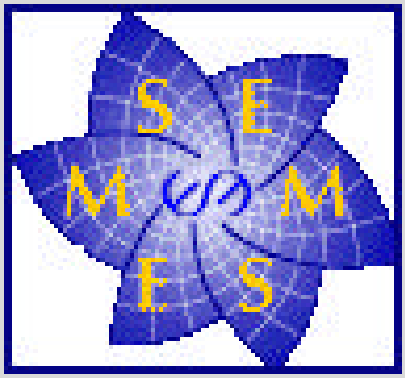
# EMS

**European Mathematical Society**
**Société Mathématique Européenne**

http://www.emis.de/





## IREM

**Institut de Recherche sur l'Enseignement des Mathématiques**

http://www-math.univ-fcomte.fr/DEPARTEMENT/CTU/IREM/INTERNAT.HTM

Reference levels in mathematics
in Europe at age16

Niveaux de référence en mathématiques
à 16 ans en Europe

## Luxembourg meeting

## May 11-12, 2001

# BUNDLE of PROPOSED REFERENCE QUESTIONS

# Part 2

**Presented by**

**Antoine BODIN and Lucia GRUGNETTI**

*English version*


**http://www.univ-lyon1.fr/apmep/**

**A**ssociation des **P**rofesseurs de **M**athématiques de l'**E**nseignement **P**ublic

**Observatoire EVAPM**

Équipe associée à l'INRP


http://www.inrp.fr

## EMS Project on Reference Levels in Mathematics - Reference questions
## April 19, 2001 - A. Bodin & L. Grugnetti

| Reference question N° | Individual or Group work | | Field ("Big ideas") *(Ref. PISA)* | | | | Mathematical competency *(Ref PISA)* | | | | | | | | Level of mathematisation | | | Target population | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | I | G | P1 | P2 | P3 | P4 | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | CC1 | CC2 | CC3 | T1 | T2 | T3 |
| | Individual | Group work | Quantity | Space and shape | Change and relationships | Uncertainty | Mathematical thinking skill | Mathematical argumentation skill | Modelling skill | Problem posing and solving skill | Representation skill | Symbolic, formal and technical | Communication skill | Aids and tools skill | Reproduction & routine | Integration & connection | Complex reasoning, insight & | for all | for further studies with | for further studies with |
| EMS001 | | X | 1 | | | 2 | 2 | | 1 | | | | | | | X | | X | | |
| EMS002 | X | | | 1 | | | 1 | | | 2 | | | | | | | X | X | | |
| EMS003 | | X | | 1 | | | | | | 1 | | | | | | | X | | X | |
| EMS004 | | X | 1 | 2 | 3 | | 1 | | | 2 | | | | | | X | | X | | |
| EMS005 | X | | | 1 | | | 2 | | | | 1 | | | | | X | | | X | |
| EMS006 | | X | | | | 1 | 2 | 3 | 1 | | | | | | | | X | X | | |
| EMS007 | X | | | | | 1 | 2 | | 1 | | | | | | | X | | X | | |
| EMS008 | | X | | | | 1 | | | 1 | | | | | 2 | | | X | | X | |
| EMS009 | X | X | | 1 | 2 | | | 2 | 1 | | | | | | | | X | X | | |
| EMS010 | | X | 1 | | | | 2 | | 1 | | | | | | | | X | X | | |
| EMS011 | X | | | 1 | | | | | 1 | | | | | | | X | | X | | |
| EMS012 | X | | | | 1 | | | | | 1 | | | | | | X | | X | | |
| EMS013 | X | | | | 1 | | | | 1 | 2 | | | | | | X | | X | | |
| EMS014 | X | | | | 1 | | | | 1 | 2 | | | | | | X | | X | | |
| EMS015 | X | | | | 1 | | | | | | | 1 | | | | X | | X | | |
| EMS016 | X | | | 1 | | | 1 | | | | | | | | | X | | X | | |
| EMS017 | X | | | | 1 | | | | | 1 | | | | | | X | | X | | |
| EMS018 | X | | | | 1 | | 1 | | | | | | | | | X | | X | | |
| EMS019 | | X | | | 1 | | | | | 1 | | | | | | X | | X | | |
| EMS020 | X | | 1 | | 2 | | | | 1 | | | | | | | X | | X | | |
| EMS021 | | X | | 1 | 2 | | | 1 | | | | | | | | | X | X | | |
| EMS022 | X | | | 1 | | | | | 1 | | | | | | | X | | X | | |
| EMS023 | | X | 1 | | | | 1 | | | | | | | | | X | | X | | |
| EMS024 | X | | | | 1 | | | 1 | 2 | | | | | | | X | | X | | |
| EMS025 | X | | | | 1 | | | | | | | 1 | | | | X | | X | | |
| EMS026 | | X | | | 2 | 1 | | 1 | 2 | | | | | | | X | | X | | |
| EMS027 | X | | 2 | 1 | | | | | | 1 | | | | | X | | | X | | |
| EMS028 | X | X | 1 | | | | 2 | | | | | 1 | | | X | | | X | | |
| EMS029 | X | X' | | | 1 | | | | | 1 | | | | | | X | | X' | X | |
| EMS030 | X | | | | 1 | | 2 | | 1 | | | | | | | X | | X | | |
| EMS031 | X | | | | 1 | | 1 | | | | | | | | | X | | | X | |
| EMS032 | X | | | | 1 | | | | 1 | | | | | | X | | | X | | |
| EMS033 | | X | | | 1 | | 2 | | 1 | | | | | | | X | | | | X |
| EMS034 | X | | | | 1 | | 1 | | | | | | | | | X | | | X | |
| EMS035 | X | | | | 1 | | 1 | | | | | | | | | X | | X | | |
| EMS036 | X | | | 1 | | | | | | | 1 | | | | | X | | X | | |
| EMS037 | X | | 1 | | | | 1 | | | | | | | | | X | | X | | |
| EMS038 | X | | | 1 | 2 | | | 2 | 1 | | | | | | | X | | X | | |
| EMS039 | X | | | 1 | | | 2 | | | | 1 | | | | | X | | | X | |
| EMS040 | X | | | | 1 | | | | | | | 1 | | | X | | | | X | |
| EMS041 | X | | | 1 | | | | 1 | | | | | | | X | | | | | X |
| EMS042 | | X | 1 | 2 | 3 | | 1 | | 2 | | 3 | | | | | X | | X | | |
| EMS043 | X | | | 1 | | | | 1 | | | | | | | | | X | | | X |
| EMS044 | X | | | | 1 | | | | | | | 1 | | | | X | | | X | |
| EMS045 | X | | | | 1 | | | | 1 | | | | | | | X | | X | | |
| EMS046 | X | | | 1 | | | | 1 | | | | | | | | | X | | | X |
| EMS047 | X | | 1 | | | | | | 1 | | | | | | X | | | X | | |
| EMS048 | X | | 1 | | | | | | 1 | | | | | | | X | | X | | |
| EMS049 | X | | | 1 | | | | | | | 1 | | | | | X | | X | | |
| EMS050 | | X | | | 1 | | 1 | | | | | 2 | | | | X | | | | X |
| EMS051 | X | | 1 | 2 | | | | | 1 | | | | | | X | | | X | | |
| EMS052 | X | | | | 1 | | | | 1 | | | | | | | X | | X | | |
| EMS053 | | X | 2 | | 1 | | 2 | | 1 | | | | | | | X | | | X | |
| EMS054 | | X | | 2 | 1 | | 1 | 2 | | | | | | | | X | | | X | |
| EMS055 | X | | | X | | | | | X | | | | | | | X | | X | | |
| EMS056 | | X | X | X | | | X | | X | | | | | | | X | | X | | |
| EMS057 | X | | | X | | | | | | | X | | | | | X | | | X | |
| EMS058 | X | | | X | | | X | | | | | | | | | X | | X | | |
| EMS059 | X | | | | X | | X | | | | | | | | | X | | X | | |
| EMS060 | X | | 1 | | | | | | | | 1 | | | | | X | | X | | |
| EMS061 | X | | 1 | 2 | | | | | 1 | | | | | | | X | | X | | |
| EMS062 | X | | | | 1 | | | | | | 1 | | | | | X | | X | | |
| EMS063 | X | | | | | 1 | | | | | 1 | | 2 | | | X | | X | | |
| EMS064 | X | | | | 1 | | | | | 1 | | | | | | X | | | X | |
| EMS065 | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |

*April 19, 2001 - A. Bodin & L. Grugnetti*

**EMS Reference question N° 001** **Heartbeats**

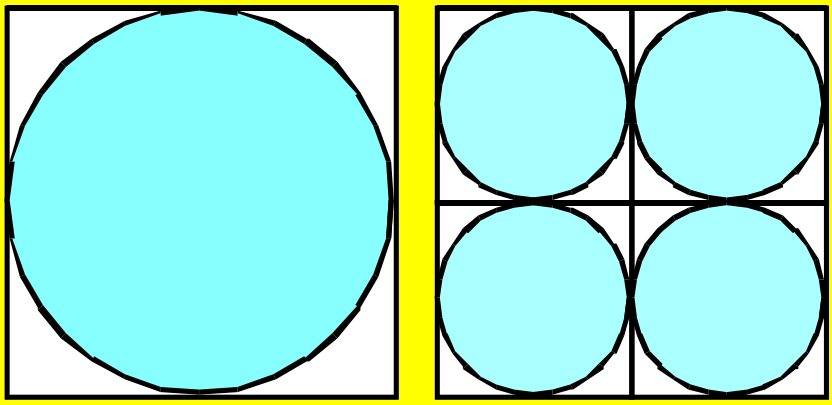
Estimate the number of heartbeats during a normal human life.

