitates hand authoring. Macros that allow for substitution of parameters facilitate such usage even further.

Extension of Content Markup By defining macros for semantic objects, for example a binomial coefficient, or a Bessel function, one can in effect extend the content markup for MathML. Such a macro could include an explicit semantic binding, or such a binding could be easily added by an external applications. Narrowly defined disciplines should be able to easily introduce standardize content markup by using standard macro packages. For example, the OpenMath project could release macro packages for attaching OpenMath content markup up.

Rendering and Style Control Another basic way in which macros are often used is to provide a way of controlling style and rendering behavior by replacing high-level macro definitions. This is especially important for controlling the rendering behavior of MathML content tags in a context sensitive way. Such a macro capability is also necessary to provide a way of attaching renderings to user-defined XML extensions to the MathML core.

Accessibility Reader-controlled style sheets are important in providing accessibility to MathML. For example, a reader listening to a voice renderer might by default hear a bit of MathML presentation markup read as 'D sub x sup 2 of f'. Knowing the context to be multi-variable calculus, the reader may wish to use a style sheet

or macro package that instructs the renderer to render this <msubsup> element as ‘second derivative with respect to x of f’.

7.3.2 XML Extensions to MathML

The set of elements and attributes specified in the MathML specification are necessary for rendering common mathematical expressions. It is recognized that not all mathematical notation is covered by this set of elements, that new notations are continually invented, and that sub-communities within mathematics often have specialized notations; and furthermore that the explicit extension of a standard is a necessarily slow and conservative process. This implies that the MathML standard could never explicitly cover all the presentational forms used by every sub-community of authors and readers of mathematics, much less encode all mathematical content.

In order to facilitate the use of MathML by the widest possible audience, and to enable its smooth evolution to encompass more notational forms and more mathematical content (perhaps eventually covered by explicit extensions to the standard), the set of tags and attributes is open-ended, in the sense described in this section.

MathML is described by an XML DTD, which necessarily limits the elements and attributes to those occurring in the DTD. Renderers desiring to accept non-standard elements or attributes, and authors desiring to include these in documents, should accept or produce documents that conform to an appropriately extended XML DTD that has the standard MathML DTD as a subset.

MathML-compliant renderers are allowed, but not required, to accept non-standard elements and attributes, and to render them in any way. If a renderer does not accept some or all non-standard tags, it is encouraged either to handle them as errors as described above for elements with the wrong number of arguments, or to render their arguments as if they were arguments to an `mrow`, in either case rendering all standard parts of the input in the normal way.

Chapter 8

Document Object Model for MathML

Issue (questions): Certain issues previously identified have now been given tentative solutions. Again, we would solicit any input regarding these solutions - particularly any strong disagreement!

1. Is a *MathMLRowElement* desirable? The major intent of this hypothetical element is now answered by the MathMLDocumentFragment interface. This interface is essentially the DocumentFragment interface, with the convenience of assuming all child nodes are MathMLElements.
2. Is the *MathMLElement::getMathElement()* method sufficiently useful to justify its existence? The answer here appears to be No! Also, the MathMLElement::insertMathElement() method is felt to be inappropriate. These potential methods have been omitted, not so much in favor of the base Node class's get and insert methods, but in favor of more specific methods which have been added throughout the MathML DOM interfaces. The sense is that getting and setting child elements of MathML elements by index can be misleading, depending on the type of element being represented. It is still possible to obtain the child Nodes of a MathMLElement as a NodeList, and to operate on those in the way indicated by getMathElement() and insertMathElement(). To provide more specific methods on MathMLElement would imply a preference for these methods, whereas in fact use of such a general solution should probably be avoided.
3. Some potential layers of object hierarchy have not been stipulated here, in view of the limited scope of the Level 1 DOM. Particularly glaring is the absence of a *MathMLPresentationElement* / *MathMLContentElement* dichotomy. On further reflection, we feel that it is wise to include these interfaces now to emphasize the heirarchy of objects, and the differing behaviors they may have.
4. The *MathMLMultiScriptsElement* interface. This has been by now blessed by a total absence of controversy or comment.

8.1 Introduction

This document extends the Core API of the DOM Level 1 to describe objects and methods specific to MathML elements in documents. The functionality needed to manipulate hierarchical document structures, elements, and attributes will be found in the core document;

functionality that depends on the specific elements defined in MathML will be found in this document.

The goals of the MathML-specific DOM API are:

- To specialize and add functionality that relates specifically to MathML elements.
- To provide convenience mechanisms, where appropriate, for common and frequent operations on MathML elements.

This document includes the following specializations for MathML:

- A `MathMLElement` interface derived from the core interface `Element`. MathMLElement specifies the operations and queries that can be made on any MathML element. Methods on `MathMLElement` include those for the retrieval and modification of attributes that apply to all MathML elements.
- Specializations for all MathML elements that have attributes that extend beyond those specified in the `MathMLElement` interface. For all such attributes, the derived interface for the element contains explicit methods for setting and getting the values.
- Special methods for insertion and retrieval of children of MathML elements. While the basic methods available from the `Node` and `Element` interfaces must clearly remain available, it is felt that in many cases they may be misleading. Thus, for instance, the `MathMLFractionElement` interface provides for access to `numerator` and `denominator` attributes; a call to `getDenominator()` is less ambiguous from a calling application's perspective than a call to `Node::replaceNode(newNode, Node::childNodes().item(2))`

MathML specifies rules that are invisible to generic XML processors and validators. The fact that MathML DOM objects are required to respect these rules, and to throw exceptions when those rules are violated, is an important reason for providing a MathML-specific DOM extension.

There are basically two kinds of additional MathML grammar and syntax rules. One kind involves placing additional criteria on attribute values. For example, it is not possible in pure XML to require that an attribute value be a positive integer. The second kind of rule specifies more detailed restrictions on the child elements (for example on ordering) than are given in the DTD. For example, it is not possible in XML to specify that the first child be interpreted one way, and the second in another. The MathML DOM objects are required to provide this interpretation.

MathML ignores whitespace occurring outside token elements. Non-whitespace characters are not allowed there. Whitespace occurring within the content of token elements is 'trimmed' from the ends (i.e. all whitespace at the beginning and end of the content is removed), and 'collapsed' internally (i.e. each sequence of 1 or more whitespace characters is replaced with one blank character). The MathML DOM elements perform this whitespace trimming as necessary. In MathML, as in XML, 'whitespace' means blanks, tabs, newlines, or carriage returns, i.e. characters with hexadecimal Unicode codes U+0020, U+0009, U+000a, or U+000d, respectively.

8.1.1 MathML DOM Extensions

It is expected that a future version of the MathML DOM may deal with issues which are not resolved here. Some of these are described here.

8.1.1.1 Style Issues

Issue (level-scopes): The interfaces described to represent MathML elements include access to a number of attributes (in the sense of XML) belonging to those elements. The intent of these methods in the core MathML interfaces (the ‘get’/ ‘set’ pairs) is only to access explicitly specified attributes of the elements, and specifically not to access implicit values which may be application-specific. Calls to these interfaces to get attributes that have not been explicitly specified should return nothing (an empty `DOMString`). It seems important to belabor this distinction in light of the nature of the MathML elements and their attributes; all of the attributes defined for MathML presentation elements are declared in the DTD with a default value of `#IMPLIED`, for instance. This is particularly relevant for the interface of the `mo` element, where the `form` attribute may be inferred from context if not given explicitly, but other attributes are normally collected from an operator dictionary available to a renderer. The variety of applications which may need to implement the MathML DOM may sometimes be concerned with validation, computation or other aspects of the document to the exclusion of rendering or editing; such applications do not need to resolve `#IMPLIED` attributes, and thus there is no access to such resolution implied in this version of the MathML DOM. Methods for obtaining the current actual values of certain style attributes are considered desirable due to the need to make frequent calls to discover style information and the current script level and display style. Mathematics is characterized by recursive nesting of objects, frequently with implications for the calculation of style parameters such as font size. As anyone who’s implemented math rendering knows, there’s a constant need for this information, and it must be obtained very quickly. However, we feel that introducing methods now for dealing with these issues would be premature. CSS and XSL support for mathematics is still evolving, and the mechanisms for handling style issues in MathML documents may well evolve with them. Additionally, these issues also apply to the core XML DOM. Thus far (XML DOM level 2), issues such as privacy with regard to user-side style sheets have resulted in no core DOM methods being defined for obtaining the cascaded, computed or actual style values for a specific element, with DOM access being limited to providing the style declarations which are in effect. If a future iteration of the XML DOM were to expand this access, the methods used there would apply to the MathML DOM as well, and render any specifications we might make now obsolete.

Appendix A

Parsing MathML

MathML documents should be validated using the XML DTD below. Note in particular that the standard XML attribute `xml:space` is not used, so whitespace characters in element content (that is, outside the presentation token elements `mi`, `mo`, `mn`, `mtext`, `mspace`, `mtext`, `ms`, the content token elements `ci`, `cn` and `annotation`) are not significant.

If a MathML fragment is parsed without a DTD, in other words as a well-formed XML fragment, it is the responsibility of the processing application to treat these whitespace characters as not significant.

An SGML parser (such as `nsgmls`) can be used to validate MathML. In this case an SGML declaration defining the constraints of XML applicable to an SGML parser must be used. See the note on SGML and XML.

A.1 The MathML DTD

Here we give the main body of the DTD. The full DTD is available as a zip archive.

The entity declarations for characters are referenced at the end of the DTD. These are linked to the character tables for each entity set.

A list of the combined MathML set of character names, ordered by name or by Unicode value are also available.

Appendix B

Content Markup Validation Grammar

```
Informal EBNF grammar for Content Markup structure validation
=============================================================
// Notes
//
// This defines the valid expression trees in content markup
//
// ** it does not define attribute validation -
// ** this has to be done on top
//
// Presentation_tags is a placeholder for a valid
// presentation element start tag or end tag
//
// #PCDATA is the XML parsed character data
//
// symbols beginning with '_' for example _mmlarg are internal symbols
// (recursive grammar usually required for recognition)
//
// all-lowercase symbols for example 'ci' are terminal symbols
// representing MathML content elements
//
// symbols beginning with Uppercase are terminals
// representating other tokens
//
// revised sb 3.nov.97, 16.nov.97 and 22.dec.1997
// revised sb 6.jan.98, 6.Feb.1998 and 4.april.1998
// whitespace  definitions  including presentation_tags
Presentation_tags ::= "presentation"       //placeholder
Space    ::= #x09 | #xoA | #xoD | #x20     //tab, lf, cr, space characters
S       ::= (Space | Presentation_tags)*   //treat presentation as space
// only for content validation
// characters
Char     ::= Space | [#x21 - #xFFFD]
           | [#x00010000 - #x7FFFFFFF]   //valid XML chars
// start and end tag functions
// start(\%x) returns a valid start tag for the element \%x
```

```
// end(\%x) returns a valid end tag for the element \%x
// empty(\%x) returns a valid empty tag for the element \%x
//
// start(ci)   ::= "<ci>"
// end(cn)            ::= "</cn>"
// empty(plus)  ::= "<plus/>"
//
// The reason for doing this is to avoid writing a grammar
// for all the attributes. The model below is not complete
// for all possible attribute values.
_start(\%x)     ::= "<\%x" (Char - '>')* ">"
// returns a valid start tag for the element \%x
_end(\%x)       ::= "<\%x" Space* ">"
// returns a valid end tag for the element \%x
_empty(\%x)     ::= "<\%x" (Char - '>')* "/>"
// returns a valid empty tag for the element \%x
_sg(\%x)        ::= S _start(\%x)
// start tag preceded by optional whitespace
_eg(\%x)        ::= _end(\%x) S
// end tag followed by optional whitespace
_ey(\%x)        ::= S _empty(\%x) S
// empty tag preceded and followed by optional whitespace
// mathml content constructs
// allow declare within generic argument type so we can insert it anywhere
_mmlall        ::= _container | _relation | _operator | _qualifier | _other
_mmlarg        ::= declare* _container declare*
_container     ::= _token | _special | _constructor
_token         ::= ci | cn | csymbol
_special       ::= apply | lambda | reln | fn
_constructor    ::= interval | list | matrix | matrixrow | set | vector
_other         ::= condition |  declare | sep
_qualifier     ::= lowlimit | uplimit | bvar | degree | logbase
// relations
_relation      ::= _genrel | _setrel | _seqrel2ary
_genrel        ::= _genrel2ary | _genrelnary
_genrel2ary    ::= ne
_genrelnary    ::= eq | leq | lt | geq | gt
_setrel        ::=  _seqrel2ary | _setrelnary
_setrel2ary    ::=  in | notin | notsubset | notprsubset
_setrelnary    ::= subset | prsubset
_seqrel2ary    ::= tendsto
//operators
_operator      ::= _funcop | _sepop | _arithop | _calcop
              | _seqop | _trigop | _statop | _lalgop
              | _logicop | _setop
_funcop        ::= _funcop1ary | _funcopnary
_funcop1ary    ::= inverse    | ident
_funcopnary    ::= fn| compose   // general user-defined function is n-ary
// arithmetic operators
// (note minus is both 1ary and 2ary)
```

```
_arithop       ::= _arithop1ary | _arithop2ary | _arithopnary  | root
_arithop1ary    ::= abs | conjugate | exp | factorial | minus
_arithop2ary    ::= quotient | divide | minus | power | rem
_arithopnary    ::= plus | times | max | min | gcd
// calculus
_calcop        ::= _calcop1ary | log | int | diff | partialdiff
_calcop1ary     ::= ln
// sequences and series
_seqop         ::= sum | product | limit
// trigonometry
_trigop        ::= sin | cos | tan | sec | csc | cot | sinh
                 | cosh | tanh | sech | csch | coth
                 | arcsin | arccos | arctan
// statistics operators
_statop        ::= _statopnary | moment
_statopnary     ::= mean | sdev | variance | median | mode
// linear algebra operators
_lalgop        ::= _lalgop1ary | _lalgopnary
_lalgop1ary     ::= determinant | transpose
_lalgopnary     ::= selector
// logical operators
_logicop       ::= _logicop1ary | _logicopnary | _logicop2ary | _logicopquant
_logicop1ary    ::= not
_logicop2ary    ::= implies
_logicopnary    ::= and | or | xor
_logicopquant   ::= forall | exists
// set theoretic operators
_setop         ::= _setop2ary | _setopnary
_setop2ary      ::= setdiff
_setopnary      ::= union | intersect
// operator groups
_unaryop       ::=  _func1ary | _arithop1ary | _trigop | _lalgop1ary
                  | _calcop1ary | _logicop1ary
_binaryop      ::=  _arithop2ary | _setop2ary | _logicop2ary
_naryop        ::=  _arithopnary | _statopnary  | _logicopnary
                  | _lalgopnary | _setopnary | _funcopnary
_ispop         ::= int | sum | product
_diffop        ::= diff | partialdiff
_binaryrel      ::= _genrel2ary | _setrel2ary |  _seqrel2ary
_naryrel       ::= _genrelnary | _setrelnary
//separator
sep           ::=  _ey(sep)
// leaf tokens  and data content of leaf elements
// note _mdata includes Presentation constructs here.
_mdatai        ::= (#PCDATA | Presentation_tags)*
_mdatan        ::= (#PCDATA | sep | Presentation_tags)*
ci            ::=  _sg(ci) _mdatai _eg(ci)
cn            ::=  _sg(cn) _mdatan _eg(cn)
// condition -  constraints constraints. contains either
// a single reln (relation), or
```

```
// an apply holding a logical combination of relations, or
// a set (over which the operator should be applied)
condition      ::= _sg(condition)  reln | apply | set  _eg(condition)
// domains for integral, sum , product
_ispdomain     ::= (lowlimit  uplimit?)
              | uplimit
              | interval
              | condition
// apply construct
apply         ::= _sg(apply) _applybody _eg(apply)
_applybody     ::= ( _unaryop _mmlarg  )
//1-ary ops
                           | (_binaryop _mmlarg _mmlarg)
//2-ary ops
                           | (_naryop _mmlarg*)
//n-ary ops, enumerated arguments
                           | (_naryop bvar* condition _mmlarg)
//n-ary ops, condition defines argument list
                           | (_ispop  bvar? _ispdomain? _mmlarg)
//integral, sum, product
                           | (_diffop bvar* _mmlarg)
//differential ops
                           | (log logbase? _mmlarg)
//logs
                           | (moment degree? _mmlarg*)
//statistical moment
                           | (root degree? _mmlarg)
//radicals - default is square-root
                           | (limit bvar* lowlimit? condition? _mmlarg)
//limits
                           | (_logicopquant bvar+ condition? (reln | apply))
//quantifier with explicit bound variables
// equations and relations - reln uses lisp-like syntax (like apply)
// the bvar and condition are used to construct a "such that" or
// "where" constraint on the relation
reln          ::= _sg(reln) _relnbody _eg(reln)
_relnbody      ::= ( _binaryrel bvar* condition? _mmlarg _mmlarg )
                 | ( _naryrel bvar* condition? _mmlarg*  )
// fn construct
fn                   ::= _sg(fn) _fnbody _eg(fn)
_fnbody        ::=  Presentation_tags | container
// lambda construct     - note at least 1 bvar must be present
lambda         ::=  _sg(lambda) _lambdabody _eg(lambda)
_lambdabody    ::=  bvar+ _container  //multivariate lambda calculus
//declare construct
declare        ::= _sg(declare) _declarebody _eg(declare)
_declarebody   ::= ci (fn | constructor)?
// constructors
interval       ::=  _sg(interval) _mmlarg _mmlarg _eg(interval)
//start, end define interval
```

```
set        ::=  _sg(set) _lsbody _eg(set)
list       ::=  _sg(list) _lsbody _eg(list)
_lsbody        ::=  _mmlarg*                //enumerated arguments
                  | (bvar* condition _mmlarg)   //condition constructs arguments
matrix         ::= _sg(matrix) matrixrow* _eg(matrix)
matrixrow      ::= _sg(matrixrow) _mmlall* _eg(matrixrow)
//allows matrix of operators
vector         ::= _sg(vector) _mmlarg* _eg(vector)
//qualifiers - note the contained _mmlarg could be a reln
lowlimit       ::= _sg(lowlimit) _mmlarg  _eg(lowlimit)
uplimit        ::= _sg(uplimit) _mmlarg _eg(uplimit)
bvar           ::= _sg(bvar) ci degree? _eg(bvar)
degree         ::= _sg(degree) _mmlarg _eg(degree)
logbase        ::= _sg(logbase) _mmlarg _eg(logbase)
//relations and operators
// (one declaration for each operator and relation element)
_relation      ::= _ey(\%relation)        //for example <eq/>  <lt/>
_operator      ::= _ey(\%operator)        //for example <exp/> <times/>
//the top level math element
math           ::= _sg(math) mmlall* _eg(math)
```

Appendix C

Content Element Definitions

C.1 About Content Markup Elements

Every content element must have a mathematical definition associated with it in some form. The purpose of this appendix is to provide default definitions. (An index to the definitions is provided later in this document.) For this release of MathML definitions have not been restricted to any one format. There are several reasons for allowing flexibility at this time.

1. Many mathematical constructs are not yet implemented in any computation based system. However, MathML must still allow authors to associate mathematical constructs with definitions for archival purposes and so that work on such implementations can begin.
2. The task of defining a mathematical object, and establishing an association with a particular definition does not logically depend on the existence of an implementation in a computational system. It is a perfectly legitimate mathematical activity independent of whether it is ever implemented. Providing a record of those author specified associations is integral part of the role of MathML.
3. The task of designing a machine readable language suitable for recording semantic descriptions is an onerous one that goes substantially beyond the scope of this particular recommendation. It also overlaps substantially with efforts groups such as the OpenMath Consortium. (See also: North American OpenMath Initiative, and The European OpenMath Consortium)

The feasibilty of implementing a particular object in a particular computational system and the details of particular implementations have very little to do with the requirement that there actually be a mathematical definition. An author's decision to use content elements is a decision to work with defined objects. The definitions may be as vague as claiming that, say F, is an unknown, but differentiable function from the real numbers to the real numbers, or as complicated as requiring that F to be an elaborate new function or operation as defined in some recent research paper. The primary role of MathML content elements is to provide a mechanism for recording the fact that a particular structure has a particular mathematical meaning. If a definition is implemented in a computational system, this is a bonus.

Of course, default definitions and semantics should be chosen to be as useful as possible. Where possible they should be already implemented or easy to implement and all other things being equal, an author would be well advised to use a definition that is in common use. This is no different from noting that most well written mathematical communications (in any format) benefit substantially from the author's use of widely used and understood terms.

A requirement that there be a definition is also very different from a requirement that a definition be provided in some specific manner. Before requiring a particular approach to definitions one needs to consider such issues as:

1. providing a language for defining semantics.
2. deciding if it is reasonable to require the use of such a syntax. (Authors may not have the time or expertise to provide a formal description in a new and unfamiliar language.)
3. not being constrained by the limitations of existing computational systems.

In order to leave open the discussion of such fundamental issues we have deliberately limited the support for new or author defined definitions to support for specifying an appropriate `definitionURL`. The format of the content of that URL is unspecified. It might be the URL of a mathematical paper whose whole purpose is to define a new operator, or even a simple reference to a traditional text. If the author's mathematical operator matches exactly with an operator in a particular computational system, an appropriate definition might be a MathML `semantics` element establishing a correspondence between two encodings. Whatever is chosen, the only essential feature is that the definition be provided.

This rest of this appendix provides detailed descriptions of the default semantics associated with each of the MathML content elements. Since this is exactly the role intended for the encodings under development by the OpenMath Consortium and one of our goals is to foster international cooperation in such standardization efforts we have presented the default definitions in a format modeled on OpenMath's content dictionaries. While the actual details differ somewhat from the OpenMath specification, the underlying principles are the same and this is being used as input to ongoing discussions underway with the OpenMath Consortium aimed at ensuring that the OpenMath encodings will be capable of conveying the necessary information.

C.1.1 The Structure of an MML `definition`.

Each MathML element is described using an XML format. The top element is `MMLdefinition` The sub-elements identify the various parts of the description and include:

name `PCDATA` providing the name of the MathML element.

description A text-based description of the object that an element represents. This will often include cross-references to more traditional texts or papers or existing papers on the Web.

functorclass Each MathML element must be classified according to its mathematical role.

- **punctuation** Some elements exist simply as an aid to parsing. For example the `sep` element is used to separate the `CDATA` defining a rational number into two parts in a manner that is easily parsed by an XML application. These objects are refered to as punctuation.
- **modifier** Some elements exist simply to modify the properties of an existing element or mathematical object. For example the `declare` construct is used to reset the default attribute values, or to associate a name with a specific instance of an object. These kinds of elements are referred to as modifiers and the result is of the same type, but with different attributes.
- **constructor** The remaining objects that 'contain' sub-elements are all object constructors of some sort or another. They combine the sub-elements into a compound mathematical object such as a constant, set, list, or an expression representing a function application. For example, the `lambda` element is actually a constructor that constructs a function definition from a list of

variables and an expression, while the `fn` element is a constructor that, in effect, sets the type of an object to function and if necessary, provides an external definition.