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Heartbeats EMS 001** |
| Origin of the question | Proposed by Vinicio Villani (ITALY) |
| Problematic field («Big idea») | P1 |
| Main contents supposed to be covered | Orders of magnitude, powers of 10, rounding |
| Competencies supposed to be implied | C3-C1 |
| Complexity class | Class 2 |
| Target group | Target 1 (for all) |
| Type of setting | Groupwork |

**EMS Reference question N° 002** **Circle tessellation**

Let Q be a square whose sides have a length of ***1 m***, and let C be the inscribed circle.

If one subdivides Q into smaller squares and considers the respective inscribed circles, one gets the figures below :



Increasing as you can imagine the number of subdivisions, does the area of the shaded part (I.E. THE PART COVERED BY THE CIRCLES) increase, decrease, or remain always the same ?

What about this question in space ?

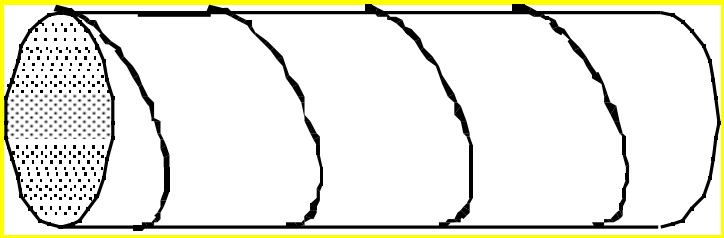
| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Circle tessellation EMS 002** |
| Origin of the question | Proposed by Vinicio Villani (ITALY) |
| Problematic field («Big idea») | P2 |
| Main contents supposed to be covered | either similarity for a synthetic answer, or simple algebraic calculations |
| Competencies supposed to be implied | C1 – C4 |
| Complexity class | Class 3 |
| Target group | Target 1 (for all) |
| Type of setting | Individual work |

**EMS Reference question N° 003** **The string**

A string is wound symmetrically around a circular rod. The string goes exactly 4 times around the rod. The circumference of the rod is 4 cm and its length is 12 cm.

Find the length of the string.

Show all your work.



**TIMSS**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **The string EMS 003** |
| Origin of the question | TIMSS - pop 3 - Specialists (released item) |
| Problematic field («Big idea») | P2 |
| Main contents supposed to be covered | Cylinder, development, Pythagorean theorem |
| Competencies supposed to be implied | C4 |
| Complexity class | Class 3 |
| Target group | Target 2 |
| Type of setting | Group work |

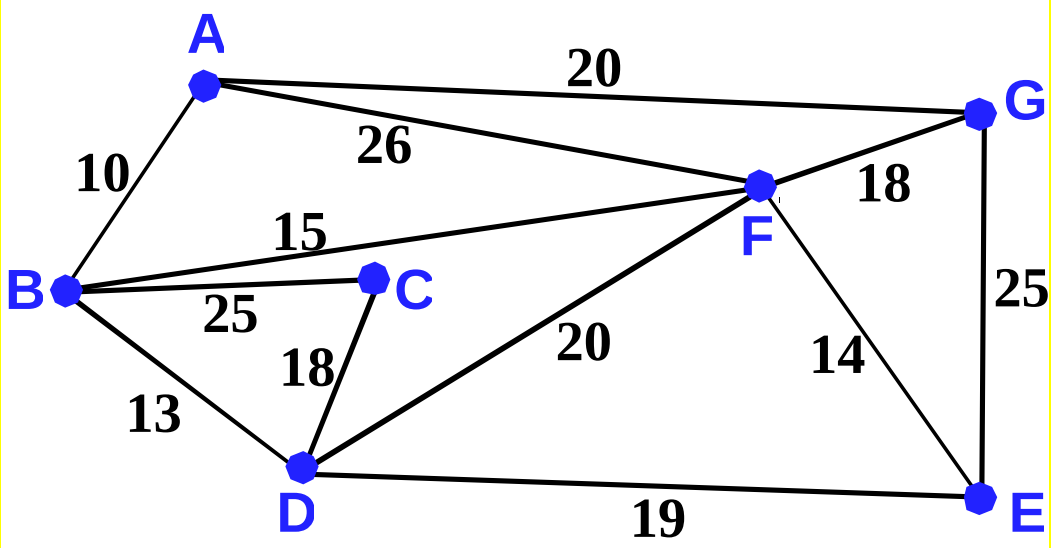
**EMS Reference question N° 004** **Paving roads**

There are seven small towns in Smith County that are connected by dirt roads, as in the diagram (the diagram is not to scale).

The distances are in kilometre. The county, which ha a limited budget, wants to pave some of the roads so that people can get from every town to every other town on paved roads, either directly or indirectly, but so that the total number of kilometres paved is minimised.

Find a network of paved roads that will fulfil the county's requirements.

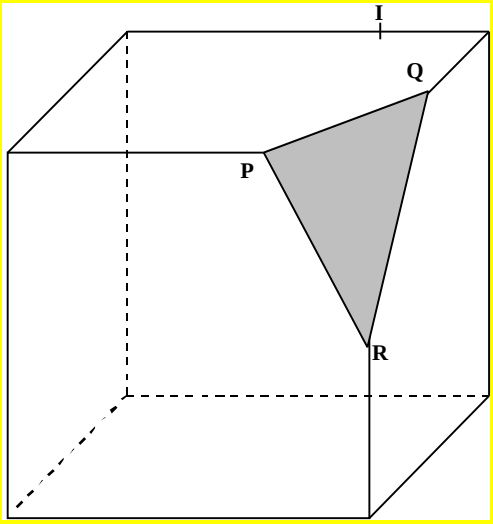
Eliminate any non-paved road from your drawing.




**NCTM**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Paving roads** | **EMS 004** |
| Origin of the question | NCTM standards 2000 | |
| Problematic field («Big idea») | P1 - P2 – P3 | |
| Main contents supposed to be covered | Optimisation | |
| Competencies supposed to be implied | C1 – C4 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Group work | |

**EMS Reference question N° 005** **Cutting a cube**




Here is a representation of a truncated cube (the cube has been sliced along a plane containing points P, Q and R.

Construct precisely on the figure the intersection of this cube with the plan containing point I that is parallel to the plane PQR.

**APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Paving roads** | **EMS 005** |
| Origin of the question | EVAPM/APMEP | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Parallelism in space | |
| Competencies supposed to be implied | C5 – C1 | |
| Complexity class | Class 2 | |
| Target group | Target 2 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | *medium* |
| Expected present achievement rate at 16 | | *Total success : 10% - partial success 50%* |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | *Total success : 14% - partial success 50%*) |

**EMS Reference question N°006** **Drawing straws**

Five people are drawing straws.

***Among 5 straws, 4 are of the same length while 1 is shorter than the others are.***

***The straws are presented in such a way the players can't get any cue about their respective lengths.***

***One after one, each player draws one of the straws.***

***The winner is the one that would have drawn the shorter straw.***

The last person that should take the last straw claims she is disadvantaged.

What do you think ?

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Drawing straws EMS 006** |
| Origin of the question | Proposed by Michel Henry (IREM Besançon) |
| Problematic field («Big idea») | P4 |
| Main contents supposed to be covered | Probability |
| Competencies supposed to be implied | C3-C1-C2 |
| Complexity class | Class 3 |
| Target group | 1 |
| Type of setting | Classroom work |

**EMS Reference question N°007** **Throwing dices 1**

One throws together 3 indistinguishable dices.

Is-it more likely to get three identical faces or to get a 4 – 2 – 1 ?

Would your answer be the same if the dices were throw separately ?

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Throwing dices EMS 007** |
| Origin of the question | Proposed by Michel Henry (IREM Besançon) |
| Problematic field («Big idea») | P4 (Uncertainty) |
| Main contents supposed to be covered | Probability |
| Competencies supposed to be implied | C3 - C1 |
| Complexity class | Class 2 |
| Target group | 1 |
| Type of setting | Individual work |

**Comments**

The question is not precise on purpose.

In the second part of the question, we are expecting a student to be able to considering by himself different possible cases : looking for the set { 4 ; 1 ; 2 }, or looking for the sequence (1 ; 2 ; 3 ).

Out of examinations there is often some advantage to let some uncertainty in the questioning.

**EMS Reference question N°008** **Throwing dices 2**

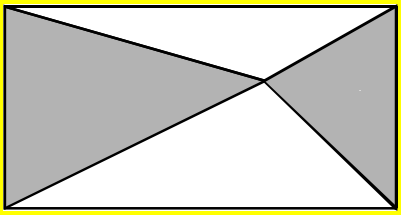
How many times must you plan to throw a dice to have 95% of chance to get one six (at least) ?

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Throwing dices 2 EMS 008** |
| Origin of the question | Proposed by Michel Henry (IREM Besançon) |
| Problematic field («Big idea») | P4 (Uncertainty) |
| Main contents supposed to be covered | Probability |
| Competencies supposed to be implied | C3 - C8 |
| Complexity class | Class 3 |
| Target group | 2 |
| Type of setting | Group work |

**EMS Reference question N°009** **The Inheritance**

Two brothers inherit land in a rectangular shape.

To divide it into equal area, a neighbour suggests that they should plant at any point on the terrain and traces of right segments that go from this stake to the four summits of the terrain.

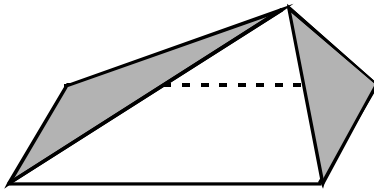


One of the brothers will take parts in grey on the figure, the other the part in white.

**Do the two parts really equal ?**

Justify your reasoning

Investigate what happens if the figure is a pyramid (for instance a roof of a house) seen from above.



**RMT**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **The inheritance** | **EMS 009** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 2000) | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Triangle area - Pythagorean theorem. | |
| Competencies supposed to be implied | C3_C2 | |
| Complexity class | Class 2 | |
| Target group | Target 1 (for all) | |
| Type of setting | Individual work for the first part – group work for the second part. | |

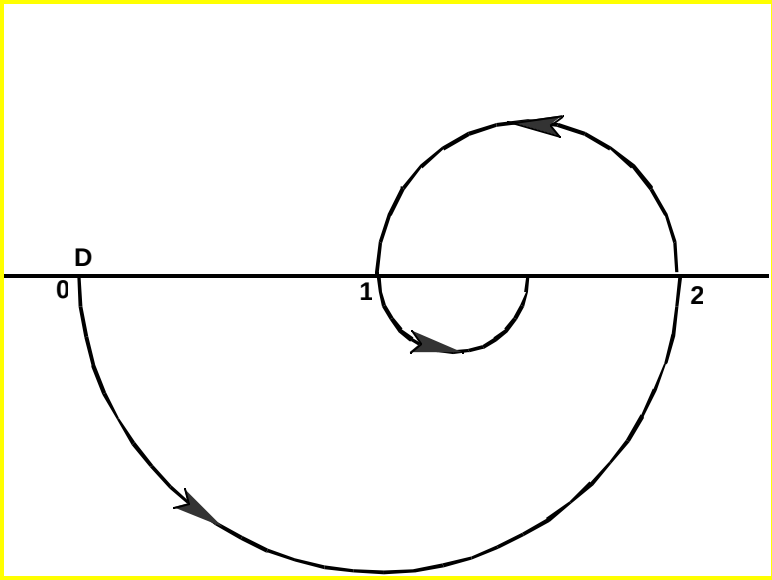
| COUNTRY | | ITALY |
|---|---|---|
| Fitness to curriculum | | |
| Expected present achievement rate at 16 | | *60%* |
| **Try out of the question** | *Context of the trial* | *RMT at 14 Italy and in Switzerland (only first part)* |
| | *Number of students* | |
| | *Results* | *Concerning proof : difficult at 14* |

The space situation is open for several kinds of simulations : using dynamical geometry software or spreadsheet.

**P1 _ C3_C1** **EMS RQ 010**

**EMS Reference question N°010** **A strange spiral**

From a starting point a semicircle of radius 1 is described. It is then continued into a semicircle of radius 1/2, and so on, such that each semicircle has a radius half of the preceding semicircle.




What is the distance from the starting point (D) to the end point ?

What is the length of the path ?

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **A strange spiral EMS 010** |
| Origin of the question | Proposed by François Jaquet (Switzerland) |
| Problematic field («Big idea») | P1 |
| Main contents supposed to be covered | Length of a circle - Infinity Sum |
| Competencies supposed to be implied | C3_C1 |
| Complexity class | Class 3 |
| Target group | Target 1 |
| Type of setting | Group work |

**EMS Reference question N°011** **The fly**

The small rectangle on the right is a photograph of the big one on the left.

At the moment in which the photograph was taken, a fly (*) placed itself on the big rectangle.

The photographer made sure he erased it while developing the photograph.

(*)The fly is symbolised by  .

**Put the fly onto the photograph.**

**Explain your method.**

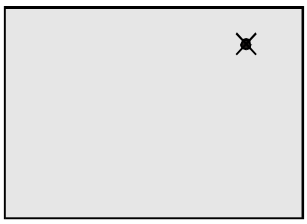
**RMT**

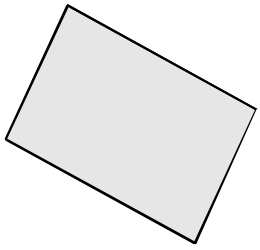
| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **The fly EMS 011** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 2000) |
| Problematic field («Big idea») | P2 |
| Main contents supposed to be covered | Enlargement (homothety) - Proportionality |
| Competencies supposed to be implied | C3 |
| Complexity class | Class 2 |
| Target group | Target 1 (for all) |
| Type of setting | Individual work |

Comments

It would be better to give the drawing on a sheet big enough to allow geometrical constructions as well as computing procedure.

In a second time the two rectangles might be given non-parallels.





**EMS Reference question N°012** **Grape gathering**

It is a period of grape gathering !

Each grape gatherer receives a sum of 60 Euro and a case of grape for an 8 hours working day.

On a particular day, after having worked for 5 hours, Paolo had to go home.

For his work he received 30 Euro and a case of grape.

**What is the value of a case of grape ?**

Explain your reasoning.

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Grape gathering EMS 012** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 1998) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Proportionality, equations |
| Competencies supposed to be implied | C4 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°013** **Neighbours**

At a diner, all the chairs around a big round table are occupied.

7 women have a woman at their right.

12 women have a man at their right.

3 men out of 4 have a woman on their right.

**How many are they on the whole (men plus women) ?**

Explain your method.

**RMT**

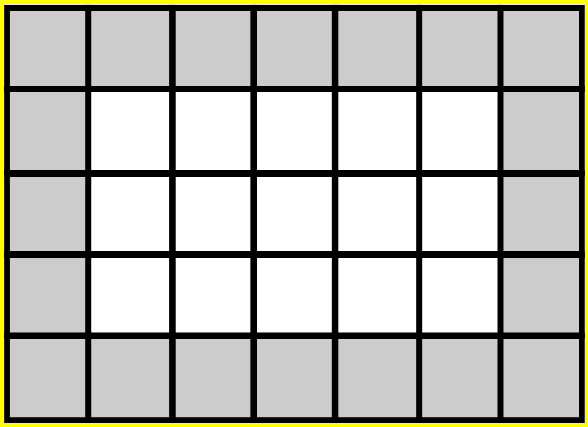
| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Neighbours** | **EMS 013** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 2000) | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Logical reasoning | |
| Competencies supposed to be implied | C4 – C3 | |
| Complexity class | Class 2 | |
| Target group | Target 1 (for all) | |
| Type of setting | Individual work | |

**EMS Reference question N°014** **Borders**

Mombo Carpet makes squared carpets.

He would like to create an “equality” model that has as much grey squares on the border as white squares into the interior.

His apprentice Amal proposed the model in the figure that is unfortunately not convenient, because of 15 white squares into the interior and 20 grey squares on the border.



**Is-it possible to create carpets having as much grey squares on the border as white squares into the interior ?**

Explain your answer.

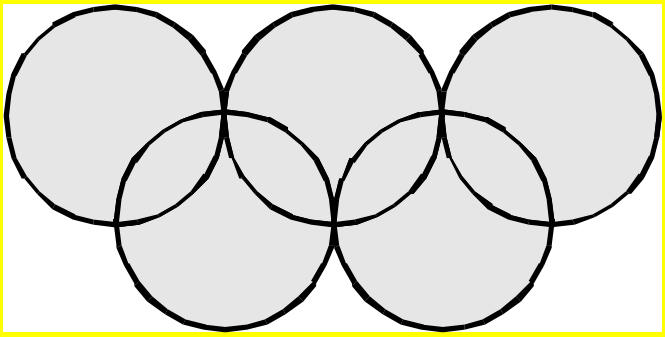
**RMT**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Borders EMS 014** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 1996) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C3 -C4 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°015** **Numbers and circles**

May we put the numbers 1, 2, …,9 in the places formed by the five circles of the following scheme, in order to obtain the same sum inside each circle ?

Note that symmetrical placements of the numbers are not considered different.



| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Numbers and circles EMS 015** |
| Origin of the question | Question proposed by Panayiotis Vlamos (Greece) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C6 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°016** **The cat**

The great old geometer would like to construct the perpendicular bisector of segment AB, when his little cat jumps into the table and takes place as in figure.

Can he draw some parts of the desired line without disturbing the cat ? (his compass and ruler is big).


| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **The cat** | **EMS 016** |
| Origin of the question | Proposed by Sandor Dobos (Hungary) | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Bisector of a segment | |
| Competencies supposed to be implied | C1 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

**EMS Reference question N°017** **Transports**

This Monday the firm SAVONEX has produced 291 case of soap.

For carrying out all this cases, the lorry of the firm has done several trips, always entirely full.

As it left only 3 cases, the driver decided not to do another trip and just to wait for taking them the following day.

On Tuesday, with the new production, there were on the whole 229 cases to carry out.

The lorry did 2 trips less than the previous day, all full but the last one where it left still room for 11 cases.

**How many trips the lorry has done the second day and how many cases does it take when it is full ?**

**RMT**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Transports EMS 017** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 1999) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C4 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°018** **The tunnel**

Four persons are going to get through of a narrow dark tunnel.

They have a torch that can work for 18 minutes.

They need respectively 1 , 2 , 5 , and 10 minutes for getting through the tunnel.