Issue (constructor): This explanation needs to be updated.

Any use of `apply` produces an object of type `apply` whose sub-type is determined by the first operand and its properties. The signature of a constructor indicates the type of its sub-elements and the type (and sometimes subtype) of the resulting object.

function (operator) The MathML objects represented by empty XML elements are functions or operators. These function definitions are parameterized by their XML attribute values and are used as the first argument to an `apply` or `reln`. Functions are classified according to how they are used. For example the empty `sin` element represents the unary mathematical function sine. In every case, element attributes may be used to further qualify the object. The `plus` element is an nary operator. The result of using a function or operator is an expression which represents an object in a certain algebraic domain.

parameter Another class of objects are the named parameters. For example, these named objects are used to identify bounds of integration, or differentiation variables.

MMLattribute Some of the XML attributes of a MathML content element have a direct impact on the mathematical semantics of the object. For example the `type` attribute of the `cn` element is used to determine what type of constant (integer, real, etc.) is being constructed. Only those attributes that affect the mathematical properties of an object are listed here and typically they also appear explicitly in the signature.

signature The signature is a systematic representation that associates the different possible combinations of attributes and function arguments to the different kinds of mathematical objects that are constructed. The possible combinations of parameter and argument types (the left-hand side) each result in an object of some type (the right-hand side). In effect, it describes how to resolve operator overloading. For constructors (including parameters), the left-hand side of the signature describes the types of the child elements and the right-hand side describes the type of object that is constructed. For functions, the left-hand side of the signature indicates the types of the parameters and arguments that would be expected when it is applied, or used to construct a relation, and the right-hand side represents the mathematical type of the object constructed by the `apply` Modifiers modify the attributes of an existing object. For example, a symbol might become a symbol of type vector. The signature must be able to record specific attribute values and argument types on the left, and parameterized types on the right.. The syntax used for signatures is of the general form:

```
[<attribute name>=<attributevalue>]( <list of argument types> )
--> <mathematical result type>(<mathematical subtype>)
```

The MMLattributes, if any, appear in the form `<name>=<value>`. They are separated notationally from the rest of the arguments by square braces. The possible values are usually taken from an enumerated list, and the signature is usually affected by selection of a specific value. For the actual function arguments and named parameters on the left, the focus is on the mathematical types involved. The function argument types are presented in a syntax similar to that used for a DTD, with the one main exception. The types of the named parameters appear in the signature as `<elementname>=<type>` in a manner analogous for that used

for attribute values. For example, if the argument is named (e.g. `bvar`) then it is represented in the signature by an equation as in:

```
[<attribute name>=<attributevalue>]( bvar=symbol,<argument list> ) -->
<mathematical result type>(<mathematical subtype>)
```

No mathematical evaluation ever takes place in MathML. Every MathML content element either refers to a defined object such as a mathematical function or it combines such objects in some way to build a new object. For purposes of the signature, the constructed object represents an object of a certain type parameterized type. For example the result of applying `plus` to arguments is an expression that represents a sum. The type of the resulting expression depends on the types of the operands, and the values of the MathML attributes.

example Examples of the use of this object in MathML and possibly other syntax are included in these elements.

property This element describes the mathematical properties of such objects. For simple associations of values with specific instances of an object, the first child is an instance of the object being defined. The second is a `value` or `approx` (approximation) element that contains a MathML description of this particular value. More elaborate conditions on the object are expressed using the MathML syntax.

C.2 Definitions of MathML Content Elements

C.2.1 Leaf Elements

C.2.1.1 *cn*

```
<MMLdefinition>
<name> cn </name>
<description>
   A numerical constant.  The mathematical type of number
   is given as an attribute.  The default type is "real".
   Numbers such as rational, complex or real, require two
   parts for a complete specification.  The parts of such
   a number are separated by an empty "sep" element.
   There are a number of pre-defined constants including:
     &pi; &Exponential; &ComplexI &true; &false; &NaN;
   the properties of some of which are outlined below.
    The &NaN; is IEEE's "Not a Number", as defined in
    IEEE 854 standard for Floating point arithmetic.
</description>
<functorclass> constant </functorclass>
<MMLattribute>
   <name> type </name>
   <value> integer | rational | complex-cartesian
           | complex-polar | real
   </value>
   <default> real </default>
</MMLattribute>
<MMLattribute>
   <name> base </name>
```

```
    <value> positive_integer </value>
    <default> 10 </default>
</MMLattribute>
<signature> [type=integer](numstring) -> constant(integer) </signature>
<signature> [base=basevalue](numstring) -> constant(integer) </signature>
<signature> [type=rational](numstring,numstring) -> constant(rational) </signatur
<signature> [type=complex-cartesian](numstring,numstring) -> constant(complex) </
<signature> [type=rational](numstring,numstring) -> constant(rational) </signatur
<signature> [type=real](&pi;) -> constant(real) </signature>
<signature> [definition](numstring,numstring) -> constant(userdefined) </signatur
<signature> (&gamma;) -> constant</signature>
<example> <cn> 245 </cn> </example>
<example> <cn type="integer"> 245 </cn> </example>
<example> <cn type="integer" base="16"> A </cn></example>
<example> <cn type="rational"> 245 <sep> 351 </cn> </example>
<example> <cn type="complex-cartesian"> 1 <sep/> 2 </cn> </example>
<example> <cn> 245 </cn> </example>
<property> <approx>
  <cn> &pi; </cn>
  <cn> 3.141592654 </cn>
</approx></property>
<property> <approx>
  <cn> &gamma; </cn>
  <cn> .5772156649 </cn>
</approx> </property>
<property> <reln><identity/>
  <cn>&ImaginaryI; </cn>
  <apply><root><cn>-1</cn><cn>2</cn></apply>
</reln>
</property>
<property> <reln><approx>
<cn> &ExponentialE; </cn><cn>2.718281828 </cn>
</reln> </property>
<property> <apply><forall/>
  <bvar><ci type=boolean>p</ci></bvar>
 apply><and/>
    <ci>p</ci><cn>&true;</cn></apply>
    <ci>p</ci>
  </apply>
</property>
<property> <apply><forall/>
  <bvar><ci type=boolean>p</ci></bvar>
  <apply><or/>
    <ci>p</ci><cn>&true;</cn></apply>
    <cn>&true;</cn>
  </apply>
</property>
  <bvar><ci type=boolean>p</ci></bvar>
  <apply><or/>
    <ci>p</ci><cn>&true;</cn></apply>
```

```
    <cn>&true;</cn>
  </apply>
</property>
<property>
    <identity>
      <apply><not/><cn> &true; </apply>
      <cn> &false; </cn>
    </identity>
</property>
<property> <reln><identity/>
  <cn base="16"> A </cn> <cn> 10 </cn> </reln> </property>
<property> <reln><identity/>
  <cn base="16"> B </cn> <cn> 11 </cn> </reln></property>
<property> <reln><identity/>
  <cn base="16"> C </cn> <cn> 12 </cn> </reln></property>
<property> <reln><identity/>
  <cn base="16"> D </cn> <cn> 13 </cn> </reln></property>
<property> <reln><identity/>
  <cn base="16"> E </cn> <cn> 14 </cn> </reln></property>
<property> <reln><identity/>
  <cn base="16"> F </cn> <cn> 15 </cn> </reln></property>
</MMLdefinition>
```

C.2.1.2 *ci*

```
<MMLdefinition>
<name> ci </name>
<description>
  A symbolic name constructor. The type attribute can
  be set to any valid MathML type.
</description>
<functorclass> constructor , unary </functorclass>
<MMLattribute>
  <name> type </name>
  <value> constant | matrix | set | vector | list | MathMLtype </value>
  <default> real </default>
</MMLattribute>
<signature> ({string|mmlpresentation}) -> symbol(constant) </signature>
<signature> [type=MathMLType]({string|mmlpresentation}) -> symbol(MathMLType) </s
<example><ci> xyz </ci> </example>
<example><ci> type="vector"> V </ci> </example>
</MMLdefinition>
```

C.2.2 Basic Content Element

C.2.2.1 *apply*

```
<MMLdefinition>
<name> apply </name>
<description>
```

```
  This is the MathML constructor for function application.
  The first argument is applied to the remaining arguments.
  It may be the case that some of the child elements are
  named elements. (See the signature.)
</description>
<functorclass> constructor , nary </functorclass>
<signature> (function,anything*) -> application </signature>
<example><apply><plus/><ci>x</ci><cn>1</cn></apply></example>
<example><apply><sin/><ci>x</ci></apply></example>
</MMLdefinition>
```

C.2.2.2 *reln*

```
<MMLdefinition>
<name> reln </name>
<description>
  This is the MathML constructor for expressing a relation between
  two or more mathematical objects.  The first argument indicates
  the type of "relation" between the remaining arguments. (See the signature.)
  No assumptions are made about the truth value of such a relation.
  Typically, the relation is used as a component in the construction
  of some logical assertion.   Relations may be combined into
  sets, etc. just like any other mathematical object.
</description>
<functorclass> constructor </functorclass>
<signature> (function,anything*) -> reln </signature>
<example><reln><and/><ci>P</ci><ci>Q</ci></reln></example>
<example><reln><lt/><ci>x</ci><ci>y</ci></reln></example>
</MMLdefinition>
```

C.2.2.3 *fn*

```
<MMLdefinition>
<name> fn </name>
<description>
  This is the MathML constructor for building new function
  names.   The "name" can be a general MathML content element.
  It identifies that object as "usable" in a function
  context.
  By setting its definitionURL value, you can
  associate it with a particular function definition.
  Use the MathML Declare to associate a name with a lambda
  construct.
</description>
<MMLattribute>
  <name>definitionURL</name>
  <value> URL </value>
  <default> none </default>
</MMLattribute>
<functorclass> constructor </functorclass>
```

```
<signature> (anything) -> function </signature>
<signature> [definitionURL=functiondef](anything) ->
  function(definitionURL=functiondef)
</signature>
<example><fn><ci>F</ci></fn></example>
<example><fn definitionURL="http://www.w3c/...">
  <lt/><ci>G</ci></fn>
</example>
<!--Declaring Id to be the identity function.-->
<example>
  <declare><fn><ci>Id</ci></fn><lambda><ci>x</ci><ci>x</ci></declare>
</example>
</MMLdefinition>
```

C.2.2.4 *interval*

```
<MMLdefinition>
<name> interval </name>
<description>
  This is the MathML constructor element for building an interval
  on the real line.  While an interval could be expressed by
  combining relations appropriately, they occur explicitly because
  of their frequence of occurrence in common use.
</description>
<MMLattribute>
  <name>type</name>
  <value> closed | open | open-closed | closed-open </value>
  <default> closed </default>
</MMLattribute>
<functorclass> constructor , binary </functorclass>
<signature> [type=intervaltype](expression,expression) -> interval </signature>
<example><reln><and/><ci>x</ci><cn>1</cn></reln></example>
<example><reln><lt/><ci>x</ci></reln></example>
</MMLdefinition>
```

C.2.2.5 *inverse*

```
<MMLdefinition>
<name> inverse </name>
<description>
  This MathML element is applied to a function in order to
  construct a new function that is to be interpreted as the
  inverse function of the original function.  For a particular
  function F,  inverse(F) composed with F behaves like the
  identity map on the domain of F and F composed with inverse(F)
  should be an identity function on a suitably restricted
  subset of the Range of F.
  The MathML definitionURL attribute should be used to resolve
  notational ambiguities, or to restrict the inverse to a
  particular domain or make it one-sided.
```

```
</description>
<MMLattribute>
  <name>definitionURL</name>
  <value> CDATA </value>
  <default> none </default>
<!--none corresponds to using the default MathML definition ...-->
</MMLattribute>
<functorclass> operator, unary </functorclass>
<signature> (function) -> function </signature>
<signature> [definitionURL=URL](function) ->
    function(definition) </signature>
<example><apply><inverse/><sin/></apply></example>
<example>
  <apply>
  <inverse definitionURL="www.w3c.org/MathML/Content/arcsin"/>
  <sin/>
  </apply>
</example>
<property><apply><forall/>
  <bvar><ci>y</ci></bvar>
  <apply><sin/>
    <apply>
      <apply><inverse/><sin/></apply>
      <ci>y</ci>
    </apply>
  </apply>
  <value><ci>y</ci></value>
</apply>
</property>
<property>
<apply>
  <apply><inverse/><sin/></apply>
  <apply>
    <sin/>
    <ci>x</ci>
  </apply>
</apply>
<value><ci>x</ci></value>
</property>
<property>F(inverse(F)(y))<value>y</value></property>
</MMLdefinition>
```

C.2.2.6 *sep*

```
<MMLdefinition>
<name> sep </name>
<description>
  This is the MathML infix constructor used to sub-divide PCDATA into
  separate components. for example, this is used in the description of
  a multipart number such as a rational or a complex number.
```

```
</description>
<functorclass> punctuation </functorclass>
<example><cn type="complex-polar">123<sep/>456</cn></example>
<example><cn>123</cn></example>
</MMLdefinition>
```

C.2.2.7 *condition*

```
<MMLdefinition>
<name> condition </name>
<description>
  This is the MathML constructor for building conditions.
  A condition differs from a relation in how it is used.
  A relation is simply an expression, while a condition
  is  used as a predicate to place a conditions on a bound
  variables.
  For a compound condition use relations or apply
  operators such as "and" or "or" or a set of
  relations).
</description>
<functorclass> constructor, unary </functorclass>
<signature> ({reln|apply|set}) -> predicate </signature>
<example>
<condition>
  <reln><lt/>
    <apply><power/>
      <ci>x</ci><cn>5</cn>
    </apply>
    <cn>3</cn>
  </reln>
</condition>
</example>
</MMLdefinition>
```

C.2.2.8 *declare*

```
<MMLdefinition>
<name> declare </name>
<description>
  This is the MathML constructor for redefining the properties and
  values with mathematical objects.  For example V may be a name
  delcared to be a vector, or V may be a name that stands for a
  particular vector.
  The attribute values of the declare statement are assigned as the
  corresponding default attribute values of the first object.
</description>
<functorclass> modifier , (unary | binary) </functorclass>
<MMLattribute>
<name>definitionURL</definition>
<value> Any valid URL </value>
```

```
</MMLattribute>
<MMLattribute>
<name>type</name><value> MathMLType </value>
</MMLattribute>
<MMLattribute>
<name>nargs</name><value> number of arguments for an object of type fn </value>
</MMLattribute>
<signature> [attributename=attributevalue](anything)   -> anything(attributevalue
<!-- The two argument form updates  the properties of the first
object to be those of the second.  The attribute values override the
properties of the "value".
-->
<signature> [attributename=attributevalue](anything,anything)   -> anything(attri
<example><reln><and/><ci>x</ci><cn>1</cn></reln></example>
<example><reln><lt/><ci>x</ci></reln></example>
</MMLdefinition>
```

C.2.2.9 *lambda*

```
<MMLdefinition>
  <name> lambda </name>
  <description> The operation of lambda calculus that makes a
  function from an expression and a variable.  The definition
  at this level uses only one variable.  Lambda is a binary
  function, where the first argument is the variable and
  the second argument is a the expression.
  Lambda( x, F ) is written as \lambda x [F] in the lambda
  calculus literature.
  The lambda function can be viewed as the inverse of function
  application.
  Although the expression F may contain x, the lambda expression
  is interpreted to be free of x.  That is, the x variable is
  a variable local to the environment of the definition of
  the function or operator.  Formally, lambda(x,F) is free of
  x, and any substitutions, evaluations or tests for x in
  lambda(x,F) should not happen.
  A lambda expression on an arbitrary function applied to a
  simple argument is equivalent to the arbitrary function.
  E.g.  lambda(x, f(x)) == f.  This is a common shortcut.
</description>
  <functorclass> Nary , Constructor </functorclass>
  <property>
    <lambda><ci>x</ci>
      <apply><fn><ci>F</ci></fn><ci>x</ci></apply>
    </lambda>
     <value> <fn><ci>F</ci></fn> </value>
  </property>
<!-- Constructing a variant of the sine function -->
   <example>
      <lambda>
```

```
       <ci> x </ci>
       <apply><sin/>
         <apply><plus/>
           <ci> x </ci>
           <cn> 3 </cn>
       </apply>
     </lambda>
   </example>
<!-- the identity operator  -->
  <example>
    <lambda><ci> x </ci> <ci> x </ci> </lambda>
  </example>
  <property>
  <reln><identity/>
    <lambda><ci>x</ci>
      <apply><fn><ci>F</ci></fn><ci>x</ci></apply>
    </lambda>
    <fn><ci>F</ci></fn>
  </reln>
  </property>
<MMLdefinition>
```

C.2.2.10 *compose*

```
<MMLdefinition>
<name> compose </name>
<description>
  This is the MathML constructor for composing functions.
  In order for a composition to be meaningful, the range of
  the first function must be the domain of the second function,
  etc.  .
  The result is a new function whose domain is the domain of
  the first function and whose range is the range of the last
  function and whose definition is equivalent to applying
  each function to the previous outcome in turn as in:
  (f @ g )( x )   ==  f( g(x) ).
 This function is often denoted by a small circle infix
 operator.
</description>
<functorclass> Nary , Operator </functorclass>
<signature> (fn*) -> fn </signature>
<example>
<apply><compose/>
      <fn><ci> f </ci></fn>
      <fn><ci> g </ci></fn>
    </apply></example>
<property>
<apply><forall>
  <bvar><ci>x</ci></bvar>
  <reln><eq/>
```

```
    <apply>
      <apply><compose/>
        <ci>f</ci>
        <ci>g</ci>
      </apply>
      <ci>x</ci>
    </apply>
    <apply><ci>f</ci>
      <apply><ci>g</ci>
        <ci>x</ci>
      </apply>
    </apply>
  </reln>
</apply>
</property>
</MMLdefinition>
```

C.2.2.11 *ident*

```
<MMLdefinition>
<name> ident </name>
<description>
  This is the MathML constructor for the identity function.
  This function has the property that
      f( x ) = x,  for all x in its domain.
</description>
<functorclass> Nary , Operator </functorclass>
<signature> (symbol) -> symbol </signature>
<example>
<apply><ident/>
       <ci> f </ci>
       <ci> x </ci>
</apply>
</example>
<property>
<apply><forall>
  <bvar><ci>x</ci></bvar>
  <reln><eq/>
    <apply><ident/>
        <ci>f</ci>
        <ci>x</ci>
    </apply>
    <ci>x</ci>
    </reln>
</apply>
</property>
</MMLdefinition>
```

C.2.3 Arithmetic, Algebra and Logic

C.2.3.1 *quotient*

```
<MMLdefinition>
<name> quotient </name>
    <description> Integer quotient, the result of integer
      division.  For arguments a and b, it returns q,
      where a = b*q+r,  |r| < |b|  and  a*r &ge; 0  (or
      the sign of r is the same as the sign of a).
     </description>
       <functorclass> Binary, Function </functorclass>
       <signature> (integer, integer) -> integer </signature>
       <signature> (symbolic, symbolic) -> symbolic </signature>
<!--
ForAll(bvar(a,b),identity(a ,b*Quotient(a,b) + Remainder(a,b))
-->
       <property>
       <apply><forall/>
         <bvar><ci>a</ci></bvar>
         <bvar><ci>b</ci></bvar>
         <reln/><eq/>
             <ci>a</ci>
             <apply><plus/>
               <apply><times/>
                 <ci>b</ci>
                 <apply><quotient/><ci>a</ci><ci>b</ci></apply>
               </apply>
               <apply><rem/><ci>a</ci><ci>b</ci></apply>
             </apply>
         <reln>
       </apply>
       </property>
     <!-- 1 = quotient(5,4) -->
     <property>
      <apply><identity/>
        <apply><quotient/>
          <ci>5</ci>
          <ci>4</ci>
        </apply>
        <ci>1</ci>
     <apply>
     </property>
</MMLdefinition>
```

C.2.3.2 *exp*

```
<MMLdefinition>
  <name> exp </name>
    <description> The exponential function.
           <Reference> M. Abramowitz and I. Stegun, Handbook of
```

```
            Mathematical Functions, [4.2]
          </Reference>
    </description>
       <functorclass> Unary, Function </functorclass>
       <signature> real -> real </signature>
       <signature> symbolic -> symbolic </signature>
       <property><reln><eq/>
         <apply><exp/><cn>0</cn></apply>
         <cn>1</cn></reln>
       </property>
       <property><apply><identity/>
         <apply><exp/><ci>x</ci></apply>
         <apply><power/>
           <cn>ExponentialE;</cn><ci>x</ci>
         </apply>
       </apply>
       </property>
       <property> exp(x) = limit( (1+x/n)^n, n, infinity ) </property>
</MMLdefinition>
```

C.2.3.3 *factorial*

```
<MMLdefinition>
  <name>
    factorial
  </name>
  <description>
    This element is used to construct factorials
    as in n! = n * (n-1) * (n-2 ) ... 1 .
  </description>
  <functorclass> Unary , function </functorclass>
  <signature> ( algebraic ) -> algebraic </signature>
  <example> <apply><factorial/><ci>n</ci></apply> </example>
  <!-- for all n > 0, n! = n*(n-1)! -->
  <property><apply><forall/>
    <bvar><ci>n<ci></bvar>
    <condition>
      <reln><gt/><ci>n</ci><cn>0</cn></reln>
    </condition>
    <reln><eq/>
      <apply><factorial/><ci>n</ci></apply>
      <apply><times/>
        <ci>n</ci>
        <apply><factorial/>
          <apply><minus/><ci>n</ci><cn>1</cn></apply>
        </apply>
      </apply>
    </reln>
  </property>
  </MMLdefinition>
```

C.2.3.4 *divide*

```
<MMLdefinition>
  <name> divide </name>
  <description>
    The MathML operator that is used to construct
    a "divided by" b.  If a and b are from an algebraic
    domain with a non-commutative times then this is defined
    as a * (b)^(-1).  The result is from the same algebraic
    domain as the operands.
  </description>
  <MMLattribute>
    <name> type </name>
    <value> non-commutative </name>
    <default> none </default>
  </MMLattribute>
  <functorclass> binary , function </functorclass>
  <signature> (complex, complex) -> complex </signature>
  <signature> (real, real) -> real </signature>
  <signature> (rational, rational) -> rational </signature>
  <signature> (integer, integer) -> rational </signature>
  <signature> (symbolic, symbolic) -> symbolic </signature>
  <example>
    <apply> <divide/>
     <ci> a </ci>
     <ci> b </ci>
    </apply>
  </example>
  <property>
    <apply><forall/>
      <bvar>a</bvar>
      <reln><eq/>
      <apply> <divide/>
        <ci> a </ci>
        <ci> 0 </ci>
      <ci>Error, Division by 0</ci>
    </apply>
  </property>
  </MMLdefinition>
```

C.2.3.5 *max*

```
<MMLdefinition>
  <name> max  </name>
  <description>
    Represent the maximum of a set of elements.  The elements
    may be given explicitly or described by membership in
    some set.  To be well defined, the elements must all be
    comparable.    </description>
  <functorclass> function </functorclass>
```

```
  <signature> ( ordered_set_element * ) -> ordered_set_element </signature>
  <signature> ( condition ) ->  ordered_set_element </signature>
  <example>
    <apply><max/><cn>2</cn><cn>3</cn> <cn>5</cn> </apply>
  </example>
  <example>
    <apply><max/>
       <condition>
         <bvar><ci>x</ci></bvar>
         <reln> <notin/>
           <ci> x </ci>
           <ci type="set"> B </ci>
         </reln>
       </condition>
     </apply>
  </example>
  </MMLdefinition>
```

C.2.3.6 *min*

```
<MMLdefinition>
  <name> min  </name>
  <description>
    Represent the minimum of a set of elements.  The elements
    may be given explicitly or described by membership in
    some set.  To be well defined, the elements must all be
    comparable.    </description>
  <functorclass> function </functorclass>
  <signature> ( ordered_set_element * ) -> ordered_set_element </signature>
  <signature> ( condition ) ->  ordered_set_element </signature>
  <example>
    <apply><min/><cn>2</cn><cn>3</cn> <cn>5</cn> </apply>
  </example>
  <example>
    <apply><min/>
       <condition>
         <bvar><ci>x</ci></bvar>
         <reln> <notin/>
           <ci> x </ci>
           <ci type="set"> B </ci>
         </reln>
       </condition>
     </apply>
  </example>
  </MMLdefinition>
```

C.2.3.7 *minus*

```
<MMLdefinition>
  <name> minus  </name>
```

```
  <description>
    The subtraction operator of a group.  </description>
  <MMLattribute>
    <name> definitionURL </name>
    <value> URL </name>
    <default> none </default>
  </MMLattribute>
  <functorclass>
    Operator , (Unary | Binary )
  </functorclass>
  <signature>(real,real) -> real</signature>
  <signature>(integer,integer) -> integer</signature>
  <signature>(rational,rational) -> rational</signature>
  <signature>(complex,complex) -> complex</signature>
<!--
  Note that complex-cartesian is a data input format,
  but the resulting data type is complex. !
-->
  <signature> (vector,vector) -> vector</signature>
  <signature>(matrix,matrix) -> matrix</signature>
  <signature>(real) -> real </signature>
  <signature>(integer) -> integer </signature>
  <signature>(complex) -> complex </signature>
  <signature>(rational) -> rational </signature>
  <signature>(vector) -> vector </signature>
  <signature>(matrix) -> matrix </signature>
 <example>
    <apply><minus/><cn>3</cn><cn>5</cn></apply>
 </example>
 <example>
    <apply><minus/><cn>3</cn></apply>
 </example>
<!-- Definition of the unary operator (-1) = -( 1 ) -->
<property>
    <reln><eq/>
       <bvar><ci>n</ci>
       <apply><minus/><cn>1</cn></apply>
       <cn>-1</cn>
    </reln>
  </property>
  </MMLdefinition>
```

C.2.3.8 *plus*

```
<MMLdefinition>
<name> plus </name>
<description> The N-ary addition operator of an
algebraic structure.
If no operands are provided, the expression represents
the additive identity.
```

```
If one operand a is provided, the expression represents
a.
If two or more operands are provided, the expression
represents the group element corresponding to a left
associative binary pairing of the operands.
Issues with regard to the "value" of mixed operands
are left up to the target system.  If the author wishes
to refer to specific type coercion rules, then
the definitionURL attribute should be used to refer
to a suitable specification.
</description>
<functorclass>  Operator , Nary </functorclass>
<signature>(real,real) -> real</signature>
<signature>(integer,integer) -> integer</signature>
<signature>(rational,rational) -> rational</signature>
<signature> (vector,vector) -> vector</signature>
<signature>(matrix,matrix) -> matrix</signature>
<signature>(complex,complex) -> complex</signature>
<signature>(symbolic,symbolic) -> symbolic </signature>
<signature> real -> real </signature>
<signature> rational -> rational </signature>
<signature> integer -> integer </signature>
<signature> symbolic -> symbolic </signature>
<signature>(real) -> real </signature>
<signature>(integer) -> integer </signature>
<signature>(complex) -> complex </signature>
<signature>(rational) -> rational </signature>
<signature>(vector) -> vector </signature>
<signature>(matrix) -> matrix </signature>
<example><apply><plus/><cn>3</cn></apply></example>
<example><apply><plus/><cn>3</cn><cn>5</cn></apply></example>
<example><apply><plus/><cn>3</cn><cn>5</cn><cn>7</cn></apply></example>
<!-- The properties for more restricted algebraic structures should
be defined using a definitionURL
-->
<property> +() = 0 </property>
<property> +(a) = a </property>
<property> ForAll(a,Commutative, a + b = b + a)</property>
</MMLdefinition>
```

C.2.3.9 *power*

```
<MMLdefinition>
<name> power </name>
<description> The powering operator </description>
<functorclass> binary, operator </functorclass>
<signature> (complex complex) -> complex </signature>
<signature> (real real) -> complex </signature>
<signature> (rational rational) -> complex </signature>
<signature> (rational integer) -> rational </signature>
```

```
<signature> (integer integer) -> rational </signature>
<signature> (symbolic symbolic) -> symbolic </signature>
<property> ForAll(a,Condition(a<>0),a^0=1) </property>
<property> ForAll(a,a^1=a) </property>
<property> ForAll(a,1^a=1) </property>
<property>ForAll(a,0^0=Undefined)</property>
   </MMLdefinition>
```

C.2.3.10 *rem*

```
<MMLdefinition>
<name> rem </name>
<description> Integer remainder, the result of integer
division.  For arguments a and b, it returns r,
where a = b*q+r,  |r| < |b|  and  a*r &ge; 0  (the
sign of r is the same as the sign of a when both are
non-zero).
</description>
<functorclass> binary, function </functorclass>
<signature> (integer integer) -> integer </signature>
<signature> (symbolic symbolic) -> symbolic </signature>
<property> a = b*rem(a,b) + rem(a,b) </property>
<property>rem(a,0) = Division_by_Zero</property>
</MMLdefinition>
```

C.2.3.11 *times*

```
<MMLdefinition>
<name> times </name>
<description> The multiplication operator of any
ring.
</description>
<functorclass> N-ary, Operator </functorclass>
<signature> (complex complex) -> complex </signature>
<signature> (real, real) -> real </signature>
<signature> (rational, rational) -> rational </signature>
<signature> (integer, integer) -> integer </signature>
<signature> (symbolic, symbolic) -> symbolic </signature>
<property>ForAll(bvars(a,b),condition(in({a,b},Commutative)),a*b=b*a)</property>
<property>ForAll(bvars(a,b,c),Associative,a*(b*c)=(a*b)*c), associativity </prope
<property> a*1=a </property>
<property> 1*a=a </property>
<property> a*0=0 </property>
<property> 0*a=0 </property>
</MMLdefinition>
```

C.2.3.12 *root*

```
<MMLdefinition>
  <name> root  </name>
```

```
  <description>
    Construct the nth root of an object.
    The first argument "a" is the object and the
    second object "n" denotes the root, as in
            ( a ) ^ (1/n)
  </description>
  <MMLattribute>
    <name> type </name>
    <value> real | complex | principle_branch </name>
    <default> real </default>
  </MMLattribute>
  <functorclass> binary , function </functorclass>
  <signature> ( anything , symbol ) -> root </signature>
  <example>
    <apply> <root/>
        <ci> a </ci>
        <ci> n </ci>
     </apply>
  </example>
  <property> Forall(bvars(a,n),root(a,n) = a^(1/n)) </property>
  </MMLdefinition>
```

C.2.3.13 *gcd*

```
<MMLdefinition>
  <name> gcd  </name>
  <description>
    This represents the greatest common divisor
    of its arguments.
  </description>
  <MMLattribute>
    <name> type </name>
    <value> anything </name>
    <default> integer </default>
  </MMLattribute>
  <functorclass> Function , Nary </functorclass>
  <signature> [type=typevalue](typevalue*) -> typevalue </signature>
  <example>
    <apply><gcd/><cn>12</cn> <cn>17</cn></apply>
  </example>
  <property>Forall(p,q,(is(p,prime) and is(q,prime)) , gcd(p,q)=1 </property>
  </MMLdefinition>
```

C.2.3.14 *and*

```
<MMLdefinition>
  <name> and  </name>
  <description>
    This is the logical "and" operator.
  </description>
```

```
<functorclass> function, Nary </functorclass>
<signature> (boolean*) -> boolean </signature>
<example> <apply><and/><ci>p</ci><ci>q</ci></apply> </example>
<property> identity(true and p , p ) </property>
<property> identity(p and q , q and p ) </property>
</MMLdefinition>
```

C.2.3.15 *or*

```
<MMLdefinition>
<name> or </name>
<description> The logical "or" operator.
</description>
<functorclass> Binary, Function </functorclass>
<signature> (boolean,boolean) -> boolean </signature>
<signature> [type=boolean](symbolic symbolic)     -> symbolic </signature>
<property> identity(true or p , true ) </property>
       ...
</MMLdefinition>
```

C.2.3.16 *xor*

```
<MMLdefinition>
<name> or </name>
<description> The logical "xor" operator.
</description>
<functorclass> Binary, Function </functorclass>
<signature> (boolean,boolean) -> boolean </signature>
<signature> [type=boolean](symbolic symbolic)     -> symbolic </signature>
<property> ...</property>
</MMLdefinition>
```

C.2.3.17 *not*

```
<MMLdefinition>
<name> not </name>
<description> The logical "not" operator.
</description>
<functorclass> Unary, Function </functorclass>
<signature> (boolean) -> boolean </signature>
<signature> [type=boolean](symbolic)     -> symbolic </signature>
<property> ... </property>
</MMLdefinition>
```

C.2.3.18 *implies*

```
<MMLdefinition>
<Name> implies </Name>
<description> The implies operator.  This represents
the construction  "A implies B".
```

```
</description>
<functorclass> Binary, relation </functorclass>
<signature> (boolean,boolean) -> boolean </signature>
<property> <apply></forall>
   <bvar><ci>A</ci></bvar>
   <bvar><ci>B</ci></bvar>
   <reln><eq/>
      <apply><implies/>
        <ci>A</ci>
        <ci>B</ci>
      </apply>
      <apply><or/>
        <ci>B</ci>
        <apply><not/>
          <ci> A </ci>
        </apply>
      </apply>
    </reln>
 </property>
</MMLdefinition>
```

C.2.3.19 *forall*

```
<MMLdefinition>
<name> forall </name>
<description> The logical "For all" quantifier.
</description>
<functorclass> Nary, Operator </functorclass>
<signature> (bvar*,condition?,(reln|apply)) -> boolean </signature>
<property> ... </property>
</MMLdefinition>
```

C.2.3.20 *exists*

```
<MMLdefinition>
<name> exists </name>
<description> The logical "There exists" quantifier.
</description>
<functorclass> Nary, Operator </functorclass>
<signature> (bvar*,condition?,(reln|apply)) -> boolean </signature>
<property> ... </property>
</MMLdefinition>
```

C.2.3.21 *abs*

```
<MMLdefinition>
<name> exists </name>
<description> The absolute value of a number.
</description>
<functorclass> Unary, Operator </functorclass>
```

```
<signature> (algebraic) -> algebraic </signature>
<property> ... </property>
</MMLdefinition>
```

C.2.3.22 *conjugate*

```
<MMLdefinition>
<name> conjugate </name>
<description> The "conjugate" arithmetic operator
used to represent the conjugate of a complex number.
</description>
<functorclass> Unary, Operator </functorclass>
<signature> (algebraic) -> algebraic </signature>
<property> ... </property>
</MMLdefinition>
```

C.2.4 Relations

C.2.4.1 *eq*

```
<MMLdefinition>
<Name> eq </Name>
<description> The equality operator. </description>
<functorclass> Nary, relation </functorclass>
<property> Commutative </property>
<signature> (symbolic symbolic) -> boolean </signature>
</MMLdefinition>
```

C.2.4.2 *neq*

```
<MMLdefinition>
<Name> neq </Name>
<description> The notequals operator. </description>
<functorclass> Nary, relation </functorclass>
<property> Commutative </property>
<signature> (symbolic symbolic) -> boolean </signature>
</MMLdefinition>
```

C.2.4.3 *gt*

```
<MMLdefinition>
<Name> gt </Name>
<description> The equality operator. </description>
<functorclass> binary, relation </functorclass>
<property> Commutative </property>
<signature> (symbolic symbolic) -> boolean </signature>
</MMLdefinition>
```

C.2.4.4 *lt*

```
<MMLdefinition>
<Name> lt </Name>
<description> The inequality equality operator "<" </description>
<functorclass> binary, relation </functorclass>
<property> Commutative </property>
<signature> (symbolic, symbolic*) -> boolean </signature>
</MMLdefinition>
```

C.2.4.5 *geq*

```
<MMLdefinition>
<Name> geq </Name>
<description> The inequality operator. >= </description>
<functorclass> Nary, relation </functorclass>
<signature> (symbolic, symbolic*) -> boolean </signature>
<property> ... Commutative ? ... </property>
</MMLdefinition>
```

C.2.4.6 *leq*

```
<MMLdefinition>
<Name> leq </Name>
<description> The inequality operator  </description>
<functorclass> Nary, relation </functorclass>
<property> Commutative </property>
<signature> (symbolic symbolic) -> boolean </signature>
</MMLdefinition>
```

C.2.5 Calculus

C.2.5.1 *ln*

```
    <MMLdefinition>
        <Name> ln </Name>
        <description> The logarithmic function.  Also called
            the natural logarithm.
            The inverse of the exponential function.
            <Reference> M. Abramowitz and I. Stegun, Handbook of
                Mathematical Functions, [4.1]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <property>
            Error( "logarithm has a singularity at 0" )
        </property>
        <signature> Intersect(real,positive) -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> ln(1) = 0 </property>
        <property> ln(exp(x)) = x, "for real x" </property>
```

```
    <property> exp(ln(x)) = x, always </property>
  </MMLdefinition>
```

C.2.5.2 *log*

```
  <MMLdefinition>
     <Name> log </Name>
     <description> The logarithmic function (base 10), or any
      any other user specified base.  Also called
        the natural logarithm.
        The inverse of the exponential function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.1]
        </Reference>
     </description>
     <functorclass> Unary, Function </functorclass>
     <signature> (real,logbase) -> real </signature>
     <signature> symbolic -> symbolic </signature>
     <property>
        Error( "logarithm has a singularity at 0" )
     </property>
   </MMLdefinition>
```

C.2.5.3 *int*

```
 <MMLdefinition>
       <Name> int </Name>
       <description>
          The definite or indefinite integral of a function or algebraic
          expression.
          There are several forms of calling sequences depending on
          the nature of the areguments, and whether or not it is a
          definite integral.
       </description>
       <functorclass> Binary , Function </functorclass>
       <signature> (function) -> function </signature>
       <signature> (algebraic,bvar) -> algebraic </signature>
       <signature> (algebraic,bvar,interval) -> algebraic </signature>
       <signature> (algebraic,bvar,condition) -> algebraic </signature>
  </MMLdefinition>
```

C.2.5.4 *diff*

```
  <MMLdefinition>
       <Name> diff </Name>
       <description>
          For expressions, this represents the derivative of
          its first argument evaluated at the second argument.
          For Unary functions (only one argument) it represents
          f’.
```

```
    </description>
    <functorclass> (Unary | Binary) , Function </functorclass>
    <signature> (algebraic,bvar) -> algebraic </signature>
    <property>Forall(x,diff( sin(x) , x ) = cos(x)) </property>
    <property>Forall(x,diff( x , x ) = 1 ) </property>
    <property>Forall(x,diff( x^2 , x ) = 2x) </property>
    <property>identity( diff(sin) , cos ) </property>
</MMLdefinition>
```

C.2.5.5 *partialdiff*

```
<MMLdefinition>
    <Name> partialdiff </Name>
    <description>
       For expressions, this represents the derivative of
       its first argument evaluated at the second argument.
       For Unary functions (only one argument) it represents
       f'.
    </description>
    <functorclass> (Binary) , Function </functorclass>
    <signature> (algebraic,bvar) -> algebraic </signature>
    <property>Forall(x,diff( sin(x*y) , x ) = cos(x)) </property>
    <property>Forall(x,y,diff( x*y , x ) = diff(x,x)*y + diff(y,x)*x ) </proper
    <property>Forall(x,a,b,diff( a + b , x ) = diff(a,x) + diff(b,x) ) </proper
    <property>identity( diff(sin) , cos ) </property>
</MMLdefinition>
```

C.2.5.6 *lowlimit*

```
<MMLdefinition>
    <Name> lowlimit </Name>
    <description> Construct a lower limit.  Limits
     are used in some integrals as alternative way
     of describing the region over which an integral
     is computed. (i.e. a connected component of the
     real line.)
     </description>
    <functorclass> Constructor </functorclass>
    <signature> (anything*) -> list </signature>
</MMLdefinition>
```

C.2.5.7 *uplimit*

```
<MMLdefinition>
    <Name> uplimit </Name>
    <description> Construct a an upper limit.  Limits
     are used in some integrals as alternative way
     of describing the region over which an integral
     is computed. (i.e. a connected component of the
     real line.)
```

```
        </description>
       <functorclass> Constructor </functorclass>
       <signature> (anything*) -> list </signature>
   </MMLdefinition>
```

C.2.5.8 *bvar*

```
   <MMLdefinition>
       <Name> bvar </Name>
       <description>
The bvar element is the container element
for the "bound variable" of an operation.
For example, in an integral it specifies the
variable of integration. In a derivative, it
indicates which variable with respect to
which a function is being differentiated.
When the bvar element is used to quantifiy a derivative,
the bvar element may contain a child degree element that
specifies the order of the derivative with respect to that
variable. The bvar element is also used for the internal
variable in sums and products.
   </description>
       <functorclass> Constructor </functorclass>
       <signature> (symbol) -> symbol </signature>
       <example> <bvar><ci>x</ci></bvar></example>
   </MMLdefinition>
```

C.2.5.9 *degree*

```
   <MMLdefinition>
       <Name> degree </Name>
       <description> A parameter used by some
       MathML data-types to specify that, for example,
       a bound variable is repeated several times.
       </description>
       <functorclass> Constructor </functorclass>
       <signature> (algebraic) -> algebraic </signature>
       <example> <degree><ci>x</ci></degree></example>
       <property> ... </property>
   </MMLdefinition>
```

C.2.6 Theory of Sets

C.2.6.1 *set*

```
   <MMLdefinition>
       <Name> set </Name>
       <description> Construct a set. </description>
       <functorclass> Nary, Constructor </functorclass>
       <signature> (anything*) -> set </signature>
   </MMLdefinition>
```

C.2.6.2 *list*

```
<MMLdefinition>
    <Name> list </Name>
    <description> Construct a list. </description>
    <functorclass> Nary, Constructor </functorclass>
    <signature> (anything*) -> list </signature>
</MMLdefinition>
```

C.2.6.3 *union*

```
<MMLdefinition>
    <Name> union </Name>
    <description> The union of two sets. </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (set*) -> set </signature>
</MMLdefinition>
```

C.2.6.4 *intersect*

```
<MMLdefinition>
    <Name> intersection </Name>
    <description> The intersection of two sets. </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (set set) -> set </signature>
</MMLdefinition>
```

C.2.6.5 *in*

```
<MMLdefinition>
    <Name> in </Name>
    <description>
        The membership testing operation (also commonly
        called "in" or "including").  Returns true if the first
        argument is part of the second argument.  The second
        argument must be a set.
    </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (anything, set) -> boolean </signature>
</MMLdefinition>
```

C.2.6.6 *notin*

```
<MMLdefinition>
<Name> notin </Name>
<description>
    The membership exclusion operation (also commonly
    called "notin" or "including").
    It is defined as "not in".
</description>
```

```
    <functorclass> Binary, Function </functorclass>
    <signature> (anything set) -> boolean </signature>
</MMLdefinition>
```

C.2.6.7 *subset*

```
<MMLdefinition>
<Name> subset </Name>
<description>
  Boolean function whose value is determined by
  whether or not one set is a subset of another.
 </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (set*) -> boolean </signature>
</MMLdefinition>
```

C.2.6.8 *prsubset*

```
<MMLdefinition>
<Name> prsubset </Name>
<description>
  Boolean function whose value is determined by
  whether or not one set is a proper subset of another.
 </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (set, set) -> boolean </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.6.9 *notsubset*

```
<MMLdefinition>
<Name> notsubset </Name>
<description>
  Boolean function whose value is the complement
  of "subset".
 </description>
    <functorclass> Binary, Function </functorclass>
    <signature> (set, set) -> boolean </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.6.10 *notprsubset*

```
<MMLdefinition>
<Name> notprsubset </Name>
<description>
  Boolean function whose value is the complement
  of "proper subset".
 </description>
```

```
    <functorclass> Binary, Function </functorclass>
    <signature> (set, set) -> boolean </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.6.11 *setdiff*

```
<MMLdefinition>
<Name> setdiff </Name>
<description>
  Function indicating the difference of two sets.
</description>
<functorclass> Binary, Function </functorclass>
<signature> (set, set) -> set </signature>
<property>...</property>
</MMLdefinition>
```

C.2.7 Sequences and Series

C.2.7.1 *sum*

```
<MMLdefinition>
<Name> sum </Name>
<description>
The sum element denotes the summation operator. Upper and lower
limits for the sum, and more generally a domains for the bound variables
are specified using uplimit, lowlimit or a condition on the bound
variables. The index for the summation is specified by a bvar element.
The sum element takes the attribute definition that can be used to
override the default semantics.
</description>
<functorclass> Unary, Function </functorclass>
<signature> (bvar*,((lowlimit,uplimit)|condition),algebraic) -> sum </signature>
<signature> ... </signature>
</MMLdefinition>
```

C.2.7.2 *product*

```
<MMLdefinition>
<Name> product </Name>
<description>
The product element denotes the product operator. Upper and lower
limits for the product, and more generally a domains for the bound
variables are specified using uplimit, lowlimit or a condition on the
bound variables. The index for the product is specified by a bvar
element.
The product element takes the attribute definition that can be used
to override the default semantics.
</description>
<functorclass> Unary, Function </functorclass>
```

```
<signature> (bvar*,((lowlimit,uplimit)|condition),algebraic)   -> product </signa
<signature> ... </signature>
<signature> ... </signature>
</MMLdefinition>
```

C.2.7.3 *limit*

```
<MMLdefinition>
<Name> limit </Name>
<description>
The sum element denotes the summation operator.
Upper and lower limits for the sum, and more
generally a domains for the bound variables are
specified using uplimit, lowlimit or a condition
on the bound variables. The index for the summation is
specified by a bvar element.
</description>
<functorclass> Nary, Function </functorclass>
<signature> (bvar*,(lowlimit | condition*),algebraic) -> limit </signature>
</MMLdefinition>
```

C.2.7.4 *tendsto*

```
<MMLdefinition>
<Name> tendsto </Name>
<description> tendsto is used to specify how a limit is
computed. It accepts a type attribute that determines the
manner in which it tends to a value.
</description>
<functorclass> binary, Function </functorclass>
<signature> (symbol,anything) -> condition(limit) </signature>
<signature> [type=direction](symbol,anything) ->   condition(limit) </signature>
</MMLdefinition>
```

C.2.8 Trigonometry

C.2.8.1 *sin*