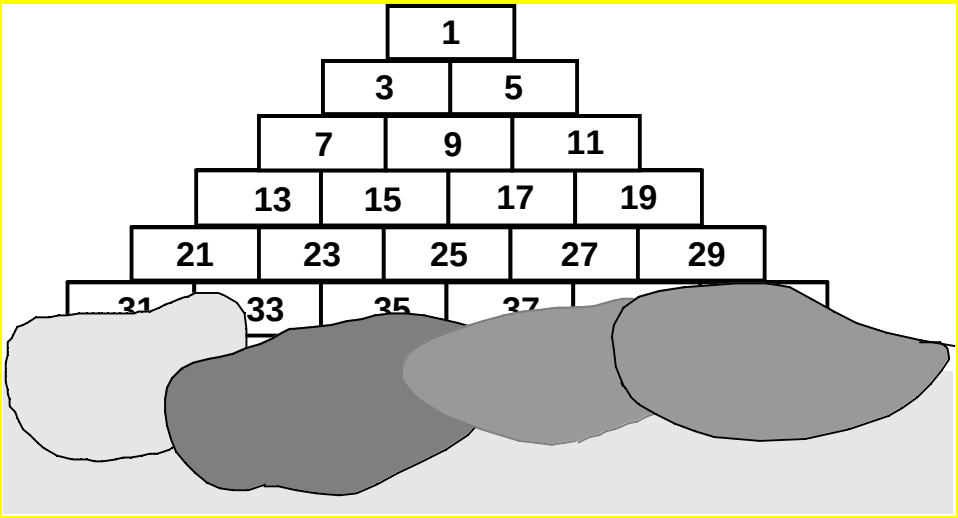
Without the torch, they can't go.

The tunnel is narrow so at most two of them can go together.

**Is-it possible to get all of them to the other side ?**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **The tunnel EMS 018** |
| Origin of the question | Proposed by Sandor Dobos (Hungary) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Logical reasoning |
| Competencies supposed to be implied | C1 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°019** **The pyramid**




This pyramid of numbers continues under the clouds.

The total sum of the numbers of the first level is 29 791

**How many levels does this pyramid of numbers have ?**

Explain your method

**RMT**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **The pyramid EMS 019** |
| Origin of the question | Inspired by PISA 2000 (?) – proposed by François Jaquet |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Numbers |
| Competencies supposed to be implied | C4 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Group work |

**EMS Reference question N°020** **Martha boxes**

Martha used to arrange her construction set of equal dimension cubes in a cardboard box with a square basis.

When lining up the cubes, the box was full and it was any space left.

With time the box became torn off, and Martha had to replace it.

She found o box of the same height but with a rectangular basis.

In her new box she can line up exactly a quarter more of her cubes along the length and exactly a quarter of her cubes less along the width.

At the end, when her new box is full, it left 12 cubes off the box.

**Could you find the total number of Martha's cubes ?**

**Explain you reasoning.**

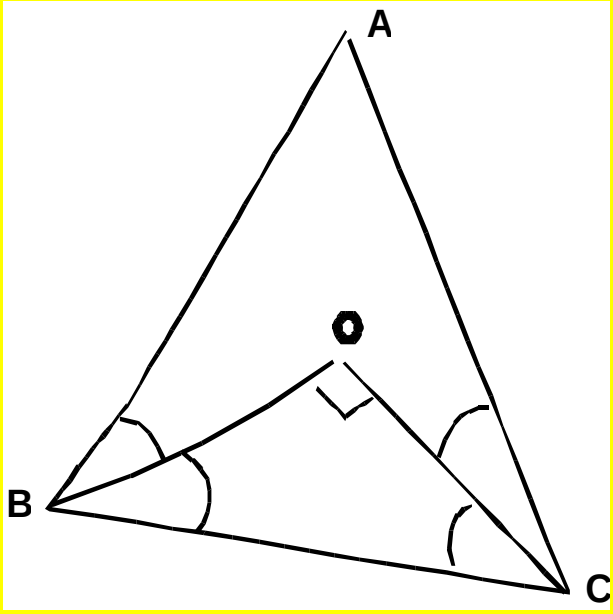
**RMT**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Martha boxes EMS 020** |
| Origin of the question | Proposed by Lucia Grugnetti and François Jaquet (from RMT 1999) |
| Problematic field («Big idea») | P1 – P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C3 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°021** **Bisectors**

Michel wants to draw a triangle of which the bisectors of angles B and C are perpendicular.

Can he succeed ?




| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Bisectors EMS 21** |
| Origin of the question | Proposed by Philippe R. Richard (Spain) |
| Problematic field («Big idea») | P2 – P3 |
| Main contents supposed to be covered | Sum of the angles of a triangle |
| Competencies supposed to be implied | C2 |
| Complexity class | Class 3 |
| Target group | Target 1 |
| Type of setting | Group work |

**EMS Reference question N°022** **The stick**

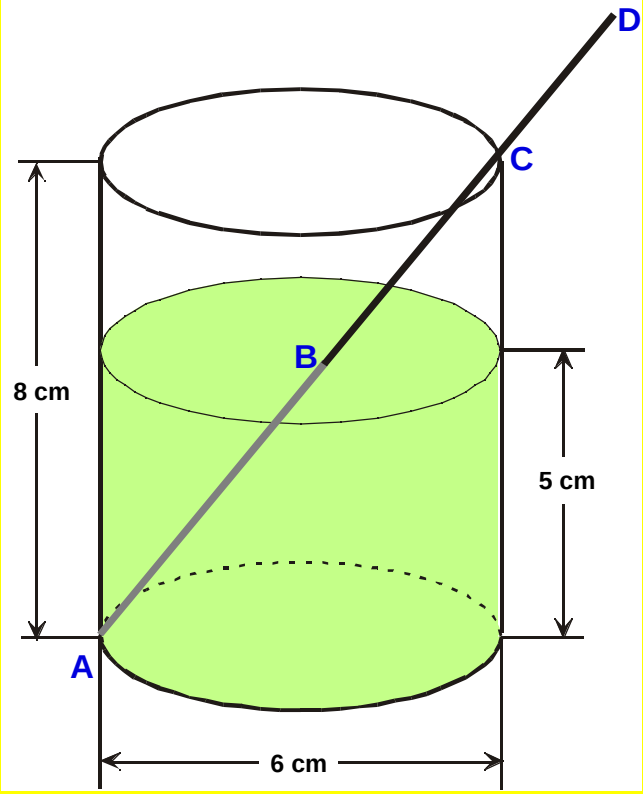
A container is 8 cm high and 6 cm of diameter.

The liquid in it is 5 cm high.

A stick, AD, 15 cm long, is plunged into a container cylindrical according to the figure.

The stick is plunged for length AC is maximum. It meets the liquid on point B.

**Which is length AB ? *(submerged part of the stick).***




| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **The stick** | **EMS 0** |
| Origin of the question | Proposed by Christos Chasiostis (Greece) | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Proportionality in geometry - Tales theorem | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

**EMS Reference question N°023** **Running 1 km**

Could you run 1 km in 1 minute?

What about someone else ?

Explain your answer

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Running 1 km** | **EMS 023** |
| Origin of the question | Proposed by Tony Gardiner (England) | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Proportionality | |
| Competencies supposed to be implied | C1 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Group work | |

**EMS Reference question N°024** **Driving school**

When in learning for a driver's licence at Roy and Roger's Driving School the theory lessons and the obligatory driving lesson together cost SEK 2300. An extra driving lesson costs SEK 220 each time.

a) Sara has just got her driver's licence. She paid a total of SEK 4060 to the driving school. How many extra driving lessons did she have?

b) Write the law that describes how much you have to pay all together to the driving school if you take a complete course for a driver's licence and have x extra driving lessons.

c) Write comments on the law you wrote.

**Sweden National Examination**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Driving school EMS 024** |
| Origin of the question | Sweden National Examination |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C2 - C3 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°025** **Magic computation**

A teacher said to his students:

Think of a number and add 15 to it. Multiply the sum by 4 and then subtract 8 from your result. Divide the difference by 4 and finally subtract 12 from your quotient. If you tell me what answer you came up with, I will tell you what number you were thinking of.

**a) Monica comes up with 5 as her answer.**
**What number was she thinking of ?**

**b) Show that the teacher's method is correct for all numbers.**

**Sweden National Examination**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Magic computation EMS 025** |
| Origin of the question | Sweden National Examination |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Equations |
| Competencies supposed to be implied | C6 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°026** **Cars statistics**

ASA has been given an assignment by the newspaper to write an article on how prices of used cars vary. She has chosen the Volvo 245 and BMW 300-series for her investigation. In an advertisement magazine, she found the prices for different cars of different year models. The prices are listed in the tables below. She now needs your help.

a) For the purpose of ASA´s assignment, show the prices for the different year models of Volvo 245 in an appropriate diagram.

b) ASA has heard that ”on the average the price of Volvos decreased with 8 000 kr each year.” Does this ”rule of thumb” agree with the values which are included in the investigation?

c) Can you find a similar or another ”rule of thumb” for the BMW?

d) If you were going to buy one of the car models, how could this investigation be of help to you?

| VOLVO 245 | |
|---|---|
| **Year of model** | **Price(kr)** |
| 1992 | 79 900<br>96 000 |
| 1991 | 93 000<br>94 000<br>77 000<br>59 000 |
| 1990 | 89 000<br>66 900<br>67 000 |
| 1989 | 66 000 |
| 1988 | 42 000<br>60 000<br>65 000<br>35 000 |
| 1987 | 49 000<br>45 000<br>37 000 |
| 1986 | 35 500<br>29 500<br>36 000<br>40 000<br>40 000<br>37 000 |
| 1985 | 38 000<br>34 000<br>25 000<br>32 500<br>20 000<br>32 500 |

| BMW 300 | |
|---|---|
| **Year of model** | **Price(kr)** |
| 1992 | 156 000<br>179 000<br>198 000<br>167 000 |
| 1991 | 149 000<br>105 000<br>112 000<br>136 000 |
| 1990 | 78 000<br>94 000<br>80 000 |
| 1989 | 63 000<br>75 000<br>74 000<br>77 500<br>89 000 |
| 1988 | 59 500<br>52 500<br>65 000<br>60 000<br>65 000 |
| 1987 | 49 000<br>48 000<br>45 000<br>59 000<br>55 000<br>50 000 |
| 1986 | 45 000<br>44 000<br>42 000 |
| 1985 | 40 000<br>39 000<br>42 000 |

**Sweden National Examination**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Cars statistics** **EMS 026** |
| Origin of the question | Sweden National Examination |
| Problematic field («Big idea») | P4_P3 |
| Main contents supposed to be covered | Statistics - Mean |
| Competencies supposed to be implied | C2 – C3 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Group work |

**EMS Reference question N°027** **American Pizza**

A round American pizza for one person has a diameter of 21 cm.

How large should the diameter be if the pizza is for two people?

**Sweden National Examination**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **American Pizza** | **EMS 027** |
| Origin of the question | Sweden National Examination | |
| Problematic field («Big idea») | P2_P1 | |
| Main contents supposed to be covered | Disc area | |
| Competencies supposed to be implied | C4 | |
| Complexity class | Class 1 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

**EMS Reference question N°028** **PI Value**

Throughout history, mathematicians have tried to find a standard approximation for $\pi$.

Here are some of the values that have been used :

| *Indians* | *Egyptians* | *Romans* | *Greeks* |
|---|---|---|---|
| $\sqrt{10}$ | $\frac{256}{81}$ | $3\frac{1}{8}$ | $\frac{22}{7}$ |

a) Which value is closest to $\pi$ and which is farthest from $\pi$ ?

b) If we use the Egyptian value of $\pi$ to computing the circumference of a circle with a diameter of 125 m, what is the error ?

Give a value of the error rounded to 1 cm.

| **EMS REFERENCE QUESTION IDENTITY CARD** | |
|---|---|
| **NAME and Number of the Question :** | **PI Value EMS 028** |
| Origin of the question | Adapted from Sweden National Examination |
| Problematic field («Big idea») | P1 |
| Main contents supposed to be covered | Numbers - circle length |
| Competencies supposed to be implied | C1 – C6 |
| Complexity class | Class 1 |
| Target group | Target 1 |
| Type of setting | Individual or group work |

**EMS Reference question N°09** **Changing places**

| | |
|---|---|
| Choose a two-digit number. | 84 |
| Let the two digits exchange places. | 48 |
| Compute the difference between the larger and the smaller of the two numbers. | 84 - 48 = 36 |
| Let the digits exchange places. | 63 |
| Compute the difference between the larger and the smaller of the two numbers. | 63 - 36 = 27 |
| Let the digits exchange places. | |

**Continue as long as you can.**

**What do you notice from the numbers you get ?**

**What happens if you start with another two-digit number ?**

**INVESTIGATE!**

**Sweden National Examination**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Changing places** | **EMS 00** |
| Origin of the question | Sweden National Examination | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Numbers | |
| Competencies supposed to be implied | C4 | |
| Complexity class | Class 2 | |
| Target group | Target 1 (group work) – Target 2 (individual work) | |
| Type of setting | Individual or group work | |

**EMS Reference question N°030** **World population**

At present the world population P is estimated in 6 000 000 000 individuals.

According to recent data, the yearly increase of the world population amounts to 1,7 %.

Under the assumption that this increase rate remains the same also in the future, write down a formula expressing the world population P1, P2, ..., Pn, expected after 1, 2, ..., n years.

When the population will get the double ?

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **World population EMS 030** |
| Origin of the question | Question proposed by Vinicio Villani (ITALY) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Percentages - Equations |
| Competencies supposed to be implied | C3_C1 |
| Complexity class | Class 2 |
| Target group | Target 1 |
| Type of setting | Individual work |

**EMS Reference question N°031** **A polygon**

A polygon is called "regular" if all its sides are equal and also all its angles are equal.

Hence a polygon is "irregular" (= not regular) if and only if :

All its sides and all its angles are different

All its sides or all its angles are different

At least two sides are different and at least two angles are different

At least two sides are different or at least two angles are different

**Which is or (which are) the good answer(s) ?**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **A polygon EMS 031** |
| Origin of the question | Question proposed by Vinicio Villani (ITALY) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Logical reasoning |
| Competencies supposed to be implied | C1 |
| Complexity class | Class 2 |
| Target group | Target 2 |
| Type of setting | Individual work |

**EMS Reference question N°032** **Lollipops**

A child has bought 10 lollipops, all at the same unit price.

If each lollipop had cost 5 cent less, he would have got 2 lollipops more for the same total cost.

**What is the price of 1 lollipop ?**

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Lollipops** | **EMS 032** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Equations | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class1 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 55% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 54% |

**EMS - Reference question N°033** **Meeting trains**

On any full hour, a train leaves DETROIT to CHICAGO (i.e. on 0h, 1h, 2h,…etc…).

The trip lasts 6 hours.

Under the same conditions, on any full hour, a train leaves CHICAGO to DETROIT.

**If you took the train in DETROIT to get to CHICAGO, how many trains coming from Chicago**.

***One doesn't take into account the trains meet in Detroit or in Chicago stations.***

**Question EVAPM/APMEP**

| **EMS REFERENCE QUESTION IDENTITY CARD** | |
|---|---|
| **NAME and Number of the Question :** | **Meeting trains EMS 033** |
| Origin of the question | EVAPM/APMEP – France (Take over a classic) |
| Problematic field («Big idea») | P3 |
| Main contents supposed to be covered | Graphical representations - Logical reasoning |
| Competencies supposed to be implied | C3_C1 |
| Complexity class | Class2 |
| Target group | Population 3 |
| Type of setting | Group work |

| **TRIAL COUNTRY** | | **FRANCE** |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 10% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 10% |

**EMS - Reference question N°034** **Balls**

Some balls, coloured WHITE, BLACK or RED, have been distributed onto three boxes labelled A, B and C.

**Using the information given below, you are ask to find the number of balls of each colour in each of the boxes.**

- In box B, there are 5 red balls and there are the same number of black balls than in box A.
- There is no white ball in box C.
- The number of black balls in box C is the same as the number of white balls in box A.
- In box C, there are the same numbers of red balls than in box B.
- In box A, the number of red balls is the same than the number of black balls.
- In box C, there are 12 balls on the whole.
- On the whole there are 7 red balls in boxes A and B.
- In box B, there are as much white balls as in box C.

**Try to shape your answer as clearly as possible and don't forget to explain and justify your solution**.

**Question EVAPM/APMEP**

| **EMS REFERENCE QUESTION IDENTITY CARD** | | |
|---|---|---|
| **NAME and Number of the Question :** | **Balls** | **EMS 034** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Logical reasoning | |
| Competencies supposed to be implied | C1 | |
| Complexity class | Class2 | |
| Target group | Population 2 | |
| Type of setting | Individual work | |

| **TRIAL COUNTRY** | | **FRANCE** |
|---|---|---|
| Fitness to curriculum | | Low |
| Expected present achievement rate at 16 | | Results: 75% - Justifications : 50% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | results : 75% - Justifications : 50% |

**EMS - Reference question N°035** **Logic**

Three people of three different nationalities live the three first houses in a given street.

Each house has a different colour and each person has a different job.

A - The French lives in the red house.

B - The German is a musician.

C - The English lives in the house in the middle.

D - The red house is next to the green house.

E – The writer lives in the first house on the left.

**Which is the writer's nationality and who lives in the yellow house ?**

Don't forget to explain your reasoning.

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Logic** | **EMS 035** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Logical reasoning | |
| Competencies supposed to be implied | C1 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

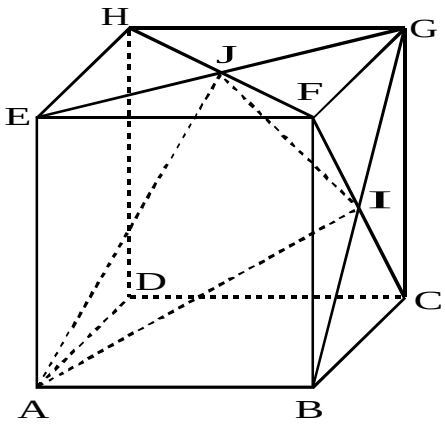
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Low |
| Expected present achievement rate at 16 | | Results : 70% - Justifications : 40% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | Results : 70% - Justifications : 40% |

**EMS - Reference question N°036** **Cube and triangles**

ABCDEFGH is a cube.

Segments [FC] and [GB] cut each other point I.

Segments [HF] and [EG] cut each other point J.




Circle the good answers

· Triangle EGB is rectangle in G RIGHT WRONG

· Triangle IAJ is isosceles RIGHT WRONG

· Triangle AEJ is rectangle in E RIGHT WRONG

· Triangle AEJ is isosceles RIGHT WRONG

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Cube et triangles EMS 036** |
| Origin of the question | EVAPM/APMEP – France |
| Problematic field («Big idea») | P2 |
| Main contents supposed to be covered | Space geometry |
| Competencies supposed to be implied | C5 |
| Complexity class | Class2 |
| Target group | Population 1 |
| Type of setting | Individual work |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 60% |

**EMS - Reference question N°037** **A test**

A same test has been given to two group-classes.

The first class with 20 students has obtained 12.30 as a mean score.

The second class with 30 students has obtained 14.80 as a mean score.

**Which is the mean score of the group formed with the 50 students from these two classes ?** ***(Tick on the correct answer)***

12,55

13,30

13,55

13,80

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **A test** | **EMS 037** |
| Origin of the question | SIMS . Adapted by EVAPM/APMEP – France | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Statistics - mean | |
| Competencies supposed to be implied | C1 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 33% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 33% |

**EMS - Reference question N°038** **Triangle coordinates**

The rectangular co-ordinates of three points are :

A ( 2 ; 4 ) ; B( 8 ; 3 ) ; C(10 ; 12 )

Is triangle ABC a right triangle ?