```
<MMLdefinition>
    <Name> sin </Name>
    <description> The circular trigonometric function sine
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property> sin(0) = 0 </property>
    <property> sin(integer*Pi) = 0 </property>
```

```
    <property> sin((Z+1/2)*Pi) = (-1)^Z, "for integer Z" </property>
    <property> -1 <= sin(real) </property>
    <property> sin(real) <= 1 </property>
    <property> sin(3*x)=-4*sin(x)^3+3*sin(x), "triple angle formula"
       <Reference> ditto, [4.3.27] </Reference>
    </property>
 </MMLdefinition>
```

C.2.8.2 *cos*

```
<MMLdefinition>
    <Name> cos </Name>
    <description> The cosine function.
       <Reference> M. Abramowitz and I. Stegun, Handbook of
          Mathematical Functions, [4.3]
       </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property> cos(0) = 1 </property>
    <property> cos(integer*Pi+Pi/2) = 0 </property>
    <property> cos(Z*Pi) = (-1)^Z, "for integer Z" </property>
    <property> -1 <= cos(real) </property>
    <property> cos(real) <= 1 </property>
 </MMLdefinition>
```

C.2.8.3 *tan*

```
 <MMLdefinition>
    <Name> tan </Name>
    <description> The tangent function.
       <Reference> M. Abramowitz and I. Stegun, Handbook of
          Mathematical Functions, [4.3]
       </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property> tan(integer*Pi) = 0 </property>
    <property> tan(x) = sin(x)/cos(x) </property>
 </MMLdefinition>
```

C.2.8.4 *sec*

```
<MMLdefinition>
    <Name> sec </Name>
    <description> The secant function.
       <Reference> M. Abramowitz and I. Stegun, Handbook of
          Mathematical Functions, [4.3]
```

```
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> sec(x) = 1/cos(x) </property>
    </MMLdefinition>
```

C.2.8.5 *csc*

```
    <MMLdefinition>
        <Name> csc </Name>
        <description> The cosecant function.
            <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
                Mathematical Functions, [4.3]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> csc(x) = 1/sin(x) </property>
    </MMLdefinition>
```

C.2.8.6 *cot*

```
    <MMLdefinition>
        <Name> cot </Name>
        <description> The cotangent function.
            <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
                Mathematical Functions, [4.3]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> cot(integer*Pi+Pi/2) = 0 </property>
        <property> cot(x) = cos(x)/sin(x) </property>
    </MMLdefinition>
```

C.2.8.7 *sinh*

```
    <MMLdefinition>
        <Name> sinh </Name>
        <description> The hyperbolic sine function.
            <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
                Mathematical Functions, [4.3]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
```

```
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.8 *cosh*

```
<MMLdefinition>
    <Name> sinh </Name>
    <description> The hyperbolic sine function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.9 *tanh*

```
<MMLdefinition>
    <Name> tanh </Name>
    <description> The hyperbolic tangent function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.10 *sech*

```
<MMLdefinition>
    <Name> sech </Name>
    <description> The hyperbolic secant function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.11 *csch*

```
<MMLdefinition>
    <Name> csch </Name>
    <description> The hyperbolic cosecant function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.12 *coth*

```
<MMLdefinition>
    <Name> coth </Name>
    <description> The hyperbolic cotangent function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.3]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property>...</property>
</MMLdefinition>
```

C.2.8.13 *arcsin*

```
<MMLdefinition>
    <Name> arcsin </Name>
    <description> The inverse of the sine function.
        <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
            Mathematical Functions, [4.4]
        </Reference>
    </description>
    <functorclass> Unary, Function </functorclass>
    <signature> real -> real </signature>
    <signature> symbolic -> symbolic </signature>
    <property> sin(arcsin(x)) = x </property>
    <property> arcsin(sin(x)) = x, "for x between -Pi/2 and Pi/2" </property>
</MMLdefinition>
```

C.2.8.14 *arccos*

```
<MMLdefinition>
    <Name> arccos </Name>
```

```
        <description> The inverse of the cosine function.
            <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
                Mathematical Functions, [4.4]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> cos(arccos(x)) = x </property>
        <property> arccos(cos(x)) = x, "for x between 0 and Pi" </property>
    </MMLdefinition>
```

C.2.8.15 *arctan*

```
    <MMLdefinition>
        <Name> arctan </Name>
        <description> The inverse of the tangent function.
            <Reference> M.&amp;nbsp;Abramowitz and I.&amp;nbsp;Stegun, Handbook of
                Mathematical Functions, [4.4]
            </Reference>
        </description>
        <functorclass> Unary, Function </functorclass>
        <signature> real -> real </signature>
        <signature> symbolic -> symbolic </signature>
        <property> tan(arctan(x)) = x </property>
        <property> arctan(tan(x)) = x, "for x between -Pi/2 and Pi/2" </property>
    </MMLdefinition>
```

C.2.9 Statistics

C.2.9.1 *mean*

```
<MMLdefinition>
  <Name> mean </Name>
  <description>
    Given  k unspecified scalar arguments they are treated as equiprobable
    values of a random variable and the mean is computed as:
      mean( a1, a2, ...  an)   Sum( ai, i=1... n )/ n.
    (see section 7.7 in CRC's Standard Mathematical tables and Formulae).
    More generally, if the first argument is a symbol X of type
    "discrete_random_variable", this is the 1st moment of the
    random variable X and is defined as
    E[ X ] = Sum( x*f(x), x in S )
    where the probability that x = x_i is  P( x = x_i) = f(x_i) .
    The arguments are either all data, all discrete random variables,
    or all continuous random variables.
    The generalizes to continuous distributions and
    k dimenions following the definitions provided in the reference:
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [7.1.2] and [7.7]
```

```
    </Reference>
  </description>
  <MMLattribute>
    <name>type</name>
    <values> random_variable | continuous_random_variable | data </value>
    <default> data </default>
  </MMLattribute>
  <functorclass>Nary , Operator </functorclass>
  <signature>(scalar*) -> scalar</signature>
  <signature>(scalar(type=data)*) -> scalar</signature>
  <signature>(symbol(type=random_variable)*) -> scalar</signature>
  <signature>(symbol(type=continuous_random_variable)*) -> scalar</signature>
  <property> </property>
</MMLdefinition>
```

C.2.9.2 *sdev*

```
<MMLdefinition>
  <Name> sdev </Name>
  <description>
    This represents the standard deviation.
    Given  k unspecified scalar arguments they are treated as equiprobable
    values of a random variable and the "standard deviation" is
    computed as the square root of the second moment about the mean  U.
      sdev( a1, a2, ...  an)^2  = E( (X - U)^2 ).
    If the first argument is a symbol X of type
    "discrete_random_variable", then all arguments are treated as
    discrete random variables, instead of data and the second moment
    about the mean is computed as
      Sum( ( x_i - U )^2 * f(x_i) , x_i  in S )
    as
    where the probability that x = x_i is  P( x = x_i) = f(x_i) .
    The arguments are either all data, all discrete random variables,
    or all continuous random variables.
    The generalizes to continuous distributions and to
    k dimenions following the definitions found in:
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [7.1.2] and [7.7]
    </Reference>
  </description>
  <MMLattribute>
    <name>type</name>
    <values> random_variable | continuous_random_variable | data </value>
    <default> data </default>
  </MMLattribute>
  <functorclass>Nary , Operator </functorclass>
  <signature>(scalar*) -> scalar</signature>
  <signature>(scalar(type=data)*) -> scalar</signature>
  <signature>(symbol(type=discrete_random_variable)*) -> scalar</signature>
  <signature>(symbol(type=continuous_random_variable)*) -> scalar</signature>
```

```
  <property> </property>
</MMLdefinition>
```

C.2.9.3 *variance*

```
<MMLdefinition>
  <Name> variance </Name>
  <description>
    This computes the second centered moment, also known as the variance.
    Given  k unspecified scalar arguments they are treated as equiprobable
    values of a random variable and the "variance" is
    computed as the second moment about the mean  U.
      variance( a1, a2, ...  an)  = E( (X - U)^2 ).
    If the first argument is a symbol X of type
    "discrete_random_variable", then all arguments are treated as
    discrete random variables, instead of data and the second moment
    about the mean is computed as  in section [7.7] (see reference below.)
      Sum( ( x_i - U )^2 * f(x_i) , x_i  in S )
    as
    where the probability that x = x_i is  P( x = x_i) = f(x_i) .
    The arguments are either all data, all discrete random variables,
    or all continuous random variables.
    The generalizes to continuous distributions and to
    k dimenions following the definitions found in:
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [7.1.2] and [7.7]
    </Reference>
  </description>
  <MMLattribute>
    <name>type</name>
    <values> random_variable | continuous_random_variable | data </value>
    <default> data </default>
  </MMLattribute>
  <functorclass>Nary , Operator </functorclass>
  <signature>(scalar*) -> scalar</signature>
  <signature>(scalar(type=data)*) -> scalar</signature>
  <signature>(symbol(type=discrete_random_variable)*) -> scalar</signature>
  <signature>(symbol(type=continuous_random_variable)*) -> scalar</signature>
</MMLdefinition>
```

C.2.9.4 *median*

```
<MMLdefinition>
  <Name> median </Name>
  <description>
    This represents the median of n data values.
    If n =2k + 1  then the mode is x_k.
    If n = 2k then the median is (x_k + x_(k+1)/2).
    (Note this discription assumes that the data has been
     sorted into ascending order.)
```

```
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996,  [7.7]
    </Reference>
  </description>
  <functorclass>Nary , Operator</functorclass>
  <signature>(scalar*) -> scalar</signature>
</MMLdefinition>
```

C.2.9.5 *mode*

```
<MMLdefinition>
  <Name> mode </Name>
  <description>
    This represents the mode of  n data values.
    The mode is the data value that occurs with the
    greatest frequency.
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996,  [7.7]
    </Reference>
  </description>
  <functorclass>Nary , Operator</functorclass>
  <signature>(scalar*) -> scalar</signature>
</MMLdefinition>
```

C.2.9.6 *moment*

```
<MMLdefinition>
  <Name> moment </Name>
  <description>
    This computes the ith moment of a set of data, or a random variable..
    Given  k  scalar arguments of unspecified type, they are treated
    as equiprobable  values of a random variable. and the "moments" are
    computed as the second moment about the mean  U.
      moment( degree=i, scalar*)= E( X^i ).
    If the first data argument x1 is a symbol X of type
    "discrete_random_variable", then all arguments are treated as
    discrete random variables, instead of data and the ith moment
    about the mean is computed as
      Sum( (x)^i * f(x) , x in S )
where the probability that x = x_i is P( x = x_i) = f(x_i) .
    The arguments are either all data, all discrete random variables,
    or all continuous random variables.
    The generalizes to continuous distributions and to
    k dimenions following the definitions found in:
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [7.1.2]
    </Reference>
  </description>
  <MMLattribute>
    <name>type</name>
```

```
    <values> random_variable | continuous_random_variable | data </value>
    <default> data </default>
  </MMLattribute>
  <functorclass>Nary , Operator </functorclass>
  <signature>(degree,scalar*) -> scalar</signature>
  <signature>(degree,scalar(type=data)*) -> scalar</signature>
  <signature>(degree,symbol(type=discrete_random_variable)*) -> scalar</signature
  <signature>(degree, symbol(type=continuous_random_variable)*) -> scalar</signat
</MMLdefinition>
```

C.2.10 Lineary Algebra

C.2.10.1 *vector*

```
<MMLdefinition>
  <Name> vector </Name>
  <description>
    A vector is an ordered n-tuple of values
    representing an element of an n-dimensional
    vector space.  The "values" are all from the
    same ring, typically real or complex.  They may
    be numbers, symbols, or general algebraic expressions.
    The type attribute can be used to specify the type of
    vector that is represented.
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [2.4]
    </Reference>
  </description>
  <MMLattribute>
    <name> type </name>
    <value>  real | complex | symbolic | anything </value>
    <default> real </default>
  </MMLattribute>
  <MMLattribute>
    <name> other </name>
    <value>  row | column </value>
    <default> row </default>
  </MMLattribute>
  <functorclass> constructor , N-ary </functorclass>
  <signature>
    ((cn|ci|apply)*) -> vector(type=real)
  </signature>
  <signature>
    [type=vectortype]((cn|ci|apply)*) -> vector(type=vectortype)
  </signature>
  <!-- Note that there is a notational need for expressing a sequence
       v1, v2, ... vn with an in-explicit value of n .  Also, in the
       following property, it should be clarified that b,v1, and v2 are all
       elements of the same ring. -->
  <property>  <!-- scalar multiplication-->
```

```
      <apply><forall/>
        <bvar><ci>b</ci></bvar>
        <bvar><ci>v1</ci></bvar>
        <bvar><ci>v2</ci></bvar>
        <reln>
          <apply><times/>
            <ci>ci>b</ci>
            <vector><ci>ci>v1</ci><ci>ci>v2</ci></vector>
            </apply>
            <vector>
              <apply><ci>b</ci><ci>v1</ci></apply>
              <apply><ci>b</ci><ci>v2</ci></apply>
            </vector>
        </reln>
      </apply>
  </property>
  <property> vector addition </property>
  <property> distributive over scalars</property>
  <property> associativity.</property>
  <property> Matrix * column vector </property>
  <property> row vector * Matrix </property>
  </property>
</MMLdefinition>
```

C.2.10.2 *matrix*

```
<MMLdefinition>
  <Name> matrix </Name>
  <description>
    This is the constructor for a matrix.  The matrix is
    constructed from matrix rows.  The type and properties
    spell out the normal interaction with vectors and
    scalars.
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [2.5.1]
    </Reference>
  </description>
  <MMLattribute>
    <name>type</name>
    <value>real | complex | integer | symbolic | anything </value>
    <default> real </default>
  </MMLattribute>
  <functorclass>constructor , N-ary </functorclass>
  <signature>(matrixrow*) -> matrix</signature>
  <signature>
    [type=matrixtype](matrixrow*) ->
       matrix(type=matrixtype)</signature>
  <property>scalar multiplication </property>
  <property>Matrix*column vector</property>
  <property>Addition</property>
```

```
  <property>Matrix*Matrix</property>
</MMLdefinition>
```

C.2.10.3 *matrixrow*

```
<MMLdefinition>
  <Name> matrixrow </Name>
  <description>
    This is a constructor for describing the rows of a matrix.
    This only occurs inside a matrix.  Its "type" is determined
    from the containing matrix element.
  </description>
  <functorclass>constructor , N-ary</functorclass>
  <signature>(cn|ci|apply)->matrixrow </signature>
</MMLdefinition>
```

C.2.10.4 *determinant*

```
<MMLdefinition>
  <Name>determinant</Name>
  <description>The "determinant" of a matrix.
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [2.5.4]
    </Reference>
  </description>
  <functorclass>Unary, operator</functorclass>
  <signature>(matrix)-> scalar </signature>
  </MMLdefinition>
```

C.2.10.5 *transpose*

```
<MMLdefinition>
  <Name> transpose </Name>
  <description>The transpose of a matrix or vector.
    <Reference> CRC Standard Mathematical Tables and Formulae,
      editor: Dan Zwillinger, CRC Press Inc., 1996, [2.4] and [2.5.1]
    </Reference>
  </description>
  <functorclass>Unary, Operator</functorclass>
  <signature>(vector)->vector(other=row)</signature>
  <signature>[other=column](vector)->vector(other=row)</signature>
  <signature>[other=row](vector)->vector(other=column)</signature>
  <signature>(matrix)->matrix</signature>
  <property>transpose(transpose(A))= A</property>
  <property>transpose(transpose(V))= V</property>
</MMLdefinition>
```

C.2.10.6 *selector*

```
<MMLdefinition>
```

```
  <Name> selector </Name>
  <description>
    The operator used to extract sub-objects from vectors, matrices
    matrix rows and lists.
    Elements are accessed by providing one index element for each
    dimension.  For Matrices, sub-matrices are selected by providing
    one fewer index items.  For a matrix A and a column vector V :
     select( i,j , A ) is the  i,j th element of A.
     select(i , A )  is the matrixrow formed from the ith row of A.
     select( i , V ) is the ith element of V.
     select( V ) is the sequence of all elements of V.
     select(A) is the sequence of all elements of A, extracted row
     by row.
     select(i,L) is the ith element of a list.
     select(L) is the sequence of elements of a list.
  </description>
  <functorclass>N-ary, operator)</functorclass>
  <signature>(scalar,scalar,matrix)->scalar</signature>
  <signature>(scalar,matrix)->matrixrow</signature>
  <signature>(matrix)->scalar* </property>
  <signature>(scalar,(vector|list|matrixrow))->scalar</signature>
  <signature>(vector|list|matrixrow)->scalar*</signature>
  <property>
    Forall(
       bvar(A(type=matrix)),bvar(V(type=vector)),
       select(A) = select(V)
     )
  </property>
  <property>For all vectors V, V = vector(select(V))</property>
</MMLdefinition>
```

Appendix D

Operator Dictionary (Non-Normative)

The following table gives the suggested dictionary of rendering properties for operators, fences, separators, and accents in MathML, all of which are represented by `mo` elements. For brevity, all such elements will be called simply ‘operators’ in this Appendix.

D.1 Format of operator dictionary entries

The operators are divided into groups, which are separated by blank lines in the listing below. The grouping, and the order of the groups, is significant for the proper grouping of sub-expressions using `<mrow>` (section 3.3.1); the rule described there is especially relevant to the automatic generation of MathML by conversion from other formats for displayed mathematics, such as TEX, which do not always specify how sub-expressions nest.

The format of the table entries is: the `<mo>` element content between double quotes (start and end tags not shown), followed by the attribute list in XML format, starting with the `form` attribute, followed by the default rendering attributes which should be used for `mo` elements with the given content and `form` attribute.

Any attribute not listed for some entry has its default value, which is given in parentheses in the table of attributes in section 3.2.4.

Note that the characters & and < are represented in the following table entries by the entity references `&amp;` and `&lt;` respectively, as would be necessary if they appeared in the content of an actual `mo` element (or any other MathML or XML element).

For example, the first entry,

```
"(" form="prefix" fence="true" stretchy="true" lspace="0em" rspace="0em"
```

could be expressed as an `mo` element by:

```
<mo form="prefix" fence="true" stretchy="true" lspace="0em" rspace="0em"> ( </mo>
```

(note the lack of double quotes around the content, and the whitespace added around the content for readability, which is optional in MathML).

This entry means that, for MathML renderers which use this suggested operator dictionary, giving the element `<mo form="prefix"> ( </mo>` alone, or simply `<mo> ( </mo>` in a position for which `form="prefix"` would be inferred (see below), is equivalent to giving the element with all attributes as shown above.

D.2 Indexing of operator dictionary

Note that the dictionary is indexed not just by the element content, but by the element content and `form` attribute value, together. Operators with more than one possible form have more than one entry. The MathML specification describes how the renderer chooses ('infers') which form to use when no `form` attribute is given; see section 3.2.4.7.

Having made that choice, or with the `form` attribute explicitly specified in the `<mo>` element's start tag, the MathML renderer uses the remaining attributes from the dictionary entry for the appropriate single form of that operator, ignoring the entries for the other possible forms.

D.3 Choice of entity names

Extended characters in MathML (and in the operator dictionary below) are represented by XML-style entity references using the syntax `&character-name;`. The complete list of characters and character names is given in chapter 6. Many characters can be referred to by more than one name; often, memorable names composed of full words have been provided in MathML, as well as one or more names used in other standards, such as Unicode. The characters in the operators in this dictionary are generally listed under their full-word names when these exist. For example, the integral operator is named below by the one-character sequence `&Integral;`, but could equally well be named `&int;`. The choice of name for a given character in MathML has no effect on its rendering.

It is intended that every entity named below appears somewhere in chapter 6. If this is not true, it is an error in this specification. If such an error exists, the abovementioned chapter should be taken as definitive, rather than this appendix.

D.4 Notes on `lspace` and `rspace` attributes

The values for `lspace` and `rspace` given here range from 0 to `verythickmathspace`, which has a default value of 6/18 em. For the invisible operators whose content is `&InvisibleTimes;` or `&ApplyFunction;`, it is suggested that MathML renderers choose spacing in a context-sensitive way (which is an exception to the static values given in the following table). For `<mo>&ApplyFunction;</mo>`, the total spacing (`lspace`+`rspace`) in expressions such as 'sin *x*' (where the right operand doesn't start with a fence) should be greater than zero; for `<mo>&InvisibleTimes;</mo>`, the total spacing should be greater than zero when both operands (or the nearest tokens on either side, if on the baseline) are identifiers displayed in a non-slanted font (i.e. under the suggested rules, when both operands are multi-character identifiers).

Some renderers may wish to use no spacing for most operators appearing in scripts (i.e. when `scriptlevel` is greater than 0; see section 3.3.4), as is the case in TEX.

D.5 Operator dictionary entries

```
"("                              form="prefix"  fence="true" stretchy="true"  lspa
```

```
")"                          form="postfix" fence="true" stretchy="true"  lspa
"["                          form="prefix"  fence="true" stretchy="true"  lspa
"]"                          form="postfix" fence="true" stretchy="true"  lspa
"{"                          form="prefix"  fence="true" stretchy="true"  lspa
"}"                          form="postfix" fence="true" stretchy="true"  lspa
"&CloseCurlyDoubleQuote;"       form="postfix" fence="true"  lspace="0em" rspa
"&CloseCurlyQuote;"            form="postfix" fence="true"  lspace="0em" rspac
"&LeftAngleBracket;"            form="prefix"  fence="true" stretchy="true"  ls
"&LeftBracketingBar;"           form="prefix"  fence="true" stretchy="true"  ls
"&LeftCeiling;"                form="prefix"  fence="true" stretchy="true"  ls
"&LeftDoubleBracket;"           form="prefix"  fence="true" stretchy="true"  ls
"&LeftDoubleBracketingBar;"      form="prefix"  fence="true" stretchy="true"
"&LeftFloor;"                 form="prefix"  fence="true" stretchy="true"  lsp
"&OpenCurlyDoubleQuote;"        form="prefix"  fence="true"  lspace="0em" rspa
"&OpenCurlyQuote;"             form="prefix"  fence="true"  lspace="0em" rspac
"&RightAngleBracket;"           form="postfix" fence="true" stretchy="true"  l
"&RightBracketingBar;"          form="postfix" fence="true" stretchy="true"  l
"&RightCeiling;"               form="postfix" fence="true" stretchy="true"  ls
"&RightDoubleBracket;"          form="postfix" fence="true" stretchy="true"  l
"&RightDoubleBracketingBar;"     form="postfix" fence="true" stretchy="true"
"&RightFloor;"                form="postfix" fence="true" stretchy="true"  ls
"&LeftSkeleton;"               form="prefix"  fence="true"  lspace="0em" rspac
"&RightSkeleton;"              form="postfix" fence="true"  lspace="0em" rspac
"&InvisibleComma;"              form="infix"   separator="true"  lspace="0em" r
","                           form="infix"   separator="true"  lspace="0em" rsp
"&HorizontalLine;"              form="infix"   stretchy="true" minsize="0"  lsp
"&VerticalLine;"               form="infix"   stretchy="true" minsize="0"  lsp
";"                           form="infix"   separator="true"  lspace="0em" rsp
";"                           form="postfix" separator="true"  lspace="0em" rsp
":="                          form="infix"   lspace="thickmathspace" rspace="t
"&Assign;"                     form="infix"   lspace="thickmathspace" rspace="
"&Because;"                    form="infix"   lspace="thickmathspace" rspace="
"&Therefore;"                  form="infix"   lspace="thickmathspace" rspace="
"&VerticalSeparator;"           form="infix"   stretchy="true"  lspace="thickma
"//"                          form="infix"   lspace="thickmathspace" rspace="t
"&Colon;"                      form="infix"   lspace="thickmathspace" rspace="
"&"                         form="prefix"  lspace="0em" rspace="thickmathspace"
"&"                         form="postfix" lspace="thickmathspace" rspace="0em"
"*="                          form="infix"   lspace="thickmathspace" rspace="t
"-="                          form="infix"   lspace="thickmathspace" rspace="t
"+="                          form="infix"   lspace="thickmathspace" rspace="t
"/="                          form="infix"   lspace="thickmathspace" rspace="t
"->"                          form="infix"   lspace="thickmathspace" rspace="t
":"                           form="infix"   lspace="thickmathspace" rspace="t
".."                          form="postfix" lspace="mediummathspace" rspace="
"..."                         form="postfix" lspace="mediummathspace" rspace="
"&SuchThat;"                   form="infix"   lspace="thickmathspace" rspace="
"&DoubleLeftTee;"               form="infix"   lspace="thickmathspace" rspace=
"&DoubleRightTee;"              form="infix"   lspace="thickmathspace" rspace=
"&DownTee;"                    form="infix"   lspace="thickmathspace" rspace="
```

```
"&LeftTee;"                       form="infix"    lspace="thickmathspace" rspace="
"&RightTee;"                      form="infix"    lspace="thickmathspace" rspace="
"&Implies;"                       form="infix"   stretchy="true"  lspace="thickmat
"&RoundImplies;"                   form="infix"    lspace="thickmathspace" rspace=
"|"                              form="infix"   stretchy="true"  lspace="thickmath
"||"                             form="infix"    lspace="mediummathspace" rspace="
"&Or;"                            form="infix"   stretchy="true"  lspace="mediummat
"&&"                        form="infix"    lspace="thickmathspace" rspace="thickmat
"&And;"                            form="infix"   stretchy="true"  lspace="mediumma
"&"                           form="infix"    lspace="thickmathspace" rspace="thick
"!"                              form="prefix"   lspace="0em" rspace="thickmathspa
"&Not;"                           form="prefix"   lspace="0em" rspace="thickmathsp
"&Exists;"                         form="prefix"   lspace="0em" rspace="thickmathsp
"&ForAll;"                         form="prefix"   lspace="0em" rspace="thickmathsp
"&NotExists;"                      form="prefix"   lspace="0em" rspace="thickmaths
"&Element;"                        form="infix"    lspace="thickmathspace" rspace="
"&NotElement;"                     form="infix"    lspace="thickmathspace" rspace=
"&NotReverseElement;"               form="infix"    lspace="thickmathspace" rspace
"&NotSquareSubset;"                form="infix"    lspace="thickmathspace" rspace=
"&NotSquareSubsetEqual;"            form="infix"    lspace="thickmathspace" rspace
"&NotSquareSuperset;"              form="infix"    lspace="thickmathspace" rspace=
"&NotSquareSupersetEqual;"          form="infix"    lspace="thickmathspace" rspace
"&NotSubset;"                     form="infix"    lspace="thickmathspace" rspace="
"&NotSubsetEqual;"                 form="infix"    lspace="thickmathspace" rspace=
"&NotSuperset;"                   form="infix"    lspace="thickmathspace" rspace=
"&NotSupersetEqual;"               form="infix"    lspace="thickmathspace" rspace=
"&ReverseElement;"                 form="infix"    lspace="thickmathspace" rspace=
"&SquareSubset;"                   form="infix"    lspace="thickmathspace" rspace=
"&SquareSubsetEqual;"              form="infix"    lspace="thickmathspace" rspace=
"&SquareSuperset;"                 form="infix"    lspace="thickmathspace" rspace=
"&SquareSupersetEqual;"             form="infix"    lspace="thickmathspace" rspace
"&Subset;"                        form="infix"    lspace="thickmathspace" rspace="
"&SubsetEqual;"                    form="infix"    lspace="thickmathspace" rspace=
"&Superset;"                      form="infix"    lspace="thickmathspace" rspace="
"&SupersetEqual;"                  form="infix"    lspace="thickmathspace" rspace=
"&DoubleLeftArrow;"                form="infix"   stretchy="true"  lspace="thickma
"&DoubleLeftRightArrow;"            form="infix"   stretchy="true"  lspace="thickm
"&DoubleRightArrow;"               form="infix"   stretchy="true"  lspace="thickma
"&DownLeftRightVector;"             form="infix"   stretchy="true"  lspace="thickm
"&DownLeftTeeVector;"              form="infix"   stretchy="true"  lspace="thickma
"&DownLeftVector;"                 form="infix"   stretchy="true"  lspace="thickma
"&DownLeftVectorBar;"              form="infix"   stretchy="true"  lspace="thickma
"&DownRightTeeVector;"             form="infix"   stretchy="true"  lspace="thickm
"&DownRightVector;"                form="infix"   stretchy="true"  lspace="thickma
"&DownRightVectorBar;"             form="infix"   stretchy="true"  lspace="thickm
"&LeftArrow;"                     form="infix"   stretchy="true"  lspace="thickmat
"&LeftArrowBar;"                   form="infix"   stretchy="true"  lspace="thickma
"&LeftArrowRightArrow;"             form="infix"   stretchy="true"  lspace="thickm
"&LeftRightArrow;"                 form="infix"   stretchy="true"  lspace="thickma
"&LeftRightVector;"                form="infix"   stretchy="true"  lspace="thickma
```

```
"&LeftTeeArrow;"                form="infix"   stretchy="true"  lspace="thickma
"&LeftTeeVector;"               form="infix"   stretchy="true"  lspace="thickma
"&LeftVector;"                 form="infix"   stretchy="true"  lspace="thickma
"&LeftVectorBar;"               form="infix"   stretchy="true"  lspace="thickma
"&LowerLeftArrow;"              form="infix"   stretchy="true"  lspace="thickma
"&LowerRightArrow;"             form="infix"   stretchy="true"  lspace="thickma
"&RightArrow;"                 form="infix"   stretchy="true"  lspace="thickma
"&RightArrowBar;"               form="infix"   stretchy="true"  lspace="thickma
"&RightArrowLeftArrow;"          form="infix"   stretchy="true"  lspace="thickm
"&RightTeeArrow;"               form="infix"   stretchy="true"  lspace="thickma
"&RightTeeVector;"              form="infix"   stretchy="true"  lspace="thickma
"&RightVector;"                form="infix"   stretchy="true"  lspace="thickma
"&RightVectorBar;"              form="infix"   stretchy="true"  lspace="thickma
"&ShortLeftArrow;"              form="infix"    lspace="thickmathspace" rspace=
"&ShortRightArrow;"             form="infix"    lspace="thickmathspace" rspace=
"&UpperLeftArrow;"              form="infix"   stretchy="true"  lspace="thickma
"&UpperRightArrow;"             form="infix"   stretchy="true"  lspace="thickma
"="                           form="infix"    lspace="thickmathspace" rspace="t
"<"                         form="infix"    lspace="thickmathspace" rspace="thic
">"                           form="infix"    lspace="thickmathspace" rspace="t
"!="                          form="infix"    lspace="thickmathspace" rspace="t
"=="                          form="infix"    lspace="thickmathspace" rspace="t
"<="                        form="infix"    lspace="thickmathspace" rspace="thic
">="                          form="infix"    lspace="thickmathspace" rspace="t
"&Congruent;"                  form="infix"    lspace="thickmathspace" rspace="
"&CupCap;"                     form="infix"    lspace="thickmathspace" rspace="
"&DotEqual;"                   form="infix"    lspace="thickmathspace" rspace="
"&DoubleVerticalBar;"           form="infix"    lspace="thickmathspace" rspace=
"&Equal;"                     form="infix"    lspace="thickmathspace" rspace="
"&EqualTilde;"                 form="infix"    lspace="thickmathspace" rspace="
"&Equilibrium;"                form="infix"   stretchy="true"  lspace="thickma
"&GreaterEqual;"               form="infix"    lspace="thickmathspace" rspace=
"&GreaterEqualLess;"            form="infix"    lspace="thickmathspace" rspace=
"&GreaterFullEqual;"            form="infix"    lspace="thickmathspace" rspace=
"&GreaterGreater;"              form="infix"    lspace="thickmathspace" rspace=
"&GreaterLess;"                form="infix"    lspace="thickmathspace" rspace=
"&GreaterSlantEqual;"           form="infix"    lspace="thickmathspace" rspace=
"&GreaterTilde;"               form="infix"    lspace="thickmathspace" rspace=
"&HumpDownHump;"               form="infix"    lspace="thickmathspace" rspace=
"&HumpEqual;"                 form="infix"    lspace="thickmathspace" rspace="
"&LeftTriangle;"               form="infix"    lspace="thickmathspace" rspace=
"&LeftTriangleBar;"             form="infix"    lspace="thickmathspace" rspace=
"&LeftTriangleEqual;"           form="infix"    lspace="thickmathspace" rspace=
"&le;"                        form="infix"    lspace="thickmathspace" rspace="t
"&LessEqualGreater;"            form="infix"    lspace="thickmathspace" rspace=
"&LessFullEqual;"              form="infix"    lspace="thickmathspace" rspace=
"&LessGreater;"                form="infix"    lspace="thickmathspace" rspace=
"&LessLess;"                  form="infix"    lspace="thickmathspace" rspace="
"&LessSlantEqual;"             form="infix"    lspace="thickmathspace" rspace=
"&LessTilde;"                  form="infix"    lspace="thickmathspace" rspace="
```

```
"&NestedGreaterGreater;"          form="infix"   lspace="thickmathspace" rspace
"&NestedLessLess;"                form="infix"   lspace="thickmathspace" rspace=
"&NotCongruent;"                  form="infix"   lspace="thickmathspace" rspace=
"&NotCupCap;"                     form="infix"   lspace="thickmathspace" rspace="
"&NotDoubleVerticalBar;"          form="infix"   lspace="thickmathspace" rspace
"&NotEqual;"                      form="infix"   lspace="thickmathspace" rspace="
"&NotEqualTilde;"                 form="infix"   lspace="thickmathspace" rspace=
"&NotGreater;"                    form="infix"   lspace="thickmathspace" rspace="
"&NotGreaterEqual;"               form="infix"   lspace="thickmathspace" rspace=
"&NotGreaterFullEqual;"           form="infix"   lspace="thickmathspace" rspace
"&NotGreaterGreater;"             form="infix"   lspace="thickmathspace" rspace=
"&NotGreaterLess;"                form="infix"   lspace="thickmathspace" rspace=
"&NotGreaterSlantEqual;"          form="infix"   lspace="thickmathspace" rspace
"&NotGreaterTilde;"               form="infix"   lspace="thickmathspace" rspace=
"&NotHumpDownHump;"               form="infix"   lspace="thickmathspace" rspace=
"&NotHumpEqual;"                  form="infix"   lspace="thickmathspace" rspace=
"&NotLeftTriangle;"               form="infix"   lspace="thickmathspace" rspace=
"&NotLeftTriangleBar;"            form="infix"   lspace="thickmathspace" rspace
"&NotLeftTriangleEqual;"          form="infix"   lspace="thickmathspace" rspace
"&NotLess;"                       form="infix"   lspace="thickmathspace" rspace="
"&NotLessEqual;"                  form="infix"   lspace="thickmathspace" rspace=
"&NotLessFullEqual;"              form="infix"   lspace="thickmathspace" rspace=
"&NotLessGreater;"                form="infix"   lspace="thickmathspace" rspace=
"&NotLessLess;"                   form="infix"   lspace="thickmathspace" rspace=
"&NotLessSlantEqual;"             form="infix"   lspace="thickmathspace" rspace=
"&NotLessTilde;"                  form="infix"   lspace="thickmathspace" rspace=
"&NotNestedGreaterGreater;"       form="infix"   lspace="thickmathspace" rspac
"&NotNestedLessLess;"             form="infix"   lspace="thickmathspace" rspace=
"&NotPrecedes;"                   form="infix"   lspace="thickmathspace" rspace=
"&NotPrecedesEqual;"              form="infix"   lspace="thickmathspace" rspace=
"&NotPrecedesSlantEqual;"         form="infix"   lspace="thickmathspace" rspace
"&NotPrecedesTilde;"              form="infix"   lspace="thickmathspace" rspace=
"&NotRightTriangle;"              form="infix"   lspace="thickmathspace" rspace=
"&NotRightTriangleBar;"           form="infix"   lspace="thickmathspace" rspace
"&NotRightTriangleEqual;"         form="infix"   lspace="thickmathspace" rspace
"&NotSucceeds;"                   form="infix"   lspace="thickmathspace" rspace=
"&NotSucceedsEqual;"              form="infix"   lspace="thickmathspace" rspace=
"&NotSucceedsSlantEqual;"         form="infix"   lspace="thickmathspace" rspace
"&NotSucceedsTilde;"              form="infix"   lspace="thickmathspace" rspace=
"&NotTilde;"                      form="infix"   lspace="thickmathspace" rspace="
"&NotTildeEqual;"                 form="infix"   lspace="thickmathspace" rspace=
"&NotTildeFullEqual;"             form="infix"   lspace="thickmathspace" rspace=
"&NotTildeTilde;"                 form="infix"   lspace="thickmathspace" rspace=
"&NotVerticalBar;"                form="infix"   lspace="thickmathspace" rspace=
"&Precedes;"                      form="infix"   lspace="thickmathspace" rspace="
"&PrecedesEqual;"                 form="infix"   lspace="thickmathspace" rspace=
"&PrecedesSlantEqual;"            form="infix"   lspace="thickmathspace" rspace
"&PrecedesTilde;"                 form="infix"   lspace="thickmathspace" rspace=
"&Proportion;"                    form="infix"   lspace="thickmathspace" rspace=
"&Proportional;"                  form="infix"   lspace="thickmathspace" rspace=
```

```
"&ReverseEquilibrium;"            form="infix"   stretchy="true"  lspace="thickm
"&RightTriangle;"                 form="infix"    lspace="thickmathspace" rspace=
"&RightTriangleBar;"              form="infix"    lspace="thickmathspace" rspace=
"&RightTriangleEqual;"            form="infix"    lspace="thickmathspace" rspace
"&Succeeds;"                     form="infix"    lspace="thickmathspace" rspace="
"&SucceedsEqual;"                 form="infix"    lspace="thickmathspace" rspace=
"&SucceedsSlantEqual;"            form="infix"    lspace="thickmathspace" rspace
"&SucceedsTilde;"                 form="infix"    lspace="thickmathspace" rspace=
"&Tilde;"                        form="infix"    lspace="thickmathspace" rspace="
"&TildeEqual;"                    form="infix"    lspace="thickmathspace" rspace=
"&TildeFullEqual;"                form="infix"    lspace="thickmathspace" rspace=
"&TildeTilde;"                    form="infix"    lspace="thickmathspace" rspace=
"&UpTee;"                        form="infix"    lspace="thickmathspace" rspace="
"&VerticalBar;"                   form="infix"    lspace="thickmathspace" rspace=
"&SquareUnion;"                   form="infix"   stretchy="true"  lspace="mediumm
"&Union;"                        form="infix"   stretchy="true"  lspace="mediumma
"&UnionPlus;"                     form="infix"   stretchy="true"  lspace="mediumma
"-"                              form="infix"    lspace="mediummathspace" rspace="
"+"                              form="infix"    lspace="mediummathspace" rspace="
"&Intersection;"                  form="infix"   stretchy="true"  lspace="mediumm
"&MinusPlus;"                     form="infix"    lspace="mediummathspace" rspace=
"&PlusMinus;"                     form="infix"    lspace="mediummathspace" rspace=
"&SquareIntersection;"            form="infix"   stretchy="true"  lspace="medium
"&Vee;"                          form="prefix"  largeop="true" movablelimits="tru
"&CircleMinus;"                   form="prefix"  largeop="true" movablelimits="tr
"&CirclePlus;"                    form="prefix"  largeop="true" movablelimits="tr
"&Sum;"                          form="prefix"  largeop="true" movablelimits="tru
"&Union;"                         form="prefix"  largeop="true" movablelimits="tru
"&UnionPlus;"                     form="prefix"  largeop="true" movablelimits="tr
"lim"                            form="prefix"  movablelimits="true"  lspace="0em"
"max"                            form="prefix"  movablelimits="true"  lspace="0em"
"min"                            form="prefix"  movablelimits="true"  lspace="0em"
"&CircleMinus;"                   form="infix"    lspace="thinmathspace" rspace="
"&CirclePlus;"                    form="infix"    lspace="thinmathspace" rspace="
"&ClockwiseContourIntegral;"       form="prefix"  largeop="true" stretchy="true"
"&ContourIntegral;"               form="prefix"  largeop="true" stretchy="true"
"&CounterClockwiseContourIntegral;"  form="prefix"  largeop="true" stretchy="true
"&DoubleContourIntegral;"          form="prefix"  largeop="true" stretchy="true"
"&Integral;"                      form="prefix"  largeop="true" stretchy="true"  l
"&Cup;"                          form="infix"    lspace="thinmathspace" rspace="th
"&Cap;"                          form="infix"    lspace="thinmathspace" rspace="th
"&VerticalTilde;"                 form="infix"    lspace="thinmathspace" rspace="
"&Wedge;"                        form="prefix"  largeop="true" movablelimits="tru
"&CircleTimes;"                   form="prefix"  largeop="true" movablelimits="tr
"&Coproduct;"                     form="prefix"  largeop="true" movablelimits="tr
"&Product;"                       form="prefix"  largeop="true" movablelimits="tru
"&Intersection;"                  form="prefix"  largeop="true" movablelimits="tr
"&Coproduct;"                     form="infix"    lspace="thinmathspace" rspace="t
"&Star;"                         form="infix"    lspace="thinmathspace" rspace="th
"&CircleDot;"                     form="prefix"  largeop="true" movablelimits="tr
```

```
"*"                                form="infix"    lspace="thinmathspace" rspace="th
"&InvisibleTimes;"                   form="infix"    lspace="0em" rspace="0em"
"&CenterDot;"                       form="infix"    lspace="thinmathspace" rspace="t
"&CircleTimes;"                      form="infix"    lspace="thinmathspace" rspace="
"&Vee;"                            form="infix"    lspace="thinmathspace" rspace="th
"&Wedge;"                           form="infix"    lspace="thinmathspace" rspace="t
"&Diamond;"                         form="infix"    lspace="thinmathspace" rspace="t
"&Backslash;"                        form="infix"   stretchy="true"  lspace="thinmath
"/"                                form="infix"   stretchy="true"  lspace="thinmaths
"-"                                form="prefix"   lspace="0em" rspace="veryverythin
"+"                                form="prefix"   lspace="0em" rspace="veryverythin
"&MinusPlus;"                        form="prefix"   lspace="0em" rspace="veryveryth
"&PlusMinus;"                        form="prefix"   lspace="0em" rspace="veryveryth
"."                                form="infix"    lspace="0em" rspace="0em"
"&Cross;"                           form="infix"    lspace="verythinmathspace" rspac
"**"                               form="infix"    lspace="verythinmathspace" rspace
"&CircleDot;"                        form="infix"    lspace="verythinmathspace" rspac
"&SmallCircle;"                      form="infix"    lspace="verythinmathspace" rspa
"&Square;"                          form="prefix"   lspace="0em" rspace="verythinmat
"&Del;"                            form="prefix"   lspace="0em" rspace="verythinmat
"&PartialD;"                        form="prefix"   lspace="0em" rspace="verythinmat
"&CapitalDifferentialD;"              form="prefix"   lspace="0em" rspace="verythinm
"&DifferentialD;"                    form="prefix"   lspace="0em" rspace="verythinma
"&Sqrt;"                           form="prefix"  stretchy="true"  lspace="0em" rspa
"&DoubleDownArrow;"                  form="infix"   stretchy="true"  lspace="verythi
"&DoubleLongLeftArrow;"               form="infix"   stretchy="true"  lspace="veryth
"&DoubleLongLeftRightArrow;"           form="infix"   stretchy="true"  lspace="veryt
"&DoubleLongRightArrow;"              form="infix"   stretchy="true"  lspace="veryth
"&DoubleUpArrow;"                    form="infix"   stretchy="true"  lspace="verythi
"&DoubleUpDownArrow;"                 form="infix"   stretchy="true"  lspace="veryth
"&DownArrow;"                       form="infix"   stretchy="true"  lspace="verythin
"&DownArrowBar;"                     form="infix"   stretchy="true"  lspace="verythi
"&DownArrowUpArrow;"                  form="infix"   stretchy="true"  lspace="veryth
"&DownTeeArrow;"                     form="infix"   stretchy="true"  lspace="verythi
"&LeftDownTeeVector;"                 form="infix"   stretchy="true"  lspace="veryth
"&LeftDownVector;"                   form="infix"   stretchy="true"  lspace="verythi
"&LeftDownVectorBar;"                 form="infix"   stretchy="true"  lspace="veryth
"&LeftUpDownVector;"                  form="infix"   stretchy="true"  lspace="veryth
"&LeftUpTeeVector;"                   form="infix"   stretchy="true"  lspace="verythi
"&LeftUpVector;"                     form="infix"   stretchy="true"  lspace="verythi
"&LeftUpVectorBar;"                   form="infix"   stretchy="true"  lspace="verythi
"&LongLeftArrow;"                    form="infix"   stretchy="true"  lspace="verythi
"&LongLeftRightArrow;"                form="infix"   stretchy="true"  lspace="veryth
"&LongRightArrow;"                   form="infix"   stretchy="true"  lspace="verythi
"&ReverseUpEquilibrium;"              form="infix"   stretchy="true"  lspace="veryth
"&RightDownTeeVector;"                form="infix"   stretchy="true"  lspace="veryth
"&RightDownVector;"                  form="infix"   stretchy="true"  lspace="verythi
"&RightDownVectorBar;"                form="infix"   stretchy="true"  lspace="veryth
"&RightUpDownVector;"                 form="infix"   stretchy="true"  lspace="veryth
"&RightUpTeeVector;"                  form="infix"   stretchy="true"  lspace="verythi
```

```
"&RightUpVector;"                form="infix"   stretchy="true"  lspace="verythi
"&RightUpVectorBar;"              form="infix"   stretchy="true"  lspace="verythi
"&ShortDownArrow;"                form="infix"    lspace="verythinmathspace" rspa
"&ShortUpArrow;"                 form="infix"    lspace="verythinmathspace" rspa
"&UpArrow;"                     form="infix"   stretchy="true"  lspace="verythin
"&UpArrowBar;"                   form="infix"   stretchy="true"  lspace="verythi
"&UpArrowDownArrow;"              form="infix"   stretchy="true"  lspace="verythi
"&UpDownArrow;"                  form="infix"   stretchy="true"  lspace="verythi
"&UpEquilibrium;"                form="infix"   stretchy="true"  lspace="verythi
"&UpTeeArrow;"                   form="infix"   stretchy="true"  lspace="verythin
"^"                            form="infix"    lspace="verythinmathspace" rspace
"<>"                          form="infix"    lspace="verythinmathspace" rspace="v
"′"                            form="postfix"  lspace="verythinmathspace" rspace
"!"                            form="postfix"  lspace="verythinmathspace" rspace
"!!"                           form="postfix"  lspace="verythinmathspace" rspace
"~"                            form="infix"    lspace="verythinmathspace" rspace
"@"                            form="infix"    lspace="verythinmathspace" rspace
"--"                           form="postfix"  lspace="verythinmathspace" rspace
"--"                           form="prefix"   lspace="0em" rspace="verythinmath
"++"                           form="postfix"  lspace="verythinmathspace" rspace
"++"                           form="prefix"   lspace="0em" rspace="verythinmath
"&ApplyFunction;"                form="infix"    lspace="0em" rspace="0em"
"?"                            form="infix"    lspace="verythinmathspace" rspace
"_"                            form="infix"    lspace="verythinmathspace" rspace
"&Breve;"                         form="postfix" accent="true"  lspace="0em" rspac
"&Cedilla;"                       form="postfix" accent="true"  lspace="0em" rspac
"&DiacriticalGrave;"               form="postfix" accent="true"  lspace="0em" rspa
"&DiacriticalDot;"                form="postfix" accent="true"  lspace="0em" rspa
"&DiacriticalDoubleAcute;"         form="postfix" accent="true"  lspace="0em" rsp
"&DiacriticalLeftArrow;"          form="postfix" accent="true" stretchy="true"
"&DiacriticalLeftRightArrow;"      form="postfix" accent="true" stretchy="true"
"&DiacriticalLeftRightVector;"     form="postfix" accent="true" stretchy="true"
"&DiacriticalLeftVector;"         form="postfix" accent="true" stretchy="true"
"&DiacriticalAcute;"              form="postfix" accent="true"  lspace="0em" rspa
"&DiacriticalRightArrow;"         form="postfix" accent="true" stretchy="true"
"&DiacriticalRightVector;"        form="postfix" accent="true" stretchy="true"
"&DiacriticalTilde;"              form="postfix" accent="true" stretchy="true"
"&DoubleDot;"                    form="postfix" accent="true"  lspace="0em" rspac
"&DownBreve;"                    form="postfix" accent="true"  lspace="0em" rspac
"&Hacek;"                       form="postfix" accent="true" stretchy="true"  ls
"&Hat;"                         form="postfix" accent="true" stretchy="true"  ls
"&OverBar;"                      form="postfix" accent="true" stretchy="true"  ls
"&OverBrace;"                    form="postfix" accent="true" stretchy="true"  ls
"&OverBracket;"                  form="postfix" accent="true" stretchy="true"  l
"&OverParenthesis;"               form="postfix" accent="true" stretchy="true"
"&TripleDot;"                    form="postfix" accent="true"  lspace="0em" rspac
"&UnderBar;"                     form="postfix" accent="true" stretchy="true"  ls
"&UnderBrace;"                   form="postfix" accent="true" stretchy="true"  l
"&UnderBracket;"                 form="postfix" accent="true" stretchy="true"  l
"&UnderParenthesis;"              form="postfix" accent="true" stretchy="true"
```