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **Triangle coordinates EMS 038** |
| Origin of the question | EVAPM/APMEP – France |
| Problematic field («Big idea») | P2_P3 |
| Main contents supposed to be covered | Coordinate geometry - Pythagore |
| Competencies supposed to be implied | C3_C2 |
| Complexity class | Class2 |
| Target group | Population 1 |
| Type of setting | Individual work |

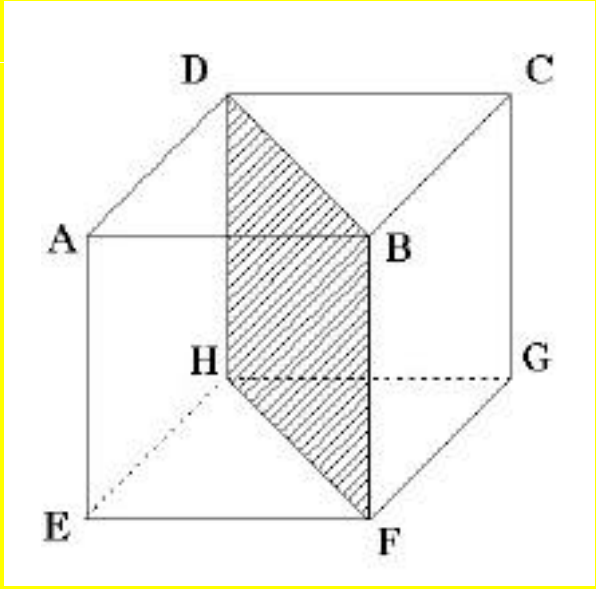
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 50% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) (et troisième 1990) |
| | *Number of students* | 100 000 |
| | *Results* | 41% en seconde – 66% en troisième 1990 |

**EMS - Reference question N°039** **Cube section**

Here is a cube draw in perspective.

As a fact, the edge of this cube is 4 cm long.

This cube is cut into two right prisms along plan DBFH.




**DRAW in its real dimensions the common face DBFH of these two prisms.**

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Cube section** | **EMS 039** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Space geometry | |
| Competencies supposed to be implied | C5_C1 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 58% |

**EMS - Reference question N°040** **An equation**

Solve this equation :

$$( 3x + 5 ) ( x - 2 ) - ( x + 4 ) ( x - 2 ) = 0$$

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **An equation** | **EMS 040** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Algèbre | |
| Competencies supposed to be implied | C6 | |
| Complexity class | Class1 | |
| Target group | Population 2 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 40% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 41% |

**EMS - Reference question N°041** **A pyramid**

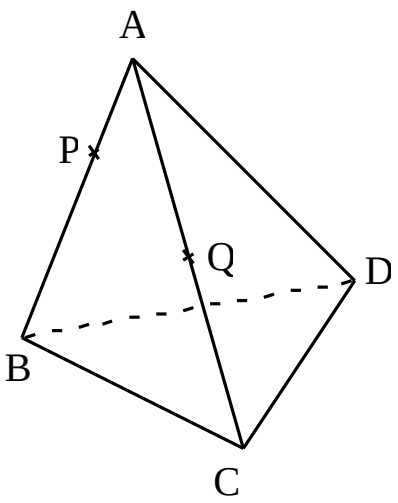
ABCD is a pyramid.

P is a point on edge [AB],

Q is a point on edge [AC].

Lines (PQ) and (BC) are not parallel.

(see figure)




**Draw the intersection between line (PQ) and plan (BCD)**

**Justify your answer.**

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **A pyramid** | **EMS 041** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Space geometry | |
| Competencies supposed to be implied | C2 | |
| Complexity class | Class1 | |
| Target group | Population 3 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | Drawing: 30% - justification : 15% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | Drawing : 30% - justification : 15% |

**EMS - Reference question N°042** **Volumes**

The figure represents four solids : a cone, a cylinder, a pyramid and a prism.

The cone is 24 $cm^3$ of volume.

The cylinder and the cone have same base area.

The pyramid and the prism have a base area double of that of the cylinder.

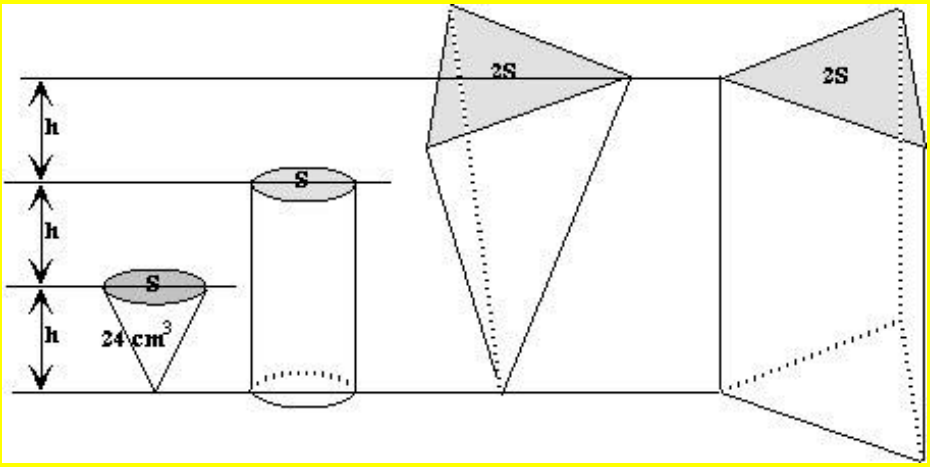
The height of the cylinder is double of that of the cone.

The heights of the pyramid and of the prism are triple of that of the cone.

What is the volume of the pyramid ?

What is the volume of the prism ?

What is the volume of the cylinder ?




**From EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Volumes** | **EMS 042** |
| Origin of the question | From de EVAPM/APMEP – France | |
| Problematic field («Big idea») | P1_P2_P3 | |
| Main contents supposed to be covered | Volumes of usual solids | |
| Competencies supposed to be implied | C1_C5 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Group work | |

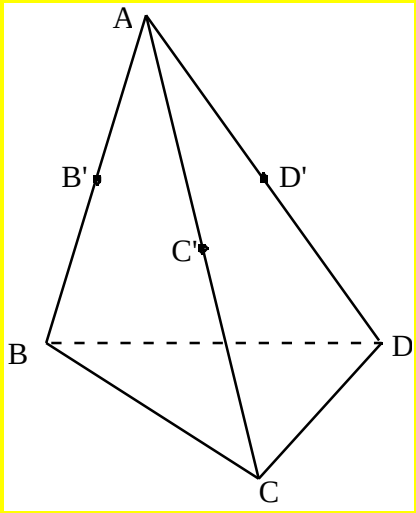
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Low |
| Expected present achievement rate at 16 | | Less than 10% if individual work |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) : the 4 solids having then same height and same base area. |
| | *Number of students* | 100 000 |
| | *Results* | 10% (20% for the pyramid) |

**EMS - Reference question N°043** **Plans and pyramid**

ABCD is a pyramid.

Point B', C' and D' are respectively the middles of segments [AB], [AC] and [AD].

Prove that plans (BCD) and (B'C'D') are parallels.




**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Plans and pyramid** | **EMS 043** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Space geometrie | |
| Competencies supposed to be implied | C2 | |
| Complexity class | Class3 | |
| Target group | Population 3 | |
| Type of setting | Group work | |

| **TRIAL COUNTRY** | | **FRANCE** |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 25% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 25% |

**EMS - Reference question N°044** **Equation system**

Equation systems A et B are given :

**A** $\begin{cases} 2x + 3y = 1 \\ 4x - 3y = 2 \end{cases}$ **B** $\begin{cases} x - y = -2 \\ 2x - 2y = 1 \end{cases}$

**For each of the following cases cross out what doesn't apply**

| System A | System B |
|---|---|
| *has - has not* | *has - has not* |
| a unique solution | a unique solution |

**Justify your answer**

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Equation system** | **EMS 044** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Equations | |
| Competencies supposed to be implied | C6 | |
| Complexity class | Class2 | |
| Target group | Population 2 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 25% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 22% |

**EMS - Reference question N°045** **Shirts and trousers**

In a department store there are some shirts and some pair of trousers on sale.

All shirts are sold the same unit price.

All pair of trousers are sold the same unit price.

John has paid 570 F for 7 shirts and 3 pair of trousers.

Sophy has paid 730 F for 3 shirts and 7 pair of trousers.

**Work out the price of one shirt and of one pair of trousers.**

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Shirts and trousers** | **EMS 045** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3_P1 | |
| Main contents supposed to be covered | Equations | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

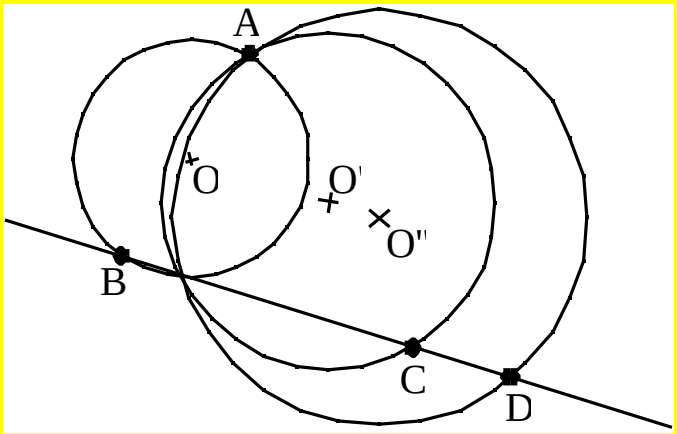
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 63% |

**EMS - Reference question N°046** **Circles**

Points B, C, D, are on a same straight line ; A is a point out of straight line (BC).