Appendix E

Document Object Model for MathML (Non-Normative)

The following sections describe the interfaces that have been defined in the Document Object Model for MathML. Please refer to chapter 8 for more information.

E.1 IDL Interfaces

E.1.1 Miscellaneous Object Definitions

Interface MathMLCollection

This interface is included by analogy with the interface HTMLCollection, and for the same reasons. (Specifically, it allows access to a list of nodes either by index or by `name` or `id` attributes. The rationale making this desirable for the HTML DOM applies also to the MathML DOM; particularly the presence of named hyperlink targets.) The documentation below is essentially copied from the definition of HTMLCollection.

IDL Definition

```
interface MathMLCollection {
  readonly attribute unsigned long length;
  Node item(in unsigned long index);
  Node namedItem(in DOMString name);
};
```

Attributes

`length` This attribute specifies the length or size of the list.

Methods

`item` This method retrieves a Node specified by ordinal index. `Node`s are numbered in tree order (depth-first traversal order).

Parameters

`index` The index of the `Node` to be fetched. The index origin is 0.

Return value

The `Node` at the corresponding position upon success. A value of `null` is returned if the index is out of range.This method raises no exceptions.

namedItem This method retrieves a Node using a name. It first searches for a Node with a matching id attribute. If it doesn't find one, it then searches for a Node with a matching name attribute, but only on those elements that are allowed a name attribute.

Parameters

name The name of the Node to be fetched.

Return value

The Node with a name or id attribute whose value corresponds to the specified string. Upon failure (e.g. no Node with this name exists), returns null This method raises no exceptions.

Interface MathMLDocumentFragment

This interface is provided as a specialization of the DocumentFragment interface. The child Nodes of this MathMLElement must be MathMLElements as well. As with the DocumentFragment object, inserting a MathMLDocumentFragment into a MathMLElement which can accept children has the effect of inserting each of the top-level child Nodes of the fragment rather than the fragment itself.

IDL Definition

```
interface MathMLDocumentFragment: MathMLElement {
};
```

E.1.2 Generic MathML Elements

Interface MathMLElement

All MathML element interfaces derive from this object, which derives from the basic DOM interface Element.

Note: At some point it is expected that CSS support for mathematics will be available. At that point, the style attribute of an HTML element should be accessed through the ElementCSSInlineStyle interface which is defined in the CSS DOM specification.

IDL Definition

```
interface MathMLElement: Element {
  attribute DOMString className;
  attribute DOMString style;
  attribute DOMString id;
  attribute DOMString other;
  attribute NamedNodeMap otherAttributes;
};
```

Attributes

className The class attribute of the element. See the discussion elsewhere in this document and the HTML definition of the class attribute.

style A string identifying the element's style attribute.(?)

id The element's identifier. See the discussion elsewhere in this document and the HTML definition of the id attribute.

other Direct access to the element's `other` attribute, as a string.

otherAttributes This attribute retrieves or sets a `NamedNodeList` representing the contents of the element's `other` attribute. This will allow more convenient access to the name-key pairs this attribute is defined to contain.

Interface MathMLmathElement

This interface represents the top-level MathML `math` element. It may be useful for interfacing between the Document Object Model objects encoding an encosing document and the MathML DOM elements which are its children. It may also be used for some purposes as a MathML DOM surrogate for a `Document` object. For instance, MathML-specific factory methods could be placed here, as could

IDL Definition

```
interface MathMLmathElement: MathMLElement {
  readonly attribute MathMLCollection declares;
};
```

Attributes

declares Provides access to the `declare` elements which are children of this `math` element, in a `MathMLCollection`

Interface MathMLSemanticsElement

This interface represents the `semantics` element in MathML.

IDL Definition

```
interface MathMLSemanticsElement: MathMLElement {
  attribute MathMLElement body;
  MathMLElement getAnnotation(in unsigned long index);
  MathMLElement setAnnotation(in MathMLElement newAnnotation, in unsigned long inc
};
```

Attributes

body This attribute represents the first child of the `semantics` element, i.e. the child giving the 'primary' content represented by the element.

Methods

getAnnotation This method gives access to the `index`th 'alternate' content associated with a `semantics` element.

Parameters

`index` The 0-based index of the annotation being retrieved.

Return value

The `MathMLAnnotationElement` or `MathMLXMLAnnotationElement` representing the `index`th `annotation` or `xml-annotation` child of the `semantics` element. Note that all child elements of a `semantics` element other than the first are required to be of one of these types.

Exceptions

`setAnnotation` This method allows setting or replacement of the `index`th 'alternate' content associated with a `semantics` element. If there is already an `annotation` or `xml-annotation` element with this index, it is replaced by `newAnnotation`

Parameters

`newAnnotation` An `MathMLAnnotationElement` or `MathMLXMLAnnotationElement` to be inserted as the `index`th `annotation` or `xml-annotation` child. `index` The 0-based index of the annotation being set.

Return value

The `MathMLAnnotationElement` or `MathMLXMLAnnotationElement` being inserted as a child of this `MathMLSemanticsElement`

Exceptions

E.1.3 Presentation Elements

Interface MathMLPresentationElement

This interface is provided to serve as a base interface for various MathML Presentation interfaces. It contains no new attributes or methods at this time; however, it is felt that the distinction between Presentation and Content MathML entities should be indicated in the MathMLElement heirarchy. In particular, future versions of the MathML DOM may add functionality on this interface; it may also serve as an aid to implementors.

IDL Definition

```
interface MathMLPresentationElement: MathMLElement {
};
```

E.1.3.1 Leaf Presentation Element Interfaces

Interface MathMLCharacterElement

This interface supports the `mchar` element section 3.2.8.

IDL Definition

```
interface MathMLCharacterElement: MathMLPresentationElement {
  attribute DOMString name;
};
```

Attributes

`name` The name of a non-ASCII character, taken from chapter 6.

Interface MathMLGlyphElement

This interface supports the `mglyph` element section 3.2.9.

IDL Definition

```
interface MathMLGlyphElement: MathMLPresentationElement {
  attribute DOMString alt;
  attribute DOMString fontfamily;
  attribute unsigned long index;
};
```

Attributes

`alt` A string giving an alternate name for the character. Represents the `mglyph`'s `alt` attribute.

`fontfamily` A string representing the font family.

`index` An unsigned integer giving the glyph's position within the font.

Interface MathMLSpaceElement

This interface extends the `MathMLPresentationElement` interface for the MathML space element `mspace`. Note that this is not derived from `MathMLPresentationTokenElement`, despite the fact that `mspace` is classified as a token element, since it does not carry the attributes declared for `MathMLPresentationTokenElement`

IDL Definition

```
interface MathMLSpaceElement: MathMLPresentationElement {
  attribute DOMString width;
  attribute DOMString height;
  attribute DOMString depth;
};
```

Attributes

`width` A string of the form 'number h-unit'; represents the `width` attribute for the `mspace` element, if specified.

`height` A string of the form 'number v-unit'; represents the `height` attribute for the `mspace` element, if specified.

`depth` A string of the form 'number v-unit'; represents the `depth` attribute for the `mspace` element, if specified.

E.1.3.2 Presentation Token Element Interfaces

Interfaces representing the MathML Presentation token elements which may have content are described here.

Interface MathMLPresentationTokenElement

This interface extends the `MathMLElement` interface to include access for attributes specific to text presentation. It serves as the base class for all MathML presentation token elements. Access to the body of the element is via the `nodeValue` attribute inherited from `Node`. Elements that expose only the core presentation token attributes are directly supported by this object. These elements are:

`mi` identifier element
`mn` number element
`mtext` text element

Issue (methodless-interfaces): Interfaces with no methods? Should we provide interfaces with no methods for `mi`, `mn`, and `mtext`? This would provide separate objects for these elements. Since the element name provides complete information, there is no pressing need for such 'interfaces'. Of course, extending this argument could lead to no MathML DOM specification at all.

IDL Definition

```
interface MathMLPresentationTokenElement: MathMLPresentationElement {
  attribute DOMString fontsize;
  attribute DOMString fontweight;
  attribute DOMString fontstyle;
  attribute DOMString fontfamily;
  attribute DOMString color;
  readonly attribute DOMString contents;
};
```

Attributes

`fontsize` The font size attribute for the element, if specified.
`fontweight` The font weight attribute for the element, if specified.
`fontstyle` The font style attribute for the element, if specified.
`fontfamily` The font family attribute for the element, if specified.
`color` The color attribute for the element, if specified.
`contents` Returns the child `Node`s of the element. These should consist only of `Text` nodes and possibly `MathMLGlyphElement` or `MathMLCharacterElement`s. It should behave the same as the base class's `Node::children` attribute; however, it is provided here for clarity.

Interface MathMLOperatorElement

This interface extends the `MathMLPresentationTokenElement` interface for the MathML operator@@ element `mo`.

IDL Definition

```
interface MathMLOperatorElement: MathMLPresentationTokenElement {
  attribute DOMString form;
  attribute DOMString fence;
  attribute DOMString separator;
```

```
  attribute DOMString lspace;
  attribute DOMString rspace;
  attribute DOMString stretchy;
  attribute DOMString symmetric;
  attribute DOMString maxsize;
  attribute DOMString minsize;
  attribute DOMString largeop;
  attribute DOMString moveablelimits;
  attribute DOMString accent;
};
```

Attributes

form The form attribute (prefix, infix or postfix) for the mo element, if specified.
fence The fence attribute (true or false) for the mo element, if specified.
separator The separator attribute (true or false) for the mo element, if specified.
lspace The lspace attribute (spacing to left) of the mo element, if specified.
rspace The rspace attribute (spacing to right) of the mo element, if specified.
stretchy The stretchy attribute (true or false) for the mo element, if specified.
symmetric The symmetric attribute (true or false) for the mo element, if specified.
maxsize The maxsize attribute for the mo element, if specified.
minsize The minsize attribute for the mo element, if specified.
largeop The largeop attribute for the mo element, if specified.
moveablelimits The moveablelimits (true or false) attribute for the mo element, if specified.
accent The accent attribute (true or false) for the mo element, if specified.

Interface MathMLStringLitElement

This interface extends the MathMLPresentationTokenElement interface for the MathML string literal element ms.

IDL Definition

```
interface MathMLStringLitElement: MathMLPresentationTokenElement {
  attribute DOMString lquote;
  attribute DOMString rquote;
};
```

Attributes

lquote A string giving the opening delimiter for the string literal; represents the lquote attribute for the ms element, if specified.
rquote A string giving the closing delimiter for the string literal; represents the rquote attribute for the ms element, if specified.

E.1.3.3 Presentation Container Interfaces

We include under the heading of Presentation Container Elements interfaces designed to represent MathML Presentation elements which can contain arbitrary numbers of child MathMLElement.

Interface MathMLStyleElement

This interface extends the `MathMLElement` interface for the MathML style element `mstyle`. While the `mstyle` element may contain any attributes allowable on any MathML presentation element, only attributes specific to the `mstyle` element are included in the interface below. Other attributes should be accessed using the methods on the base `Element` class, particularly the `Element::getAttribute` and `Element::setAttribute` methods, or even the `Node::attributes` attribute to access all of them at once. Not only does this obviate a lengthy list below, but it seems likely that most implementations will find this a considerably more useful interface to a `MathMLStyleElement`

IDL Definition

```
interface MathMLStyleElement: MathMLPresentationContainerElement {
  attribute DOMString scriptlevel;
  attribute DOMString displaystyle;
  attribute DOMString scriptsizemultiplier;
  attribute DOMString scriptminsize;
  attribute DOMString color;
  attribute DOMString background;
};
```

Attributes

`scriptlevel` A string of the form "+/- unsigned integer"; represents the `scriptlevel` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

`displaystyle` Either `true` or `false`; a string representing the `displaystyle` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

`scriptsizemultiplier` A string of the form 'number'; represents the `scriptsizemultiplier` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

`scriptminsize` A string of the form 'number v-unit'; represents the `scriptminsize` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

`color` A string representation of a color; represents the `color` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

`background` A string representation of a color or the string `transparent`; represents the `background` attribute for the `mstyle` element, if specified. See also the discussion of this attribute.

Interface MathMLPaddedElement

This interface extends the `MathMLElement` interface for the MathML spacing adjustment element `mpadded`

IDL Definition

```
interface MathMLPaddedElement: MathMLPresentationContainerElement {
  attribute DOMString width;
```

```
  attribute DOMString lspace;
  attribute DOMString height;
  attribute DOMString depth;
};
```

Attributes

width A string representing the total width of the mpadded element, if specified. See also the discussion of this attribute.

lspace A string representing the lspace attribute - the additional space to the left - of the mpadded element, if specified. See also the discussion of this attribute.

height A string representing the height above the baseline of the mpadded element, if specified. See also the discussion of this attribute.

depth A string representing the depth beneath the baseline of the mpadded element, if specified. See also the discussion of this attribute.

Interface MathMLFencedElement

This interface extends the MathMLPresentationContainerElement interface for the MathML fenced content element mfenced.

IDL Definition

```
interface MathMLFencedElement: MathMLPresentationContainerElement {
  attribute DOMString open;
  attribute DOMString close;
  attribute DOMString separators;
};
```

Attributes

open A string representing the opening-fence for the mfenced element, if specified; this is the element's open attribute.

close A string representing the opening-fence for the mfenced element, if specified; this is the element's close attribute.

separators A string representing any separating characters inside the mfenced element, if specified; this is the element's separators attribute.

Interface MathMLEncloseElement

This interface supports the menclose element section 3.3.9.

IDL Definition

```
interface MathMLEncloseElement: MathMLPresentationContainerElement {
  attribute DOMString notation;
};
```

Attributes

notation A string giving a name for the notation enclosing the element's contents. Represents the `notation` attribute of the `menclose`. Allowed values are `longdiv`, `actuarial`, `radical`.

Interface MathMLActionElement

This interface extends the `MathMLPresentationContainerElement` interface for the MathML enlivening expression element `maction`

IDL Definition

```
interface MathMLActionElement: MathMLPresentationContainerElement {
  attribute DOMString actiontype;
  attribute DOMString selection;
};
```

Attributes

actiontype A string specifying the action. Possible values include `toggle`, `statusline`, `tooltip`, `highlight`, and `menu`.

selection A string specifying an integer that selects the current subject of the action.

E.1.3.4 Presentation Schemata Interfaces

Interface MathMLFractionElement

This interface extends the `MathMLPresentationElement` interface for the MathML fraction element `mfrac`.

IDL Definition

```
interface MathMLFractionElement: MathMLPresentationElement {
  attribute DOMString linethickness;
  attribute MathMLElement numerator;
  attribute MathMLElement denominator;
};
```

Attributes

linethickness A string representing the `linethickness` attribute of the `mfrac`, if specified.

numerator The first child `MathMLElement` of the `MathMLFractionElement`, and represents the numerator of the represented fraction.

denominator The second child `MathMLElement` of the `MathMLFractionElement`, and represents the denominator of the represented fraction.

Interface MathMLRadicalElement

This interface extends the `MathMLPresentationElement` interface for the MathML radical and square root elements `mroot` and `msqrt`

IDL Definition

```
interface MathMLRadicalElement: MathMLPresentationElement {
  attribute MathMLElement radicand;
  attribute MathMLElement index;
};
```

Attributes

radicand The first child `MathMLElement` of the `MathMLRadicalElement`, representing the base of the represented radical.

index The second child `MathMLElement` of the `MathMLRadicalElement`, representing the index of the represented radical. This must be `null` for `msqrt` elements.

Interface MathMLScriptElement

This interface extends the `MathMLPresentationElement` interface for the MathML subscript, superscript and subscript-superscript pair elements `msub`, `msup`, and `msubsup`.

IDL Definition

```
interface MathMLScriptElement: MathMLPresentationElement {
  attribute DOMString subscriptshift;
  attribute DOMString superscriptshift;
  attribute MathMLElement base;
  attribute MathMLElement subscript;
  attribute MathMLElement superscript;
};
```

Attributes

subscriptshift A string representing the minimum amount to shift the baseline of the subscript down, if specified; this is the element's `subscriptshift` attribute. This must return `null` for an `msup`.

superscriptshift A string representing the minimum amount to shift the baseline of the superscript up, if specified; this is the element's `superscriptshift` attribute. This must return `null` for a `msub`.

base A `MathMLElement` representing the base of the script. This is the first child of the element.

subscript A `MathMLElement` representing the subscript of the script. This is the second child of a `msub` or `msubsup`; retrieval must return `null` for an `msup`. Exceptions on setting: the `DOMException` `NOT_FOUND_ERR` is raised when the element is a `msup`.

superscript A `MathMLElement` representing the superscript of the script. This is the second child of a `msup` or the third child of a `msubsup`; retrieval must return `null` for an `msub`. Exceptions on setting: the `DOMException` `NOT_FOUND_ERR` is raised when the element is a `msub`.

Interface MathMLUnderOverElement

This interface extends the `MathMLPresentationElement` interface for the MathML underscript, overscript and overscript-underscript pair elements `munder`, `mover`, and `munderover`.

IDL Definition

```
interface MathMLUnderOverElement: MathMLPresentationElement {
  attribute DOMString accentunder;
  attribute DOMString accent;
  attribute MathMLElement base;
  attribute MathMLElement underscript;
  attribute MathMLElement overscript;
};
```

Attributes

accentunder Either `true` or `false` if present; a string controlling whether underscript is drawn as an 'accent' or as a 'limit', if specified; this is the element's `accentunder` attribute. This must return `null` for an `mover`.

accent Either `true` or `false` if present; a string controlling whether overscript is drawn as an 'accent' or as a 'limit', if specified; this is the element's `accent` attribute. This must return `null` for an `munder`.

base A `MathMLElement` representing the base of the script. This is the first child of the element.

underscript A `MathMLElement` representing the underscript of the script. This is the second child of a `munder` or `munderover`; retrieval must return `null` for an `mover`. Exceptions on setting: the `DOMException` `NOT_FOUND_ERR` is raised when the element is a `mover`.

overscript A `MathMLElement` representing the overscript of the script. This is the second child of a `mover` or the third child of a `munderover`; retrieval must return `null` for an `munder`. Exceptions on setting: the `DOMException` `NOT_FOUND_ERR` is raised when the element is a `munder`.

Interface MathMLMultiScriptsElement

This interface extends the `MathMLPresentationElement` interface for the MathML multiscripts (including prescripts or tensors) element `mmultiscripts`.

IDL Definition

```
interface MathMLMultiScriptsElement: MathMLPresentationElement {
  attribute DOMString subscriptshift;
  attribute DOMString superscriptshift;
  attribute MathMLElement base;
  attribute NodeList prescripts;
  attribute NodeList scripts;
  readonly attribute unsigned long numprescriptcolumns;
  readonly attribute unsigned long numscriptcolumns;
  MathMLElement getPreSubScript(in unsigned long colIndex);
  MathMLElement getSubScript(in unsigned long colIndex);
  MathMLElement getPreSuperScript(in unsigned long colIndex);
  MathMLElement getSuperScript(in unsigned long colIndex);
  MathMLElement insertPreSubScriptAt(in unsigned long colIndex, in MathMLElement n
  MathMLElement insertSubScriptAt(in unsigned long colIndex, in MathMLElement newE
```

```
  MathMLElement insertPreSuperScriptAt(in unsigned long colIndex, in MathMLElemen
  MathMLElement insertSuperScriptAt(in unsigned long colIndex, inout MathMLElemen
};
```

Attributes

subscriptshift A string representing the minimum amount to shift the baseline of the subscripts down, if specified; this is the element's `subscriptshift` attribute.

superscriptshift A string representing the minimum amount to shift the baseline of the superscripts up, if specified; this is the element's `superscriptshift` attribute.

base A `MathMLElement` representing the base of the script. This is the first child of the element.

prescripts A `NodeList` representing the prescripts of the script, which appear in the order described by the expression (prescript presuperscript)*. This is the same as traversing the contents of the `NodeList` returned by `Node::childNodes()` from the `Node` following the `<mprescripts/>` (if present) to the end of the list.

scripts A `NodeList` representing the scripts of the script, which appear in the order described by the expression (script superscript)*. This is the same as traversing the contents of the `NodeList` returned by `Node::childNodes()` from the first `Node` up to and including the `Node` preceding the `<mprescripts/>` (if present).

numprescriptcolumns The number of script/subscript columns preceding (to the left of) the base. Should always be half of `getprescripts().length()`

numscriptcolumns The number of script/subscript columns following (to the right of) the base. Should always be half of `getcripts().length()`

Methods

getPreSubScript A convenience method to retrieve pre-subscript children of the element, referenced by column index .

Parameters

`colIndex` Column index of prescript (where 0 represents the leftmost prescript column).

Return value

Returns the `MathMLElement` representing the colIndex-th presubscript (to the left of the base, counting from 0 at the far left). Note that this may be the `MathMLElement` corresponding to the special element `<none/>` in the case of a 'missing' presubscript (see the discussion of `mmultiscripts`), or it may be `null` if `colIndex` is out of range for the element.This method raises no exceptions.

getSubScript A convenience method to retrieve subscript children of the element, referenced by column index.

Parameters

`colIndex` Column index of script (where 0 represents the leftmost script column, the first to the right of the base).

Return value

Returns the `MathMLElement` representing the colIndex-th subscript to the right of the base. Note that this may be the `MathMLElement` corresponding to the special element `<none/>` in the case of a 'missing' subscript (see the discussion of `mmultiscripts`), or it may be `null` if `colIndex` is out of range for the element.This method raises no exceptions.

getPreSuperScript A convenience method to retrieve pre-superscript children of the element, referenced by column index .

Parameters

`colIndex` Column index of pre-superscript (where 0 represents the leftmost prescript column).

Return value

Returns the `MathMLElement` representing the colIndex-th presuperscript (to the left of the base, counting from 0 at the far left). Note that this may be the `MathMLElement` corresponding to the special element `<none/>` in the case of a 'missing' presuperscript (see the discussion of `mmultiscripts`), or it may be `null` if `colIndex` is out of range for the element.This method raises no exceptions.

getSuperScript A convenience method to retrieve superscript children of the element, referenced by column index .

Parameters

`colIndex` Column index of script (where 0 represents the leftmost script column, the first to the right of the base)

Return value

Returns the `MathMLElement` representing the colIndex-th superscript to the right of the base. Note that this may be the `MathMLElement` corresponding to the special element `<none/>` in the case of a 'missing' superscript (see the discussion of `mmultiscripts`), or it may be `null` if `colIndex` is out of range for the element.This method raises no exceptions.

insertPreSubScriptAt A convenience method to insert a pre-subscript child at the position referenced by column index. If there is currently a pre-subscript at this position, it is replaced by `newElement`

Parameters

`colIndex` Column index of pre-subscript (where 0 represents the leftmost prescript column). `newElement` `MathMLElement` to be inserted.

Return value

The `MathMLElement` being inserted.This method raises no exceptions.

insertSubScriptAt A convenience method to insert a subscript child at the position referenced by column index. If there is currently a subscript at this position, it is replaced by `newElement`

Parameters

`colIndex` Column index of subscript (where 0 represents the leftmost script column, the first to the right of the base). `newElement` `MathMLElement` to be inserted.

Return value

The `MathMLElement` being inserted.This method raises no exceptions.

insertPreSuperScriptAt A convenience method to insert a pre-superscript child at the position referenced by column index. If there is currently a pre-superscript at this position, it is replaced by `newElement`

Parameters

`colIndex` Column index of pre-superscript (where 0 represents the leftmost prescript column). `newElement` `MathMLElement` to be inserted.

Return value

The `MathMLElement` being inserted.This method raises no exceptions.

insertSuperScriptAt A convenience method to insert a superscript child at the position referenced by column index. If there is currently a superscript at this position, it is replaced by `newElement`

Parameters

`colIndex` Column index of superscript (where 0 represents the leftmost script column, the first to the right of the base). `newElement` `MathMLElement` to be inserted.

Return value

The `MathMLElement` being inserted. This method raises no exceptions.

Interface MathMLTableElement

This interface extends the `MathMLPresentationElement` interface for the MathML table or matrix element `mtable`

IDL Definition

```
interface MathMLTableElement: MathMLPresentationElement {
  attribute DOMString align;
  attribute DOMString rowalign;
  attribute DOMString columnalign;
  attribute DOMString groupalign;
  attribute DOMString alignmentscope;
  attribute DOMString rowspacing;
  attribute DOMString columnspacing;
  attribute DOMString rowlines;
  attribute DOMString columnlines;
  attribute DOMString frame;
  attribute DOMString framespacing;
  attribute DOMString equalrows;
  attribute DOMString equalcolumns;
  attribute DOMString displaystyle;
  attribute DOMString side;
  attribute DOMString minlabelspacing;
  readonly attribute MathMLCollection rows;
  MathMLTableRowElement insertRow(in unsigned long index);
  void deleteRow(in unsigned long index);
};
```

Attributes

`align` A string representing the vertical alignment of the table with the adjacent text. Allowed values are (`top` | `bottom` | `center` | `baseline` | `axis`)[rownumber], where rownumber is between 1 and *n* (for a table with *n* rows) or -1 and -*n*.

`rowalign` A string representing the alignment of entries in each row, consisting of a space-separated sequence of alignment specifiers, each of which can have the following values: `top`, `bottom`, `center`, `baseline`, or `axis`

`columnalign` A string representing the alignment of entries in each column, consisting of a space-separated sequence of alignment specifiers, each of which can have the following values: `left`, `center`, or `right`

`groupalign` A string specifying how the alignment groups within the cells of each row are to be aligned with the corresponding items above or below them in the same column. The string consists of a sequence of braced group alignment lists. Each group alignment list is a space-separated sequence, each of which can have the following values: `left`, `right`, `center`, or `decimalpoint`.

alignmentscope A string consisting of the values `true` or `false` indicating, for each column, whether it can be used as an alignment scope.

rowspacing A string consisting of a space-separated sequence of specifiers of the form `number v-unit` representing the space to be added between rows.

columnspacing A string consisting of a space-separated sequence of specifiers of the form `number h-unit` representing the space to be added between columns.

rowlines A string specifying whether and what kind of lines should be added between each row. The string consists of a space-separated sequence of specifiers, each of which can have the following values: `none`, `solid`, or `dashed`

columnlines A string specifying whether and what kind of lines should be added between each column. The string consists of a space-separated sequence of specifiers, each of which can have the following values: `none`, `solid`, or `dashed`

frame A string specifying a frame around the table. Allowed values are `(none | solid | dashed)`.

framespacing A string of the form `number h-unit number v-unit` specifying the spacing between table and its frame.

equalrows A string with the values `true` or `false`.

equalcolumns A string with the values `true` or `false`

displaystyle A string with the values `true` or `false`

side A string with the values `left`, `right`, `leftoverlap`, or `rightoverlap`

minlabelspacing A string of the form `number h-unit`, specifying the minimum space between a label and the adjacent entry in the labeled row.

rows A `MathMLCollection` consisting of the rows of the table.

Methods

insertRow A convenience method to insert a new (empty) row in the table at the specified index.

Parameters

`index` Index at which to insert row.

Return value

Returns the `MathMLTableRowElement` representing the `mtr` element being inserted.This method raises no exceptions.

deleteRow A convenience method to delete the row of the table at the specified index.

Parameters

`index` Index of row to be deleted.

Return value

None.This method raises no exceptions.

Interface MathMLTableRowElement

This interface extends the `MathMLPresentationElement` interface for the MathML table or matrix row element `mtr`.

IDL Definition

```
interface MathMLTableRowElement: MathMLPresentationElement {
  attribute DOMString rowalign;
  attribute DOMString columnalign;
  attribute DOMString groupalign;
```

```
  readonly attribute MathMLCollection cells;
  MathMLTableCellElement insertCell(in unsigned long index);
  void deleteCell(in unsigned long index);
};
```

Attributes

rowalign A string representing an override of the row alignment specified in the containing `mtable`. Allowed values are `top`, `bottom`, `center`, `baseline`, and `axis`.

columnalign A string representing an override of the column alignment specified in the containing `mtable`. Allowed values are `left`, `center`, and `right`.

groupalign [To be changed?]

cells A `MathMLCollection` consisting of the cells of the row. Note that this collection does not include the label if this is a *`MathMLLabeledRowElement`*!

Methods

insertCell A convenience method to insert a new (empty) cell in the row.

Parameters

`index` Index at which to insert cell. Note that the count will differ from the *`index`*th child node if this is a *`MathMLLabeledRowElement`*!

Return value

Returns the `MathMLTableCellElement` representing the `mtd` element being inserted. This method raises no exceptions.

deleteCell A convenience method to delete a cell in the row.

Parameters

`index` Index of cell to be deleted. Note that the count will differ from the *`index`*th child node if this is a *`MathMLLabeledRowElement`*!

Return value

None. This method raises no exceptions.

Interface MathMLLabeledRowElement

This interface extends the `MathMLTableRowElement` interface to represent the `mlabeledtr` element section 3.5.3. Note that the presence of a label causes the *`index`*th child node to differ from the *`index`*th cell!

IDL Definition

```
interface MathMLLabeledRowElement: MathMLTableRowElement {
  attribute MathMLElement label;
};
```

Attributes

label A `MathMLElement` representing the label of this row. Note that retrieving this should have the same effect as a call to `Node::getfirstChild()`, while setting it should have the same effect as `Node::replaceChild(Node::getfirstChild())`.

Interface MathMLTableCellElement

This interface extends the `MathMLPresentationContainerElement` interface for the MathML table or matrix cell element `mtd`

IDL Definition

```
interface MathMLTableCellElement: MathMLPresentationContainerElement {
  attribute DOMString rowspan;
  attribute DOMString columnspan;
  attribute DOMString rowalign;
  attribute DOMString columnalign;
  attribute DOMString groupalign;
  readonly attribute boolean hasaligngroups;
  readonly attribute DOMString cellindex;
};
```

Attributes

`rowspan` A string representing a positive integer that specifies the number of rows spanned by this cell. The default is 1.

`columnspan` A string representing a positive integer that specifies the number of columns spanned by this cell. The default is 1.

`rowalign` A string specifying an override of the inherited vertical alignment of this cell within the table row. Allowed values are `top`, `bottom`, `center`, `baseline`, and `axis`

`columnalign` A string specifying an override of the inherited horizontal alignment of this cell within the table column. Allowed values are `left`, `center`, and `right`

`groupalign` A string specifying how the alignment groups within the cell are to be aligned with those in cells above or below this cell. The string consists of a space-separated sequence of specifiers, each of which can have the following values: `left`, `right`, `center`, or `decimalpoint`

`hasaligngroups` A string with the values `true` or `false` indicating whether the cell contains align groups.

`cellindex` A string representing the integer index (1-based?) of the cell in its containing row. [What about spanning cells? How do these affect this value?]

Interface MathMLAlignGroupElement

This interface extends the `MathMLPresentationElement` interface for the MathML group -alignment element `<malignmark/>`

IDL Definition

```
interface MathMLAlignGroupElement: MathMLPresentationElement {
  attribute DOMString groupalign;
};
```

Attributes

groupalign A string specifying how the alignment group is to be aligned with other alignment groups above or below it. Allowed values are `left`, `right`, `center`, or `decimalpoint`.

Interface MathMLAlignMarkElement

This interface extends the `MathMLPresentationElement` interface for the MathML alignment mark element `<malignmark/>`

IDL Definition

```
interface MathMLAlignMarkElement: MathMLPresentationElement {
  attribute DOMString edge;
};
```

Attributes

edge A string specifying alignment on the right edge of the preceding element or the left edge of the following element. Allowed values are `left` and `right`

E.1.4 Content Elements

Issue (content-names): We have named all of the content element interfaces MathML`name`Element where `name` is the MathML element.

Interface MathMLContentElement

This interface is provided to serve as a base interface for various MathML Content interfaces. It contains no new attributes or methods at this time; however, it is felt that the distinction between Presentation and Content MathML entities should be indicated in the MathMLElement heirarchy. In particular, future versions of the MathML DOM may add functionality on this interface; it may also serve as an aid to implementors.

IDL Definition

```
interface MathMLContentElement: MathMLElement {
};
```

E.1.4.1 Content Token Interfaces

Interface MathMLcnElement

The `cn` element is used to specify actual numeric constants.

IDL Definition

```
interface MathMLcnElement: MathMLContainerTokenElement {
  attribute DOMString type;
  attribute DOMString base;
  readonly attribute unsigned long nargs;
  attribute DOMString definitionURL;
};
```

Attributes

type Values include, but are not restricted to, `center`, `real`, `integer`, `rational`, `complex-cartesian`, `complex-polar`, and `constant`.

base A string representing an integer between 2 and 36; the base of the numerical representation.

nargs The number of `sep`-separated arguments.

definitionURL A URL pointing to an alternative definition

Interface MathMLciElement

The `ci` element is used to specify a symbolic name.

IDL Definition

```
interface MathMLciElement: MathMLContentTokenElement {
  attribute DOMString type;
};
```

Attributes

type Values include `integer`, `rational`, `real`, `float`, `complex`, `complex-polar`, `complex-cartesian`, `constant`, any of the MathML content container types (`vector`, `matrix`, `set`, `list` etc.) or their types.

E.1.4.2 Content Container Interfaces

We have added interfaces for content elements that are containers, i.e. elements that may contain child elements corresponding to arguments, bound variables, conditions, or lower or upper limits.

Interface MathMLapplyElement

The `apply` element allows a function or operator to be applied to its arguments.

IDL Definition

```
interface MathMLapplyElement: MathMLContentContainerElement {
  attribute MathMLElement operator;
  readonly attribute unsigned long nargs;
};
```

Attributes

operator The MathML element representing the function or operator that is applied to the list of arguments.

nargs An integer representing the number of arguments. This does not include the function or operator itself; note that this causes the return value to be less than the return from `Node::childNodes().length()`

Interface MathMLfnElement

The `fn` element makes explicit the fact that a more general MathML object is intended to be used in the same manner as if it were a pre-defined function such as `sin` or `plus`

IDL Definition

```
interface MathMLfnElement: MathMLContentContainerElement {
  attribute DOMString definitionURL;
  attribute DOMString encoding;
};
```

Attributes

`definitionURL` A URL pointing to a definition for this function-type element. Note that there is no stipulation about the form this definition may take!

`encoding` A string describing the syntax in which the definition located at `definitionURL` is given.

Issue (condition-reln): The specification of MathML 1.01 says that a condition contains a single `reln` element or a single `apply` element. Since `reln` is being deprecated in version 2.0, we have typed the body as `MathMLapplyElement`. Is this OK? It may be dangerous if there are documents that use `reln` in this context.

Interface MathMLlambdaElement

The `lambda` element is used to construct a user-defined function from an expression and one or more free variables.

IDL Definition

```
interface MathMLlambdaElement: MathMLContentContainerElement {
  attribute MathMLElement expression;
  readonly attribute unsigned long nvars;
};
```

Attributes

`expression` The `MathMLElement` representing the expression.

`nvars` An integer representing the number of variables in the expression.

Issue (sets): The following interface seems unsatisfactory. The first problem is that `set` is really two things - a condition set or an explicit list set. Another problem is that it's not easy to express the union of two sets as a set (although it's possible - the problem is that the union of a condition set and a list set is only awkwardly expressed as a condition set). The dual nature of the object makes the interface awkward. Access to the elements of an explicit list set seems problematic. What if another process deletes an element between the time you determine its position and when you delete it? Perhaps the delete function should take only a `MathMLElement` argument as returned by `getElement` - this would be the element in the DOM, so there would be no problem of a changing index.

Interface MathMLsetElement

The `set` element is the container element that represents a set of elements. The elements of a set can be defined either by explicitly listing the elements, or by using the `bvar` and `condition` elements.

IDL Definition

```
interface MathMLsetElement: MathMLContentContainerElement {
  readonly attribute boolean isConditionSet;
  readonly attribute unsigned long nelements;
  MathMLElement getElement(in unsigned long index);
  MathMLElement addElement(in MathMLelement element);
   deleteElement(in unsigned long index);
};
```

Attributes

`isConditionSet` This is true if the set is specified using a condition and false if the set is an explicit list.

`nelements` The number of elements if the set is an explicit list. Should this raise an exception if this is a condition set? Even if the conditions really amount to an explicit list?

Methods

`getElement` A convenience method to retrieve an element. There is no default ordering of the elements. Inserting or deleting an element is not guaranteed to leave the element in the *i*-th place unchanged even if the action takes place at a larger index.

Parameters

`index` Position of the element in the list of elements. The first element is numbered 1.

Return value

The `MathMLElement` element at position `index`. This method raises no exceptions.

`addElement` A convenience method to insert an element. The insertion may change the indices of any of the other elements. Since element equivalence is not easy to determine, it seems hard to specify that inserting the same element twice is an error.

Parameters

`element` The `MathMLElement` to be added to the set.

Return value

The `MathMLElement` being added. This is the element within the DOM.This method raises no exceptions.

`deleteElement` A convenience method to delete an element. The deletion may change the indices of any of the other elements.

Parameters

`index` Position of the element in the set

Return value

NoneThis method raises no exceptions.

Issue (lists): The following interface seems unsatisfactory. The first problem is that `list` is really two things - a condition list or an explicit list. Another problem is that it's not easy to express the union of two lists as a list (although it's possible - the problem is that the union of a condition list and an explicit list set is only awkwardly expressed as a condition list). The dual nature of the object makes the interface awkward. Should an exception be raised if an attempt is made to insert an element into a specified position in a list that is given by a condition? A priori, probably not; but allowing this would certainly seem to give rise to implementation problems! Access to the elements of an explicit list seems problematic. What if another process deletes an element between the time you determine its position and when you delete it? Perhaps the delete function should take only a MathMLElement argument as returned by getElement - this would be the element in the DOM, so there would be no problem of a changing index.

Interface MathMLlistElement

The `list`element is the container element which represents a list of elements. Elements can be defined either by explicitly listing the elements, or by using the `bvar`and `condition` elements.

IDL Definition

```
interface MathMLlistElement: MathMLContentContainerElement {
  readonly attribute boolean isConditionList;
  attribute MathMLconditionElement condition;
  readonly attribute unsigned long nelements;
  MathMLElement getElement(in unsigned long index);
  MathMLElement addElement(in unsigned long index, in MathMLelement element);
   deleteElement(in unsigned long index);
};
```

Attributes

`isConditionList` This is true if the `list`is specified using a condition and false if the `list`is an explicit list.

`condition` A `MathMLconditionElement`that determines the `list`. Setting this causes `isConditionList`to be `true`. Getting this if the list is an explicit list (if `isConditionList` is `false`) should raise an exception?

`nelements` The number of elements if the `list`is an explicit list. Should this raise an exception if this is a condition list? Even if the conditions really amount to an explicit list?

Methods

`getElement` A convenience method to retrieve an element.

Parameters

`index` Position of the element in the list of elements. The first element is numbered 1.

Return value

The `MathMLElement`element at position `index`in the list. This method raises no exceptions.

addElement A convenience method to insert an element.

Parameters

index The position in the list at which element is to be added. element The MathMLElement to be added to the list.

Return value

The MathMLElement being added. This is the new element within the DOM.This method raises no exceptions.

deleteElement A convenience method to delete an element. The deletion may change the indices of elements occurring after index in the list.

Parameters

index Position of the element in the list.

Return value

NoneThis method raises no exceptions.

E.1.4.3 Content Leaf Element Interfaces

Issue (builtin-interface): I propose that all built-in operator, relation, and function interfaces either derive from or be directly supported through the MathMLbuiltIn interface. Note that the name does not end with 'Element' because this interface does not correspond to a MathML element. The alternative is to provide an interface for every one of these elements individually. Again, this interface supports all empty elements that have only the additional definitionURL attribute. This includes elements that take qualifiers. I don't particularly like the name 'builtin'. Any better suggestions? QUESTION: Should we treat these as objects that own their arguments and provide methods for accessing those arguments? Similarly for operators taking qualifiers - we could provide access to the qualifiers. No, I suppose not. It's the apply that owns the arguments. Unless apply does the work of validating the arguments (ensuring the correct number, type, and checking any other conditions), there's no easy way to introduce this. Wouldn't it be easier to list the elements that are not supported by this interface?

Interface MathMLbuiltIn

This interface supports all of the empty built-in operator, relation, and function elements that have the definitionURL attribute in addition to the standard set of attributes. The elements supported in order of their appearance in section 4.4 are: inverse, compose, ident, quotient, exp, factorial, divide, max, min, minus, plus, power, rem, times, root, gcd, and, or, xor, not, implies, forall, exists, abs, conjugate, eq, neq, gt, lt, geq, leq, ln, log, int, diff, partialdiff, union, intersect, in, notin, subset, prsubset, notsubset, notprsubset, setdiff, sum, product, limit, tendsto, sin, cos, tan, sec, csc, cot, sinh, cosh, tanh, sech, csch, coth, arcsin, arccos, arctan, mean, sdev, variance, median, mode, moment, determinant, and transpose.

IDL Definition

```
interface MathMLbuiltIn: MathMLContentElement {
  attribute DOMString definitionURL;
  attribute DOMString arity;
  attribute DOMString symbolName;
};
```

Attributes

definitionURL A string that provides an override to the default semantics, or provides a more specific definition

arity A string representing the number of arguments. Values include 0, 1, ... and variable

symbolName A string that provides an override to the default semantics, or provides a more specific definition

E.1.4.4 Other Content Element Interfaces

Interface MathMLintervalElement

The interval element is used to represent simple mathematical intervals on the real number line. It contains either two child elements that evaluate to real numbers or one child element that is a condition for defining membership in the interval.

IDL Definition

```
interface MathMLintervalElement: MathMLContentElement {
  attribute DOMString closure;
  readonly attribute boolean isCondition;
  attribute MathMLconditionElement condition;
  attribute MathMLcnElement start;
  attribute MathMLcnElement end;
};
```

Attributes

closure A string with value open, closed, open-closed or closed-open. The default value is closed

isCondition true if this interval is defined by a condition rather than by two real number endpoints.

condition A MathML condition element in the case that the interval is defined using a condition. Setting this attribute has the side effect of setting isCondition to true. Getting this attribute raises an exception if isCondition is false

start A MathMLcnElement representing the real number defining the start of the interval. Setting this attribute has the side effect of setting isCondition to false. If end has not already been set, it becomes the same as start until set otherwise. Getting this attribute raises an exception if isCondition is true

end A MathMLcnElement representing the real number defining the end of the interval. Setting this attribute has the side effect of setting isCondition to false. If start has not already been set, it becomes the same as end until set otherwise. Getting this attribute raises an exception if isCondition is true

Interface MathMLconditionElement

The condition element is used to place a condition on one or more free variables or identifiers.