Points O, O' and O'' are the centres of circles of respective diameters [AB], [AC] and [AD].

Prove the points O, O', O'' are on a same straight line.




**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Circles** | **EMS 046** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Plane geometry | |
| Competencies supposed to be implied | C2 | |
| Complexity class | Class3 | |
| Target group | Population 3 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 20% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | 17% |

**EMS - Reference question N°047** **The loan**

One person has borrowed 1000 F as interest-free loan.

She has already paid off a sum S

She still has to pay off a sum equal to $\frac{2}{3}$ of the sum S already paid off.

**Work out sum S.**

***Show your work***

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **The loan** | **EMS 047** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Equations | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class1 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 70% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de seconde 1991 (age 16) |
| | *Number of students* | 100 000 |
| | *Results* | *68%* |

**EMS - Reference question N°048** **An increase**

After a 40% increase, an object is sold 84 F.

**What was its price before this increase ?**

***Show your work***

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **An increase** | **EMS 048** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Percentages - Equations | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

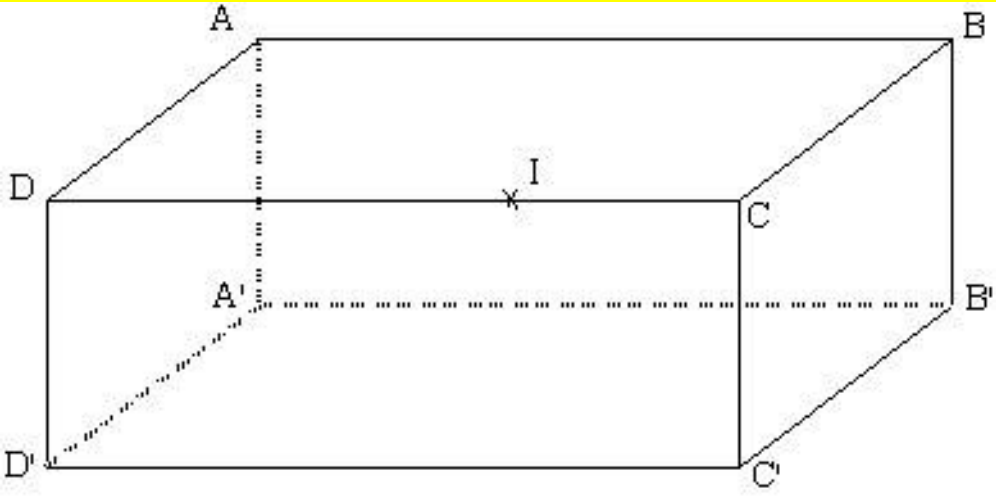
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 66% (22% en fin de troisième 1990) |

**EMS - Reference question N°049** **Plane section**

Here is a parallelepiped ABCDD'C'B'A' drew in perspective.

A point I have been marked on edge [DC].

**Draw on the figure the plane section of the parallelepiped by the plan which pass through points A, A' and I.**




**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Plane section** | **EMS 049** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P2 | |
| Main contents supposed to be covered | Space geometry | |
| Competencies supposed to be implied | C5 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 40% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1991 |
| | *Number of students* | 100 000 |
| | *Results* | 45% (28% en fin de troisième 1992) |

**EMS - Reference question N°050** **Comparisons**

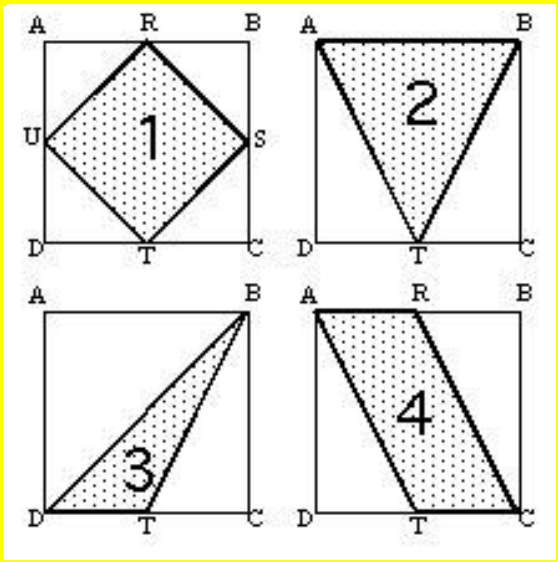
Here are 4 squares ABCD of edge a.

Points R, S, T and U are the middle of the edges.

Let us consider the three broken lines drawn in bold of which the respective lengths are noted l1, l2, l3, l4 while the areas of the coloured surfaces are marked S1, S2, S3, S4

Indexes 1, 2, 3, and 4, go with figures bearing the same numbers.

Is it true that : ***(Tick out any good answers)***




l1 < l2 < l3

l1 < l3 < l4

Two out of the four lengths are equals

Three out of the three areas are equals

**Question ESIEE- Paris**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Comparisons** | **EMS 050** |
| Origin of the question | ESIEE – adapted by EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Areas and length | |
| Competencies supposed to be implied | C1_C6 | |
| Complexity class | Class2 | |
| Target group | Population 3 | |
| Type of setting | Group work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Low |
| Expected present achievement rate at 16 | | Less than 5% if individual work |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 07% |

**EMS - Reference question N°051** **Oil tank**

An oil tank has a storage capacity of 2500 litres. It is shaped as a rectangle parallelepiped, 2m high and 1m wide.

**What is the height of this oil tank ?**

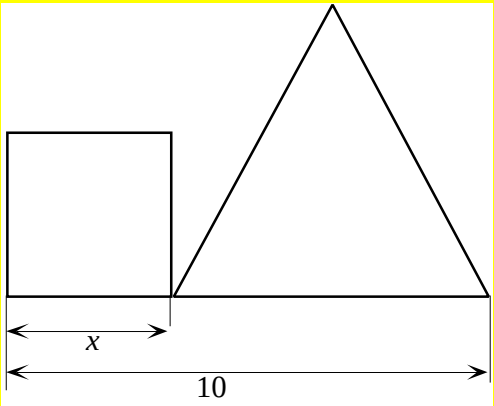
**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Oil tank** | **EMS 051** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P1_P2 | |
| Main contents supposed to be covered | Space geometry | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class1 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 40% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 49% (33% en fin de troisième 1992) |

**EMS - Reference question N°052** **Square and triangle**

Work out the x value for the square and the equilateral triangle had the same perimeter.




**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **C Square and triangle** | **EMS 052** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Equations | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class2 | |
| Target group | Population 1 | |
| Type of setting | Individual work | |

| **TRIAL COUNTRY** | | **FRANCE** |
|---|---|---|
| Fitness to curriculum | | Haute |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 65% |

**EMS - Reference question N°053** **A museum**

In its first year of public opening a museum was visited by 250 000 people.

Along the following years an increase of 8% of visitors a year has been observed.

a) Under these conditions, what was the number of visitors during the second year?

What was the total number of visitors during the two first years.

b) Under these conditions, what was the number of visitors during the 5th year?

What was the total number of visitors during the five first years ?

c) Under these conditions, what will be the number of visitors during the **n*th*** year?

What would be the total number of visitors during the **n** first years?

d) 2 000 000 entrance tickets have been printed out.

Under the announced conditions, would this number of tickets sufficient for the 10 first years?

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **A museum** | **EMS 053** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3_P1 | |
| Main contents supposed to be covered | Percentages - Equations | |
| Competencies supposed to be implied | C3_C1 | |
| Complexity class | Class2 | |
| Target group | Population 2 | |
| Type of setting | Group work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | De 60% (a) à 10% (d) |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | De 80% (a) à 15% (d) |

**EMS - Reference question N°054** **Parallelograms**

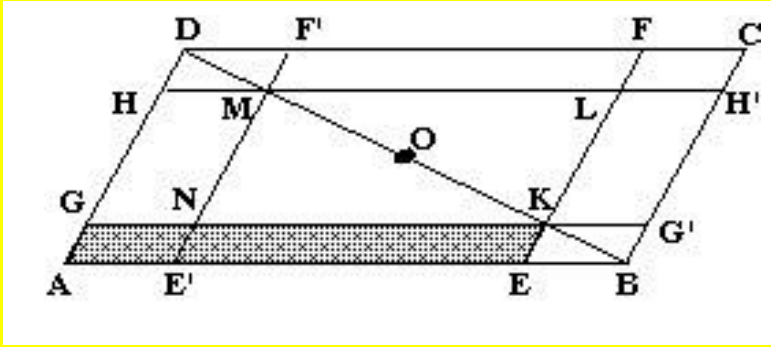
ABCD is a parallelogram of centre O.

K is a point variable on segment [BD].

Point M is symmetrical to point K about point O.

Quadrilaterals AEKG and MH'CF' are parallelograms (see figure).

The area of the parallelogram AEKG is noted S.




***The aim of this problem is to lead you to prove, by any mean, that the area of parallelogram AEKG is maximum for a particular position of point K.***

a) Prove that 7 other parallelograms in the figure have an area equal to S.

b) Is it true that : Area (ABCD) - area (KLMN) = 4S ?

b) How K should be chosen for S is maximum ?