IDL Definition

```
interface MathMLconditionElement: MathMLContentElement {
  attribute MathMLapplyElement condition;
};
```

Attributes

`condition` A `MathMLapplyElement` that represents the condition.

Interface MathMLdeclareElement

The `declare` construct has two primary roles. The first is to change or set the default attribute values for a specific mathematical object. The second is to establish an association between a 'name' and an object.

IDL Definition

```
interface MathMLdeclareElement: MathMLContentElement {
  attribute DOMString type;
  attribute DOMString scope;
  attribute unsigned long nargs;
  attribute DOMString occurrence;
  attribute DOMString definitionURL;
  attribute MathMLciElement identifier;
  attribute MathMLElement constructor;
};
```

Attributes

`type` A string indicating the type of the identifier. It must be compatible with the type of the `constructor`, if a `constructor` is present. The type is inferred from the `constructor` if present, otherwise it must be specified.

`scope` A string with values `local` or `global`

`nargs` If the identifier is a function, this attribute specifies the number of arguments the function takes.

`occurrence` A string with the values `prefix`, `infix` or `function-model`. [What about `postfix`?]

`definitionURL` A URL specifying an alternative definition. [Is 'alternative' correct here?]

`identifier` A `MathMLciElement` representing the name being declared.

`constructor` An optional `MathMLElement` providing an initialial value for the object being declared.

Interface MathMLvectorElement

`vector` is the container element for a vector.

IDL Definition

```
interface MathMLvectorElement: MathMLContentElement {
  readonly attribute unsigned long ncomponents;
  MathMLElement getComponent(in unsigned long index);
  MathMLElement insertComponent(in MathMLelement component);
   deleteComponent(in unsigned long index);
};
```

Attributes

`ncomponents` The number of components in the vector.

Methods

`getComponent` A convenience method to retrieve a component.

Parameters

`index` Position of the component in the list of components. The first element is numbered 1.

Return value

The `MathMLElement` component at the position specified by `index`. This method raises no exceptions.

`insertComponent` A convenience method to insert a component. If there is already a component at the position specified by `index`, it is replaced.

Parameters

`component` The `MathMLElement` that is to be the `index`th component of the vector.

Return value

The `MathMLElement` that is added. This is the new element within the DOM.This method raises no exceptions.

`deleteComponent` A convenience method to delete an element. The deletion changes the indexes of the following components.

Parameters

`index` Position of the component in the vector. The position of the first component is 1

Return value

NoneThis method raises no exceptions.

Interface MathMLmatrixElement

The `matrix` element is the container element for `matrixrow` elements.

IDL Definition

```
interface MathMLmatrixElement: MathMLContentElement {
  readonly attribute unsigned long nrows;
  MathMLmatrixrowElement getRow(in unsigned long index);
  MathMLmatrixrowElement insertRow(in MathMLrowElement row, in unsigned long index
   deleteRow(in unsigned long index);
};
```

Attributes

nrows The number of rows in the represented matrix.

Methods

getRow A convenience method to retrieve a specified row.

Parameters

index Position of the row in the list of rows. The first row is numbered 1.

Return value

The MathMLmatrixrowElement representing the index th row.This method raises no exceptions.

insertRow A convenience method to insert a row. If there is already a row at the specified index, it is replaced.

Parameters

row MathMLmatrixrowElement to be inserted into the matrix. index Unsigned integer giving the row position at which the row is to be inserted.

Return value

The MathMLmatrixrowElement added. This is the new element within the DOM.This method raises no exceptions.

deleteRow A convenience method to delete a row. The deletion changes the indices of the following rows.

Parameters

index Position of the row to be deleted in the list of rows

Return value

NoneThis method raises no exceptions.

Issue (matrix-vector): matrix, matrixrow, and vector How to we convert between these elements? The specification states that vectors are equivalent to single column or single row matrices in appropriate contexts. What about matrixrow's? It would help tremendously to have some form of compatibility. Is there any requirement that the number of elements be the same for each row of a matrix? If so, do we need exceptions to handle the cases where there is an attempt to add incompatible rows to a matrix?

Interface MathMLmatrixrowElement

The matrixrow element is the container element for the elements of a matrix

IDL Definition

```
interface MathMLmatrixrowElement: MathMLContentElement {
  readonly attribute unsigned long nelements;
  MathMLElement getElement(in unsigned long index);
  MathMLElement insertElement(in MathMLElement element, in unsigned long index);
   deleteElement(in unsigned long index);
};
```

Attributes

nelements The number of elements in the row.

Methods

`getElement` A convenience method to retrieve an element by index.

Parameters

`index` Position of the element in the row. The first element is numbered 1.

Return value

The `MathMLElement` element at index `index` in the row.This method raises no exceptions.

`insertElement` A convenience method to insert an element. If there is already an element at the specified index, it is replaced by the new element.

Parameters

`element` The `MathMLElement` to be inserted in the row. `index` The index at which `element` is to be inserted in the row.

Return value

The `MathMLElement` created by the insertion. This is the new element within the DOM.This method raises no exceptions.

`deleteElement` A convenience method to delete an element. The deletion changes the indices of the following elements.

Parameters

`index` Position of the element to be deleted in the row.

Return value

NoneThis method raises no exceptions.

Appendix F

Glossary (Non-Normative)

Several of the following definitions of terms have been borrowed or modified from similar definitions in documents originating from W3C or standards organisations. See the individual definitions for more information.

Argument A child of a presentation layout schema. That is, 'A is an argument of B' means 'A is a child of B and B is a presentation layout schema'. Thus, token elements have no arguments, even if they have children (which can only be `malignmark`)

Attribute A parameter used to specify some property of an SGML or XML element type. It is defined in terms of an attribute name, attribute type, and a default value. A value may be specified for it on a start-tag for that element type.

Axis The axis is an imaginary alignment line upon which a fraction line is centered. Often, operators as well as characters that can stretch, such as parentheses, brackets, braces, summation signs etcetera, are centered on the axis, and are symmetric with respect to it.

Baseline The baseline is an imaginary alignment line upon which a glyph without a descender rests. The baseline is an intrinsic property of the glyph (namely a horizontal line). Often baselines are aligned (joined) during typesetting.

Black box The bounding box of the actual size taken up by the viewable portion (ink) of a glyph or expression.

Bounding box The rectangular box of smallest size, taking into account the constraints on boxes allowed in a particular context, which contains some specific part of a rendered display.

Box A rectangular plane area considered to contain a character or further sub-boxes, used in discussions of rendering for display. It is usually considered to have a baseline, height, depth and width.

Cascading Style Sheets (CSS) A mechanism that allows authors and readers to attach style (e.g. fonts, colors and spacing) to HTML and XML documents.

Character A member of a set of identifiers used for the organization, control or representation of text. ISO/IEC Standard 10646-1:1993 uses the word 'data' here instead of 'text'.

Character data (`CDATA`) A data type in SGML and XML for raw data that does not include markup or entity references. Attributes of type `CDATA` may contain entity references. These are expanded by an XML processor before the attribute value is processed as `CDATA`

Character or expression depth Distance between the baseline and bottom edge of the character glyph or expression. Also known as the descent.

Character or expression height Distance between the baseline and top edge of the character glyph or expression. Also know as the ascent.

Character or expression width Horizontal distance taken by the character glyph as indicated in the font metrics, or the total width of an expression.

Condition A MathML content element used to place a mathematical condition on one or more variables.

Contained (element A is contained in element B) A is part of B's content.

Container (Constructor) A non-empty MathML Content element that is used to construct a mathematical object such as a number, set, or list.

Content elements MathML elements that explicitly specify the mathematical meaning of a portion of a MathML expression (defined in chapter 4).

Content token element Content element having only `PCDATA`, `sep` and presentation expressions as content. Represents either an identifier (`ci`) or a number (`cn`).

Context (of a given MathML expression) Information provided during the rendering of some MathML data to the rendering process for the given MathML expression; especially information about the MathML markup surrounding the expression.

Declaration An instance of the declare element.

Depth (of a box) The distance from the baseline of the box to the bottom edge of the box.

Direct sub-expression (of a MathML expression 'E') A sub-expression directly contained in E.

Directly contained (element A in element B) A is a child of B (as defined in XML), in other words A is contained in B, but not in any element that is itself contained in B.

Document Object Model A model in which the document or Web page is treated as an object repository. This model is developed by the DOM Working Group (DOM) of the W3C.

Document Style Semantics and Specification Language (DSSSL) A method of specify the formatting and transformation of SGML documents. ISO International Standard 10179:1996.

Document Type Definition (DTD) In SGML or XML, a DTD is a formal definition of the elements and the relationship among the data elements (the structure) for a particular type of document.

Em A font-relative measure encoded by the font. Before electronic typesetting, an `em` was the width of an 'M' in the font. In modern usage, an `em` is either specified by the designer of the font or is taken to be the height (point size) of the font. Em's are typically used for font-relative horizontal sizes.

Ex A font-relative measure that is the height of an 'x' in the font. `ex`s are typically used for font-relative vertical sizes.

Height (of a box) The distance from the baseline of the box to the top edge of the box.

Inferred `mrow` An `mrow` element that is 'inferred' around the contents of certain layout schemata when they have other than exactly one argument. Defined precisely in section 3.1.5

Embedded object Embedded objects such as Java applets, Microsoft Component Object Model (COM) objects (e.g. ActiveX Controls and ActiveX Document embeddings), and plug-ins that reside in an HTML document.

Embellished operator An operator, including any 'embellishment' it may have, such as superscripts or style information. The 'embellishment' is represented by a layout schema that contains the operator itself. Defined precisely in section 3.2.4.

Entity reference A sequence of ASCII characters of the form `&name;` representing some other data, typically a non-ASCII character, a sequence of characters, or an exter-

nal source of data, e.g. a file containing a set of standard entity definitions such as ISO Latin 1.

Extensible Markup Language (XML) A simple dialect of SGML intended to enable generic SGML to be served, received, and processed on the Web.

Fences In typesetting, bracketing tokens like parentheses, braces, and brackets, which usually appear in matched pairs.

Font A particular collection of glyphs of a typeface of a given size, weight and style, for example 'Times Roman Bold 12 point'.

Glyph The actual shape (bit pattern, outline) of a character. ISO/IEC Standard 9541-1:1991 defines a glyph as a recognizable abstract graphic symbol that is independent of any specific design.

Indirectly contained A is contained in B, but not directly contained in B.

Instance of MathML A single instance of the toplevel element of MathML, and/or a single instance of embedded MathML in some other data format.

Inverse function A mathematical function that, when composed with the original function acts like an identity function.

Lambda expression A mathematical expression used to define a function in terms of variables and an expression in those variables.

Layout schema (plural: schemata) A presentation element defined in chapter 3, other than the token elements and empty elements defined there (i.e. not the elements defined in section 3.2 and section 3.5.5, or the empty elements `none` and `mprescripts` defined in section 3.4.7). The layout schemata are never empty elements (though their content may contain nothing in some cases), are always expressions, and all allow any MathML expressions as arguments (except for requirements on argument count, and the requirement for a certain empty element in `mmultiscripts`).

Mathematical Markup Language (MathML) The markup language specified in this document for describing the structure of mathematical expressions, together with a mathematical context.

MathML element An XML element that forms part of the logical structure of a MathML document.

MathML expression (within some valid MathML data) A single instance of a presentation element, except for the empty elements `none` or `mprescripts`, or an instance of `malignmark` within a token element (defined below); or a single instance of certain of the content elements (see chapter 4 for a precise definition of which ones).

Multi-purpose Internet Mail Extensions (MIME) A set of specifications that offers a way to interchange text in languages with different character sets, and multimedia content among many different computer systems that use Internet mail standards.

Operator, content element A mathematical object that is applied to arguments using the `apply` element.

Operator, an `mo` element Used to represent ordinary operators, fences, separators in MathML presentation. (The token element `mo` is defined in section 3.2.4).

OpenMath A general representation language for communicating mathematical objects between application programs.

Parsed character data (`PCDATA`) An SGML/XML data type for raw data occurring in a context where text is parsed and markup (for instance entity references and element start/end tags) is recognised.

Point Point is often abbreviated 'pt'. The value of 1 pt is approximately 1/72 inch. Points are typically used to specify absolute sizes for font-related objects.

Pre-defined function One of the empty elements defined in section 4.2.3 and used with the `apply` construct to build function applications.

Presentation elements MathML tags and entities intended to express the syntactic structure of mathematical notation (defined in chapter 3).

Presentation layout schema A presentation element that can have other MathML elements as content.

Presentation token element A presentation element that can contain only parsed character data or the `malignmark` element.

Qualifier A MathML content element that is used to specify the value of a specific named parameter in the application of selected pre-defined functions.

Relation A MathML content element used to construct expressions such as $a < b$.

Render Faithfully translate into application-specific form allowing native application operations to be performed.

Schema Schema (plural: schemata). See 'presentation layout schema'.

Scope of a declaration The portion of a MathML document in which a particular definition is active.

Selected sub-expression (of an `maction` element) The argument of an `maction` element (a layout schema defined in section 3.6) that is (at any given time) 'selected' within the viewing state of a MathML renderer, or by the `selection` attribute when the element exists only in MathML data. Defined precisely in the above-mentioned section.

Space-like (MathML expression) A MathML expression that is ignored by the suggested rendering rules for MathML presentation elements when they determine operator forms and effective operator rendering attributes based on operator positions in `mrow` elements. Defined precisely in section 3.2.6.

Standard Generalized Markup Language (SGML) An ISO standard (ISO 8879:1986) that provides a formal mechanism for the definition of document structure via DTDs (Document Type Definitions), and a notation for the markup of document instances conforming to a DTD.

Sub-expression (of a MathML expression 'E') A MathML expression contained (directly or indirectly) in the content of E.

Suggested rendering rules for MathML presentation elements Defined throughout chapter 3; the ones that use other terms defined here occur mainly in section 3.2.4 and in section 3.6.

TEX A software system developed by Professor Donald Knuth for typesetting documents.

Token element Presentation token element or a Content token element. (See above.)

Top-level element (of MathML) `math` (defined in chapter 7).

Typeface A typeface is a specific design of a set of letters, numbers and symbols, such as 'Times Roman' or 'Chicago'.

Valid MathML data MathML data that (1) conforms to the MathML DTD, (2) obeys the additional rules defined in the MathML standard for the legal contents and attribute values of each MathML element, and (3) satisfies the EBNF grammar for content elements.

Width (of a box) The distance from the left edge of the box to the right edge of the box.

Extensible Style Language (XSL) A style language for XML developed by W3C. See XSL FO and XSLT.

XSL Formatting Objects (XSL FO) An XML vocabulary to express formatting, which is a part of XSL.

XSL Transformation (XSLT) A language to express the transformation of XML documents into other XML documents.

Appendix G

Working Group Membership (Non-Normative)

The W3C Math Working Group is presently co-chaired by Patrick Ion of the AMS, and Angel Diaz of IBM. Contact the co-chairs if you are interested in joining the group. For the present membership see its working group home page.

Members of the Working Group responsible for MathML 2.0 are:

- Ron Ausbrooks, Mackichan Software, Las Cruces NM, USA
- Laurent Bernardin, Waterloo Maple, Inc., Waterloo ON, CAN
- Stephen Buswell, Stilo Technologies, Cardiff, UK
- David Carlisle, NAG Ltd., Oxford, UK
- Stéphane Dalmas, INRIA, Sophia Antipolis, FR
- Stan Devitt, Radical Flow Inc., Waterloo ON, CAN
- Angel Diaz, IBM Research Division, Yorktown Heights NY, USA
- Ben Hinkle, Waterloo Maple, Inc., Waterloo ON, CAN
- Stephen Hunt, MATH.EDU Inc., Champaign IL, USA
- Douglas Lovell, IBM Hawthorn Research, Yorktown Heights NY, USA
- Patrick Ion, Mathematical Reviews (American Mathematical Society), Ann Arbor MI, USA
- Robert Miner, Geometry Technologies Inc., Minneapolis MN, USA
- Ivor Philips, Boeing, Seattle WA, USA
- Nico Poppelier, Salience, Utrecht, NL
- Dave Raggett, W3C (Hewlett Packard), Bristol, UK
- T.V. Raman, IBM Almaden, Palo Alto CA, USA
- Murray Sargent III, Microsoft, Redmond WA, USA
- Neil Soiffer, Wolfram Research Inc., Champaign IL, USA
- Irene Schena, Universitá di Bologna, Bologna, IT
- Paul Topping, Design Science Inc., Long Beach CA, USA
- Stephen Watt, University of Western Ontario, London ON, CAN

Earlier active members of this second W3C Math Working Group have included:

- Sam Dooley, IBM Research, Yortown Heights NY, USA
- Robert Sutor, IBM Research, Yortown Heights NY, USA
- Barry MacKichan, MacKichan Software, Las Cruces NM, USA

At the time of release of MathML 1.0 the Math Working Group was co-chaired by Patrick Ion and Robert Miner, then of the Geometry Center. Since that time several changes in membership have taken place. In the course of the update to MathML 1.01, in addition to people listed in the original membership below, corrections were offered by David Carlisle, Don Gignac, Kostya Serebriany, Ben Hinkle, Sebastian Rahtz, Sam Dooley and others.

Members of the Math Working Group responsible for the finished MathML 1.0 Specification were:

- Stephen Buswell, Stilo Technologies, Cardiff, UK
- Stéphane Dalmas, INRIA, Sophia Antipolis, FR
- Stan Devitt, Maplesoft Inc., Waterloo ON, CAN
- Angel Diaz, IBM Research Division, Yorktown Heights NY, USA
- Brenda Hunt, Wolfram Research Inc., Champaign IL, USA
- Stephen Hunt, Wolfram Research Inc., Champaign IL, USA
- Patrick Ion, Mathematical Reviews (American Mathematical Society), Ann Arbor MI, USA
- Robert Miner, Geometry Center, University of Minnesota, Minneapolis MN, USA
- Nico Poppelier, Elsevier Science, Amsterdam, NL
- Dave Raggett, W3C (Hewlett Packard), Bristol, UK
- T.V. Raman, Adobe Inc., Mountain View CA, USA
- Bruce Smith, Wolfram Research Inc., Champaign IL, USA
- Neil Soiffer, Wolfram Research Inc., Champaign IL, USA
- Robert Sutor, IBM Research, Yorktown Heights NY, USA
- Paul Topping, Design Science Inc., Long Beach CA, USA
- Stephen Watt, University of Western Ontario, London ON, CAN
- Ralph Youngen, American Mathematical Society, Providence RI, USA

Others who had been members of the W3C Math WG for periods at earlier stages were:

- Stephen Glim, Mathsoft Inc., Cambridge MA, USA
- Arnaud Le Hors, W3C, Cambridge MA, USA
- Ron Whitney, Texterity Inc., Boston MA, USA
- Lauren Wood, Softquad, Surrey BC, CAN
- Ka-Ping Yee, University of Waterloo, Waterloo ON, CAN

Appendix H

Changes (Non-Normative)

This appendix summarises the changes with respect to the preceding version (1.01) of the MathML Specification.

- changes to chapter 1 (upto revision 1.12)
  - none
- changes to chapter 2 (upto revision 1.17)
  - added reference to XML recommendation
  - removed error in description of allowed character in attribute values
- changes to chapter 3 (upto revision 1.28)
  - the attribute `definitionURL` can have a URL or a URI as value
  - added sections about `menclose` and `meqno`
  - added attributes `beveled`, `numalign` and `denomalign` to `mfrac`, and updated text accordingly
  - made sure examples are correct, and fixed several typos
  - added sections on `mchar` and `mglyph`
  - adjusted description of `mstyle` and `mglyph`
  - added examples for actuarial notation and long division
- changes to chapter 4 (upto revision 1.25)
  - discuss changed use of `apply` and deprecation of `reln`
  - introduce `csymbol` and discuss the relation with `fn`
  - introduce the new category of elementary classifical functions
  - introduce new content elements `arg`, `real`, `imaginar`, `equivalent`, `approx`, `divergence`, `grad`, `curl`, `laplacian`, `size`, `vectorproduct`, `scalarproduct` and `outerproduct`
  - made sure examples are correct, and fixed several typos
  - the attribute `definitionURL` can have a URL or a URI as value
  - revised some of the default renderings
  - described the use of presentation markup inside `cn`
  - modified the example for root to indicate that the rendering with a radical sign is for integer degrees only
  - default rendering of `not` made to match example markup
  - added `minus` to the row for unary arithmetic in the table in section 4.2.3
- changes to chapter 5 (upto revision 1.17)
  - added description of content-faithful transformation
  - use `csymbol` and not `fn` in examples
  - define list of content that can appear in presentation
  - add attribute `xref` for cross-referencing purposes

  - made sure examples are correct, and fixed several typos
  - added abrief description of the elements OMA, OMS and OMV
- changes to chapter 6 (upto revision 1.8)
  - none
- changes to chapter 7 (upto revision 1.19)
  - rewrote introductory text in section 7.2 and all text of section 7.2.1
  - rewrote many statements in future tense to present or past tense
  - reworked the text in acknowledgement of the fact that the top-level and interface elements for MathML are now in practice the same
  - rewrote the text about linking in accordance with the new XLink draft
  - revisited the material about interactions with embedded renderers to reflect the current state of DOM implementation
  - made sure examples are correct, and fixed several typos
- changes to chapter 8 (upto revision 1.3)
  - this is a completely new chapter
  - moved IDL definitions to a new, non-normative appendix
- changes to appendix A (upto revision 1.14)
  - renamed attribute `occurence` to `occurrence`
  - added global attribute `xref`
  - add links to tables for each entity set
- changes to appendix B (upto revision 1.6)
  - none
- changes to appendix C (upto revision 1.10)
  - none
- changes to appendix D (upto revision 1.13)
  - entries in operator dictionary are parametrized
  - operator dictionary has become non-normative part of the specification
- changes to appendix E (upto revision 1.13)
  - this is a completely new appendix, containing the IDL definitions that used to be in chapter 8
- changes to appendix F (upto revision 1.13)
  - added entries for XSL, XSLT and XSL FO
- changes to appendix G (upto revision 1.11)
  - all members of first and second Math working group are listed
  - new addresses for Maple
  - removed ‘Publishers’ from affiliation of NP
- changes to appendix H (upto revision 1.13)
  - completely new appendix, based on the logs obtained from CVS
- changes to appendix I (upto revision 1.8)
  - added entry for XML recommendation
  - added entry for other W3C documents
  - changed first author of reference 5 to ‘Chaundy’
- general changes
  - text of specification now in XML form, with HTML and XHTML rendering by means of XSLT, and PDF rendering by means of XSLT and TEX
  - fixed errors in spelling and notation
  - non-normative formula images replaced by HTML equivalents where possible
  - improved cross-referencing

Appendix I

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