**Question EVAPM/APMEP**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Parallelograms** | **EMS 054** |
| Origin of the question | EVAPM/APMEP – France | |
| Problematic field («Big idea») | P3_P2 | |
| Main contents supposed to be covered | Parallelogram - Area | |
| Competencies supposed to be implied | C1_C2 | |
| Complexity class | Class2 | |
| Target group | Population 2 | |
| Type of setting | Group work | |

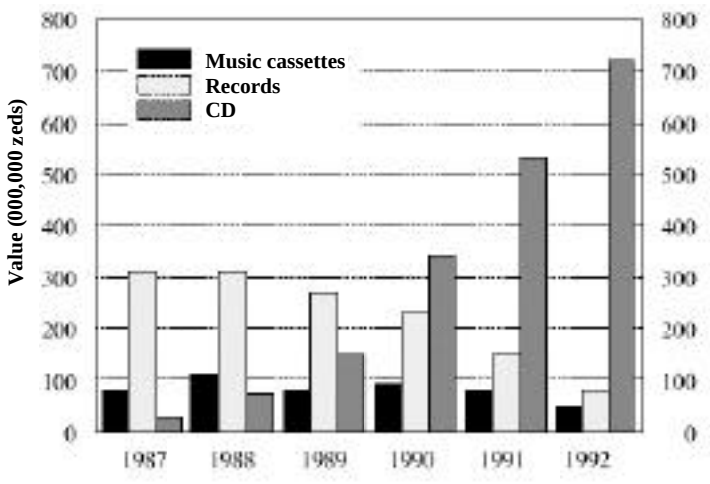
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | De 30% (a) à 10% (c) – If work individual |
| **Try out of the question** | *Context of the trial* | EVAPM fin de première 1993 |
| | *Number of students* | 100 000 |
| | *Results* | De 43% (a) à 19% (c) |

**EMS - Reference question N°060** **CDs sales**

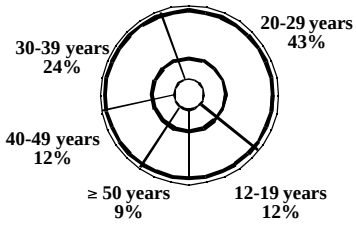
The graphs give information about sales of CDs and other sound recording media in Zedland.

Zeds are monetary units used in Zedland.

**Value of various sound recording media sold in Zedland (million of Zeds)**




**CD sales according to age in 1992**




With the aid of both graphs calculate how much money was spent by 12-19 year olds on CDs in 1992.

Show your work **TIMSS**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **CDs sales** | **EMS 060** |
| Origin of the question | TIMSS Pop 3 | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Statistics | |
| Competencies supposed to be implied | C5 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Good |
| Expected present achievement rate at 16 | | 60% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de terminale 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 57% |

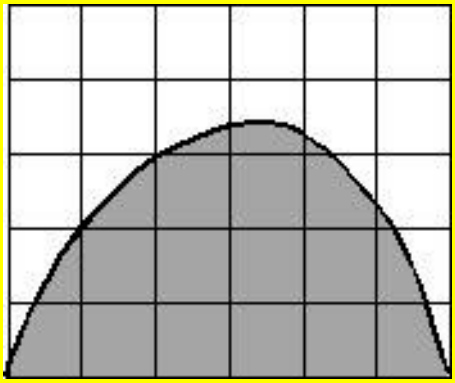
TIMSS international score : 44%

TIMSS (International Difficulty Index : 573 - 61%

**EMS - Reference question N°061** **Area**

Each of the small squares in the figure 1 is 1 square unit.

Which is the best estimate of the area of the shaded region ?



A. 10 square units ?

B. 12 square units ?

C. 14 square units ?

D. 16 square units ?

E. 18 square units ?

**TIMSS**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Area** | **EMS 061** |
| Origin of the question | TIMSS pop 3 | |
| Problematic field («Big idea») | P1 | |
| Main contents supposed to be covered | Area | |
| Competencies supposed to be implied | C3 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Low |
| Expected present achievement rate at 16 | | 70% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de terminale 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 77% |

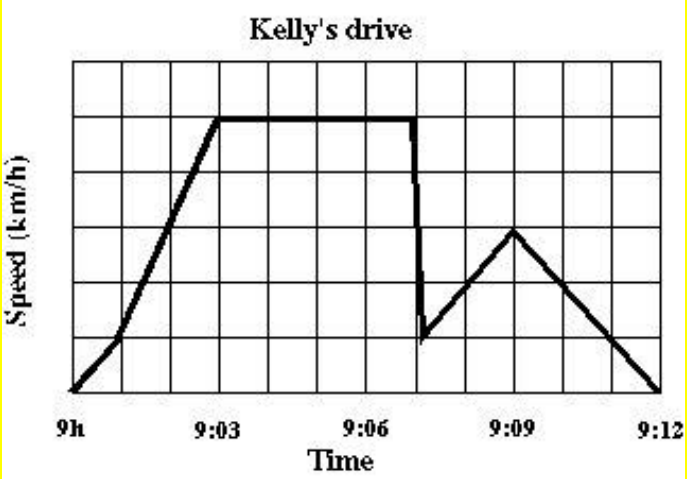
TIMSS international score : 61%

TIMSS (International Difficulty Index : 507

**EMS - Reference question N°062** **Brakes**

Kelly went for a drive in her car. During the drive, a cat ran in front of the car. Kelly slammed on the brakes and missed the cat.

Slightly shaken, Kelly decided to return home by a shorter route. The graph below is a record of the car's speed during the drive.




a) What was the maximum speed of the car during the drive ?

b) What time was it when Kelly slammed on the brakes to avoid the cat ?

**TIMSS**

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Brakes** | **EMS 062** |
| Origin of the question | TIMSS | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Functions | |
| Competencies supposed to be implied | C5 | |
| Complexity class | Class 2 | |
| Target group | Target 1 | |
| Type of setting | Individual work | |

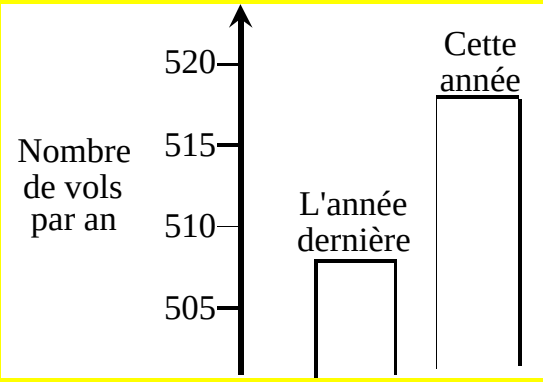
| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 90% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de terminale 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 90% |

TIMSS international score : 74%

TIMSS (International Difficulty Index : 435

**EMS - Reference question N°063** **Robberies**

A TV reporter showed this graph and said :




Do you consider the reporter's statement to be a reasonable interpretation of the graph ? Briefly explain.

**TIMSS**

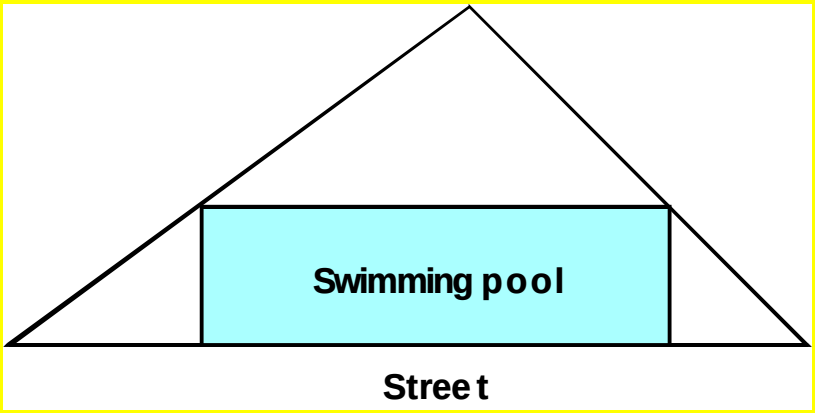
| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **EMS 00** |
| Origin of the question | TIMSS population 3 |
| Problematic field («Big idea») | P4 |
| Main contents supposed to be covered | Statistics |
| Competencies supposed to be implied | C5_C7 |
| Complexity class | Class 2 |
| Target group | Population 1 |
| Type of setting | Individual work |

| TRIAL COUNTRY | | FRANCE |
|---|---|---|
| Fitness to curriculum | | Mean |
| Expected present achievement rate at 16 | | 20% |
| **Try out of the question** | *Context of the trial* | EVAPM fin de terminale 1993 |
| | *Number of students* | 100 000 |
| | *Results* | 23% |

TIMSS international score : 19%
TIMSS (International Difficulty Index : 681

**EMS Reference question N°064** **Swimming pool**

On one triangular plot of land one wants to built a swimming pool in such a way one of its side opens directly on the street.




Where should the swimming pool be placed for its area is maximum ?

| EMS REFERENCE QUESTION IDENTITY CARD | | |
|---|---|---|
| **NAME and Number of the Question :** | **Swimming pool** | **EMS 064** |
| Origin of the question | Proposed by P. Richard (Spain) | |
| Problematic field («Big idea») | P3 | |
| Main contents supposed to be covered | Equations - Functions | |
| Competencies supposed to be implied | C4 | |
| Complexity class | Class 2 | |
| Target group | Population 2 | |
| Type of setting | Individual work | |

**EMS Reference question N°00** **EMS 00**

| EMS REFERENCE QUESTION IDENTITY CARD | |
|---|---|
| **NAME and Number of the Question :** | **EMS 00** |
| Origin of the question | |
| Problematic field («Big idea») | |
| Main contents supposed to be covered | |
| Competencies supposed to be implied | |
| Complexity class | |
| Target group | |
| Type of setting | |

*For further additions*